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CONSISTENCY IMPROVEMENT AND NORMALISATIONS IN MULTI-CRITERIA DECISION SUPPORT METHODS

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Abstract: Optimisation problems in mechanical engineering often involve several conflicting objectives and require multi-purpose optimisation (MOO) to solve. The solution of MOO problems is a set of equally good optimal solutions, which are called Pareto-optimal solutions. In order to choose one of the Pareto-optimal solutions, multi-criteria decision-making (MCDM) is required.

Keywords: *multi-criteria decision making, engineering design, VIKOR, TOPSIS, AHP*

1. IMPACT OF TOPSIS NORMALISATION PROCEDURES ON RANKING

TOPSIS is one of the compensatory methods (Technique for Order Preference by Similarity to Ideal Solution). Its main principle is that the optimal alternative is as close as possible to the ideal solution and is the furthest from the worst solution (Wang, Nabavi, & Rangaiah, 2023). The normalisation of the decision matrix is key in the method, as it compares different types of criteria, and the calculation of distances can only be done on a common scale. The values of the alternatives can be between 0 and 1, a value of 1 indicates a very good compromise, a value close to 0 indicates an alternative that is far from optimal. TOPSIS and VIKOR are based on different philosophies, as VIKOR is a compromise and conflict minimising solution, while TOPSIS ranks according to proximity to the ideal solution (Hwang & Yoon, 1981).

One of the key details of the TOPSIS method is the way the decision matrix is normalised. Normalisation can be done by vector normalisation or minimum-maximum scaling. To demonstrate the effect, benchmark data was applied, as it was used in the literature. The data of the 4 alternatives are presented in Table 1. The values of the distances measured from the best and worst points of the TOPSIS method are shown in Table 2 and 3. From these values, the relative proximity coefficient can be calculated, from which the ranking is derived. The results in Table 4 show the effect of the normalisation method on the distance from the best and worst points, and Table 5 illustrates the effect on rankings. The ranking of the best two alternatives did not change, but there was a reversal of rankings in the third and fourth places.

Table 1

TOPSIS matrix data

Alternative	C1 (Durability)	C2 (Capability)	C3 (Reliability)
A1	5	8	4
A2	7	6	8
A3	8	8	6
A4	7	4	6
Weights	0.4	0.4	0.3

Table 2

Distances from the ideal best point

Alternative	Distance
A1	0.4243
A2	0.2236
A3	0.1500
A4	0.4387

Table 3

Distances from the ideal worst point

Alternative	Distance
A1	0.4853
A2	0.6484
A3	0.7768
A4	0.3630

Table 4

Results of the TOPSIS ranking

Alternative	Vector normalization	Min-max normalization
A1	0.503720	0.485281
A2	0.658085	0.648371
A3	0.748249	0.776791
A4	0.333997	0.362977

Table 5

TOPSIS results obtained with different normalisations

Alternative	Vector normalization	Min-max normalization
A1	3	4
A2	2	2
A3	1	1
A4	4	3

When determining weight sensitivity, the results presented in Table 6 and 7 were obtained for vector normalised weights. The results obtained in the case of min-max normalisation are shown in Tables 8 and 9. From the alternatives examined, TOPSIS selected the alternative with the smallest total distance as the best. On the other hand, VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje), which is one of the rank-based procedures, would prioritise a compromise solution. The following calculations were also performed using the VIKOR method.

Table 6
Results of the weight sensitivity test of TOPSIS values for vector normalised weights

Alternative	Vector normalised weight	+10% weight	–10% weight
A1	0.503720	0.495744	0.511401
A2	0.658085	0.658304	0.657877
A3	0.748249	0.752236	0.744450
A4	0.333997	0.343414	0.324757

Table 7
Results of the TOPSIS rankings weight sensitivity test for vector normalised weights

Alternative	Original rank	Rank (+10% weight)	Rank (–10% weight)
A1	3	3	3
A2	2	2	2
A3	1	1	1
A4	4	4	4

Table 8
Results of the TOPSIS weight sensitivity test for min-max normalised weights

Alternative	Min-max normalised weight	Ranking (+10% weight)	Ranking (–10% weight)
A1	0.485281	0.472823	0.497755
A2	0.648371	0.649179	0.647573
A3	0.776791	0.782550	0.771122
A4	0.362977	0.376404	0.349251

Table 9
Results of the TOPSIS weight sensitivity test for min-max normalised weights

Alternative	Original ranking	Ranking (+10% weight)	Ranking (–10% weight)
A1	3	3	3
A2	2	2	2
A3	1	1	1
A4	4	4	4

In the first step, the VIKOR method performs scale standardisation, with which all criteria are placed on a dimension-independent scale. Then, in the second step, normalised data should be weighted. In the third step, differences are weighted, followed by the aggregation (S, R, Q). The results of the calculations are presented in Table 10.

Table 10

Results of the weight sensitivity test of the VIKOR rankings for min-max normalised weights

Alternative	**S_i**	**R_i**	**Q (0.3)**	**Q (0.5)**	**Q (0.7)**
A1	0.60	0.30	0.69	0.75	0.81
A2	0.30	0.20	0.23	0.25	0.27
A3	0.15	0.15	0.00	0.00	0.00
A4	0.65	0.40	1.00	1.00	1.00

In the method of AHP, which is one of the hierarchical methods, the main elements are the decision goal, the criteria, the sub-aspects and the alternatives. The first step in the decision-making process is the creation of the decision-making hierarchy, followed by second step with the creation of a pair comparison matrix of criteria and aspects and as a third step, the determination of the weight vectors using the eigenvector or normalisation method comes. In the fourth step, the consistency test is carried out, and the fifth step is the overall evaluation of the alternatives, where the ranking of the alternatives can be obtained by aggregating the calculated weights. In this method the decision-maker uses a scale of 1 to 9 for the pair comparison and these values are shown in Table 11.

Table 11

The Scale and Its Description

Intensity of importance	**Definition**
1	Equal importance
3	Weak importance of one over another
5	Essential or strong importance
7	Demonstrated importance
9	Absolute importance
2, 4, 6, 8	Intermediate values between the two adjacent judgements

Table 12

Mathematical definitions used in the AHP process

Definition	**Marking**	**Calculation mode**
Pairwise comparison matrix	A	$A = \begin{bmatrix} 1 & a_{1,2} & \cdots & \cdots & a_{1,j} \\ \frac{1}{a_{1,2}} & 1 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & a_{i-1,j-1} \\ \frac{1}{a_{1,j}} & \cdots & \cdots & \frac{1}{a_{i-1,j-1}} & 1 \end{bmatrix}$
each element expresses the importance of a criterion for any j criterion, where $a_{i,j} > 0$, $a_{i,j} = \frac{1}{a_{j.i}}$, $a_{i,i} = 1$		

Definition	Marking	Calculation mode
A normalised eigenvector of a matrix	w	$Aw = \lambda_{max}w$
where the highest eigenvalue of the pairwise comparison matrix is $\lambda_{max} > \lvert\lambda_i\rvert$, $w_i > 0$, i.e. the matrix has exactly one dominant eigenvalue, and all components of the eigenvector belonging to it are positive (Perron, 1907) and the pairwise comparative matrix (A) Right-hand eigenvector (w) (Perron, 1907).		
Consistency index	CI	$CI = \frac{\lambda_A - \text{n}}{n-1}$
where n is the number of criteria$n$, $\lambda_A = \lambda_{max}$ is the highest eigenvalue of the pairwise comparison matrix (A) its value is n in case of a consistent matrix, or in the case of an inconsistent matrix $\lambda_{max} > n$		
Random consistency index	RI	$CR = \frac{\text{CI}}{\text{RI}}$
where CR is consistency ratio, so the decision can be considered consistent if: $CR < 0.10$. The average CI value of random matrices.		
Weighted sum vector	S_i	$S_i = \sum_{j=1}^{n} a_{i,j}\, w_j$
Main intrinsic value	λ_{max}	$S_i = \frac{1}{n}\sum_{j=1}^{n}\frac{S_i}{w_i}$
where the n is the number of criteria, w_i the priority vector its i[th] weight.		

Suppose that if we have 3 criteria, and that the subcriterion group consists of 4 elements. In this case, the criterion matrix can be considered consistent if the weight vector w is the positive Perron eigenvector of the matrix and its main eigenvalue approximates the consistent case. In practice, the calculation of the weight vector is derived using the normalised line mean method, but the general procedure of AHP is the eigenvector method. The mathematical definitions of AHP are presented in Table 12. Table 13 presents the mathematical rules of the AHP pairwise comparison matrix.

Table 13

Practical interpretation of AHP consistency rules

Rule name	Description
Saaty intensity scale rule	Preference intensity on a scale of 1–9
Reciprocal rule	$a_{ij} = \frac{1}{a_{ji}}, a_{ii} = 1$
Transitivity	$a_{ij} \cdot a_{jk} = a_{ik}$

Rule name	Description
Unique weight vector (Positive Perron eigenvector) (Perron, 1907)	λ_{max}
Consistency ratio	$CR = \frac{\mathrm{CI}}{\mathrm{RI}}$

2. CONSISTENCY IMPROVEMENT METHODS

To accept or reject the decision-maker’s pair-by-pair comparison matrix, Saaty introduced the consistency ratio (CR), i.e. a comparative matrix is considered acceptable if the consistency ratio is less than 0.1 (Perron, 1907). Consistency guarantees that the decision can be reproduced and also improves the stability of the ranking, thus the reliability of the method. Therefore, we can find several methods in the literature to improve it. (Frish, Talmor, Hadar, Shoshany, & Shapira, 2025) using Saaty's eigenvalue definition in their article, they investigated the effect of modifying the Saaty scale on the value and how the eigenvalue changes to the maximum during iterations. As he did in his study (Saaty T., 1977) points out that the geometric mean vector is the closest consistent matrix. (Salomon & Gomes, 2024) use this thought as a basis and develop it further, so an iterative procedure was developed. In the next stage, a new step is added to the method, the expected transitive value is calculated, which is a consistent approximation of the matrix. They then adjust the highest inconsistencies until it reaches a value below 0.1. The presented literature showed that during the application of the AHP method, the authors developed new weighting procedures, which pushes the consistency of the decision matrix below the traditional limit of 0.1 by applying the RI index. These important methodological developments contribute to improving the reliability of multi-criteria analyses. However, during the review of the literature, it was found that there is no continuous, parameterised, controlled correction method in the literature that would approximate the decision-maker's judgment and rank consistency with a continuous scale.

3. NEW CONSISTENCY IMPROVEMENT METHOD: EULER RATIO CORRECTION

Since most of the methods presented are based on discrete modifications, they do not provide fine-tuned controllable transitions. Therefore, as a solution, I developed a new correction method based on Euler exponential interpolation.

The aim of the method was to produce a consistency improvement method that approaches the consistent ratio with continuous, minimal distortion, is controllable and guarantees the reduction of CR.

As a first step of the new method, the weight vector calculation is performed based on the geometric mean method, which is one of the general weight estimation procedures of the AHP. In the second step, Euler exponential interpolation is applied between the original elements and the consistent ratio.

The regulating α parameter determines the intensity of the repair. There are three cases:

- when $0 < \alpha < 10 < \alpha < 1$ then we apply real smoothing, i.e. the decision-maker's judgment is also maintained and we are also close to a consistent ratio, but with continuous correction, which can achieve a significant decrease in CR. The control parameter controls the intensity of the repair (suggested $\alpha = 0.5$).
- when $\alpha \to 1$, then we accept the judgments of the decision-maker, we do not modify or improve the consistency.
- when $\alpha \to 0$ the decision-making judgment is completely overridden, it is the overwriting of the entire decision-making judgments.

4. SUMMARY

The advantage of the method is that by reducing all deviations in a uniform proportion, the matrix becomes more consistent, step by step, gradually controlled and stable. The operation of the proposed new method is on the comparison matrix of 4 × 4 pairs in the source. The initial matrix is highly inconsistent (CR = 0.4372). With the Euler ratio correction method, the consistency ratio is increased after the first iteration. It has been reduced from 0.43 to 0.1126, an improvement that proves that the new method is suitable for reducing the consistency of the matrix. The results of the new method show that already in the second iteration, the result of the calculation is 0.026. So, the matrix satisfies CR < 0.1 condition.

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