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EFFECTS OF PROFILE SHIFTING OF A COSINE GEAR

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Abstract

Cosine gears are a special type of gear drive, in which the tooth profile of the theoretical generating rack is described by the cosine function. According to recent research results, the cosine gear drive has a lower slip coefficient and the contact and bending stresses supported by the teeth during the load are smaller than those occurring in involute gears. The main advantage of the cosine drive pair lies in the apparent higher rigidity of the gear tooth: this can reduce flank wear and increase gear life, especially in heavy load and high-speed applications. In contrast to involute profile gears, cosine gears do not show undercut root fillet, even with small teeth number.

This paper deals with cosine gear tooth profile design and the meshing of the cylindrical cosine gear pair conceived without, or with the application of the profile shifting. The mathematical model of the meshing was implemented in a Mathcad environment, showing that there exists a significant difference compared to the well-known involute gears. The simulation of the motion during the rolling of the gear pairs was realized with the Autolisp programming interface. Solid models were created and tested in the Autodesk Inventor environment.

Keywords*: cosine gear, involute gear, meshing, generating rack, profile shifting, simulation.*

1. Introduction

Machine manufacturers and designers know well that gear pairs are those machine elements from all existing types which can transmit the largest torque reported to the mass unit. Considering the tooth profile, gear transmissions can be classified as involute, cage, cycloid or circular. The most employed tooth profile is the involute which has a lot of advantages, chief among which are the invariance of the transmission ratio with the axis distance and the simplicity of manufacturing. However, it presents certain drawbacks such as limited load capacity and interference susceptibility.

In the case of involute gears, a gear-pair can be designed as elementary or Willis transmission, without profile shifting, with compensation or with generalized profile shifting **[1]**.

In the case of elementary transmission, the pitch line of the generating rack rolls without slipping on the pitch circles of the gear pair, thus pitch circles are the rolling circles too.

The most important drawbacks of the elementary gear transmission are the ($Z=17$), interference teeth number limit. the rigidity of the tooth root fillet and the bad flank-sliding conditions.

If the generating rack is pulled out with a given distance when generating the pinion and pushed inside with the same distance when meshing the gear, a compensated transmission results. Due to the displacement, a profile shifting occurs which results in a positive $+m\xi$ distance between the pitch line and the pitch circle's tangent in the case of the pinion, and in a negative $-m\xi$ distance in the case of the gear. By such a compensated gear pair the tooth thickness on the pinion increases while it decreases on the gear. In conclusion, load capacity increases on the pinion and decreases on the gear.

The most important advantage of profile shifting is avoiding the undercut. Additionally, the relative slipping of the profiles improves considerable. A compensated gear pair's axis distance equals the axis distance of a Willis type gear pair. If the pinion, due to small teeth number, falls in the undercut domain, positive profile shifting leads to avoiding this, and a tooth number smaller than 17 could be applicable.

Nowadays very precise CNC machines makes it possible to achieve gears with special profiles which exceeds classical gear mechanisms in load capacity and the quality of transmission.

This research proposes the development of a cosine profiled compensated gear pair which will compensate for the deficiencies of the involute gears regarding the minimal tooth number $Z = 17$, the tendency for interference, and which will present an increased load capacity.

2. The cosine generating rack

2.1. The geometry of the classical rack

Literature dealing with cosine gears [2] defines them by simple bending the abscissa of the cosine function together with the cosine line over the pitch circles with corresponding diameters, but not rolling without slip. The errorless contacting, the improved tooth shape and the increased load capacity was proven using CAD-modeling. In our opinion this CAD-environment application still does not deal with the reality of a manufacturing environment. Starting from these results arose the idea of manufacturing using a rack-type tool, by applying Olivier's 1st principle [3]. The profile of the generating rack is derived from the basic involute rack geometrical data (**Fig. 1.**).

In order to allow the possibility of generalization, the basic profile was defined for the unity module $m = 1$ mm. Here the standard profile angle is $\alpha = 20^\circ$, the prescribed root fillet $\rho_{of} = 0.38 \cdot m$ [4, 5].

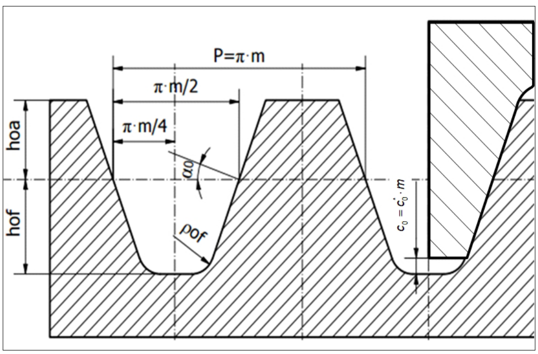

Fig. 1. *Standard rack basic profile.* [2]

From these, the module and the tooth addendum and dedendum height will be kept.

2.2. Cosine manufacturing rack with modified equation

In a previous publication [6] we have proven that a cosine profiled rack fulfills the pattern-counter-pattern principle [3] thus it allows the manufacturing of both wheels of the gear pair with the same cutting tool. Due to the non-zero curvature of the cosine profile, a free profile shifted gear pair [7]] is not achievable. The only possibility is to apply a compensated shifting thus $\Sigma\xi = 0$. If starting from the equation of the cosine gear [6],

$$y_{S1}(x_{S1}) = a \cos\left(\frac{2}{m} x_{S1}\right) \qquad (1),$$

and applying to it the profile shifting corresponding to the condition $\Sigma\xi = 0$ then it leads to the achievement of a compensated gear pair. From here it results that equation (1) has to be modified:

$$\begin{aligned} &y_S(x) = a\cos(bx) + \xi_s m, \text{ ahol} \\ &a = a_s = b_s = (h_0^* + c_0^*\, m) \\ &h_0^* = 1, c_0^* = 0{,}25, \qquad \xi_s \neq 0 \end{aligned} \qquad (2)$$

The profiles achieved from (2) by adding a positive +ξm or a negative -ξm shifting were drawn in a Mathcad environment, see **Fig. 2.** and **3.**

Due to the fact that the rack profile equation was modified, the generation of the cosine gear pair will be realized by using the mathematical model developed in [6] thus gearing equations will not be affected.

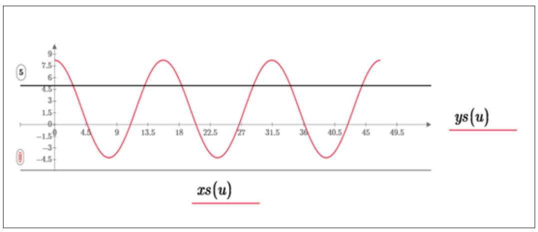

Fig. 2. *+ξm shifted rack profile.*

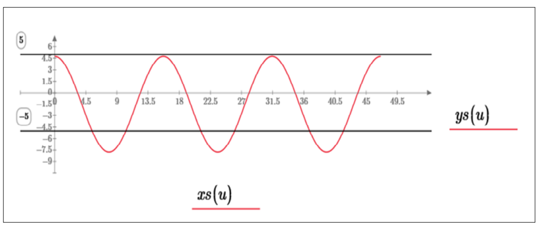

Fig. 3. *–Sξm shifted rack profile.*

2.3. Cosine gear pair meshing with shifted racks

In the following, a compensated gear pair is generated by rack shifting. This consists of pulling out the rack in the case of the pinion by a $+m\xi$ distance while in the case of the wheel, pushing it in the blank with $-m\xi$ distance. In this way the compensated gear pair results. The used frames and their relative positions during the meshing are presented in **Fig. 4.**

It should be noted regarding **Fig. 4.** that frames linked to the pinion and the wheel were taken in opposite orientation. This considerably simplifies the calculus because no negative expressions will result. As can be seen, the motion of the rack defined by the distance O_0O_{ps} can be expressed in the case of both gears by the product of the rotation angle and the pitch radius.

The necessary transformation equations are as follows:

$$\mathbf{M}_{1ps} = \mathbf{M}_{1p}\,\mathbf{M}_{Ops} \tag{3}$$

$$\mathbf{M}_{2ps} = \mathbf{M}_{2p}\,\mathbf{M}_{Ops} \tag{4}$$

The matrix transformation's detailed calculus, in the case of profile shifting, are based on the calculus presented in **[6]**, **Fig. 3**:

$$\begin{aligned} \underline{\mathbf{r}}_1 &= \mathbf{M}_{1O}\,\mathbf{M}_{Os1}\,\underline{\mathbf{r}}_{s1} \\ \mathbf{M}_{1s1} &= \mathbf{M}_{1O}\,\mathbf{M}_{Os1} \\ \underline{\mathbf{r}}_2 &= \mathbf{M}_{2O}\,\mathbf{M}_{Os2}\underline{\mathbf{r}}_{s2} \\ \mathbf{M}_{2s2} &= \mathbf{M}_{2O}\,\mathbf{M}_{Os2} \end{aligned} \tag{5}$$

Transformation matrices applied here differ from the corresponding matrices used in the case of zero profile shifting by the translation term in the fourth column. In this paper, these forms are not published.

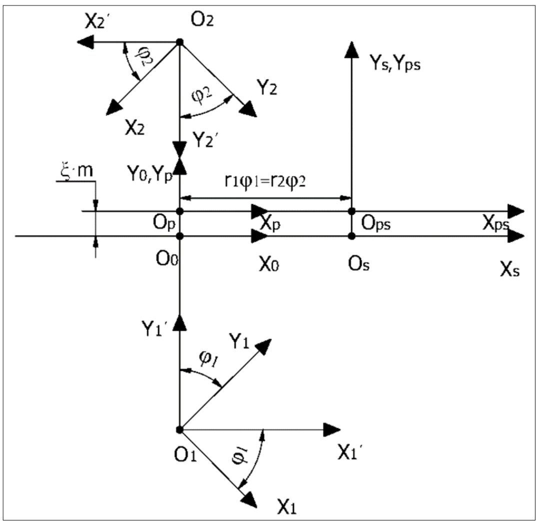

Fig. 4. *Meshing with a single rack.*

The angular velocity vectors and angular velocity origin position vectors used in the gearing equations, in the case of meshing a compensated gear pair, are as follows:

$$\underline{\boldsymbol{\omega}}_{01s}^{(1)} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix};\ \underline{\boldsymbol{A}}_{1s} = \begin{bmatrix} -(r_1 + \xi_s\,m)\,\varphi_1 \\ -(r_1 + \xi_s\,m) \\ 0 \end{bmatrix} \tag{6}$$

$$\underline{\boldsymbol{\omega}}_{02s}^{(2)} = \begin{bmatrix} 0 \\ 0 \\ i_{21} \end{bmatrix};\underline{\boldsymbol{A}}_{2s} = \begin{bmatrix} -(r_2 + \xi_s\,m)\varphi_2 \\ r_2 + \xi_s\,m \\ 0 \end{bmatrix} \tag{7}$$

The normal vector form is the same as results that used in **[6]**:

$$\underline{\boldsymbol{n}}(u) = \begin{bmatrix} \dfrac{2a}{m}\sin\left(\dfrac{2}{m}u\right) \\ 1 \\ 0 \end{bmatrix} \tag{8}$$

Completing the calculus results in the columns of the relative velocity vectors:

$$\underline{\boldsymbol{v}}_{ps}^{(ps,1)} = \begin{bmatrix} -(y_S + \xi_s\,m) \\ x_S + r_1\varphi_1 \\ 0 \end{bmatrix} \tag{9}$$

$$\underline{\boldsymbol{v}}_{ps}^{(ps,2)} = \begin{bmatrix} i_{21}(y_S + \xi_s\,m) \\ -i_{21}(x_S + r_2\varphi_2 + \xi_s\,m) \\ 0 \end{bmatrix} \tag{10}$$

After the computation the explicit forms of the gearing equations are as follows:

$$\begin{aligned} \varphi_1(u) = \frac{1}{mz_1}\Bigg(u - \left(\frac{a^2}{m}\sin\left(\frac{4u}{m}\right) + \right. \\ \left. + \xi_s m\frac{2a}{m}\sin\left(\frac{2u}{m}\right)\right)\Bigg) \end{aligned} \tag{11}$$

$$\begin{aligned} \varphi_2(u) = \frac{1}{mz_2}\Bigg(\frac{2a^2}{m}\sin\frac{4u}{m} + 4a\xi_s\sin\left(\frac{2u}{m}\right) - \\ -m\xi_s - u\Bigg) \end{aligned} \tag{12}$$

2.4. Gearing equations resulting from profile shifted racks

Gearing equations are written here with the notations and values represented in **Fig. 5. [7]**

$$\begin{aligned} \mathbf{v}_{s1}^{(1,s1)} &= \mathbf{v}_{s1}^{(1)} - \mathbf{v}_{s1}^{(s1)} = \\ &= \boldsymbol{\omega}_{01}^{(1)} \times \mathbf{r}_{s1} + \mathbf{A}_1 \times \boldsymbol{\omega}_{01}^{(1)} - \omega_1 r_1\,\mathbf{i}_{s1} \\ \mathbf{v}_{s2}^{(2,s2)} &= \mathbf{v}_{s2}^{(2)} - \mathbf{v}_{s2}^{(s2)} = \\ &= \boldsymbol{\omega}_{02}^{(2)} \times \mathbf{r}_{s2} + \mathbf{A}_2 \times \boldsymbol{\omega}_{02}^{(2)} - \omega_2 r_2\,\mathbf{i}_{s2} \end{aligned} \tag{13}$$

3. Solid models and manufacturing simulation of a compensated cosine gear pair

In order to achieve the best possible accuracy, it would be desirable to start with a point manifold of 2000 elements, but the upper limit of Autodesk Inventor is 500. Thus only 500 points were imported and then, these were linked by cubic splines. The achieved rack profile was turned into

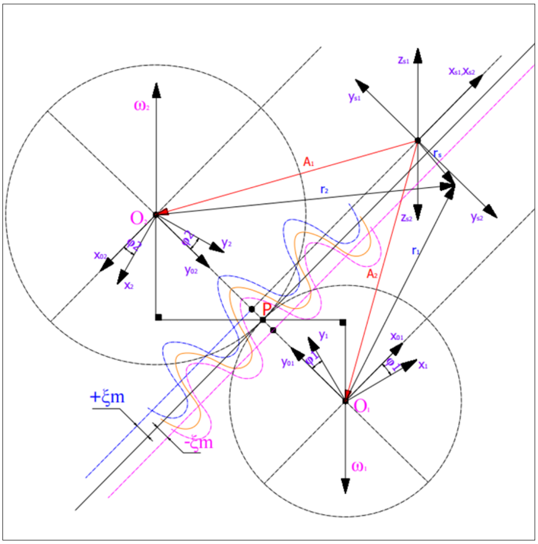

Fig. 5. *Negative and positive profile shifting in the case of a cosine gear pair.*

a generated rack. With this, in AutoLisp programming environment, three cosine and three involute gears were generated, with different teeth numbers. Knowing that the minimal realizable tooth number by an involute gear is $Z=10$, the comparison of involute and cosine profiles was set for this value. Manufacturing simulation was performed with a 0.2 mm feed of the rack.

In order to compare involute and compensated cosine gear pairs, the profile shifting values were selected from the recommendations found in the literature [9–12].

The applied profile shifting values are shown in **Table 1.**

Table 1. *Applied profile shifting values*

Z	$-\xi_{min}$	$-\xi_{max}$	$+\xi_{min}$	$+\xi_{max}$
10	×	×	×	×
19	-0.4	-0.2	0.2	0.4
27	-0.4	-0.2	0.2	0.4

All gears were modelled for the profile shifting values presented in **Table 1.** but **Fig. 6.**, 7., 8., and 9. show only the cases $\xi=0.4$, respectively $\xi=-0.4$.

As can be seen in **Fig. 6**, and 7, profile shifting has a significant influence on the shape of the tooth. In the case of the $Z=19$ tooth gear shown in **Fig. 6**, due to the negative profile shifting $\xi=-0{,}4$ 4 the resulting teeth are slimmer, having a decreased tooth addendum thickness. In this case, even a visual inspection let admit that tooth root

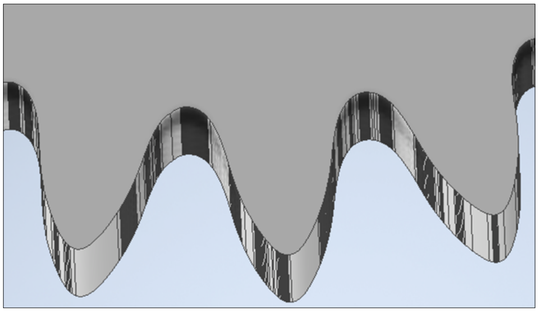

Fig. 6. *$Z=19$, $\xi=-0.4$ Cosine gear.*

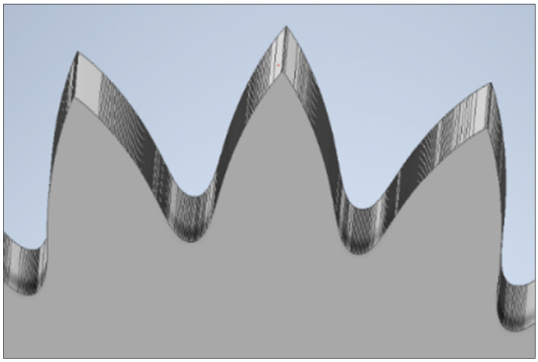

Fig. 7. *$Z=27$, $\xi=+0.4$ Cosine gear.*

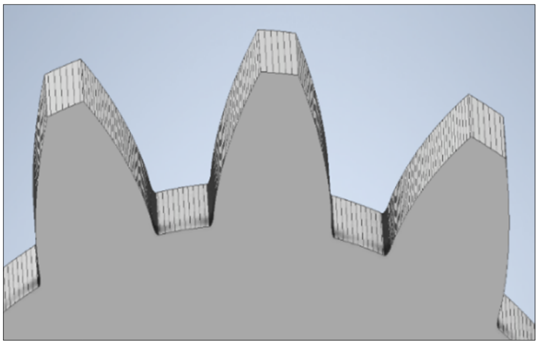

Fig. 8. *$Z=19$, $\xi=+0.4$ Involute gear.*

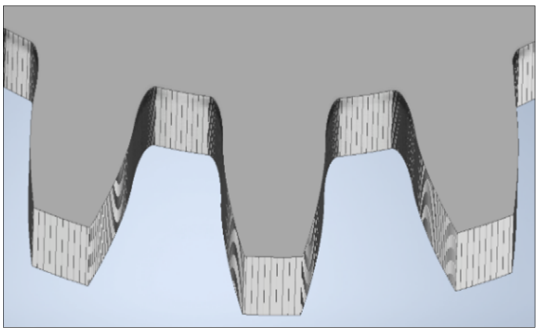

Fig. 9. *$Z=27$ $\xi=-0.4$ Involute gear.*

fillet has a robust aspect, thus tooth breaking is unlikely.

Upon inspecting the gear of $Z=27$ teeth shown in Fig. 7 it can be concluded that the resulting teeth are more robust. However, the resulting tooth tip is sharp and this leads to a decrease in the life of the transmission.

Fig. 8. and 9 shown two profile shifted involute gears, forming a compensated gear pair. As can be seen, the addendum width decreases in the case of positive and it increases in the case of negative profile shifting. Negative profile shifted tooth dedendum width decreases and this leads to decrease in load capacity. It can be seen that none of involute gears reach the robustness of cosine gears.

It can be concluded that cosine gears, whether elementary or compensated, are more convenient from the point of view of resistance and rigidity.

4. The influence of the profile shifting on the shape of a cosine tooth

Fig. 10 shows three profiles of a $Z=19$ teeth involute gear, for the cases of zero, positive and negative profile shifting.

It can be concluded that profile shifting has an important influence on the shape of the profile, the shape of the root fillet, the addendum and dedendum diameter values and the life of the gears.

5. Avoiding of undercut

As it was mentioned before, the most widespread gear type is the involute whose most significant drawback consists in the limited tooth number. Elementary involute gears are not realizable under Z=17 because the cutting tool tip excavates the root fillet undercutting it. This is clearly visible on Fig. 11. Here the undercut is so extended that the root fillet arc length is almost greater than the involute. Contrary to this, the cosine gear shown in Fig. 12. presented no undercut aspect. So, it can be stated that cosine gears tooth number can reach smaller values as involute teeth. This is the goal of one of our further studies.

6. Conclusions

Profile shifting has a significant influence on the tooth form, the load capacity and torque transmission.

During the manufacturing simulation of cosine gears it was observed that root addendum robustness increases with the teeth number, thus larger number of teeth can be more advantageous.

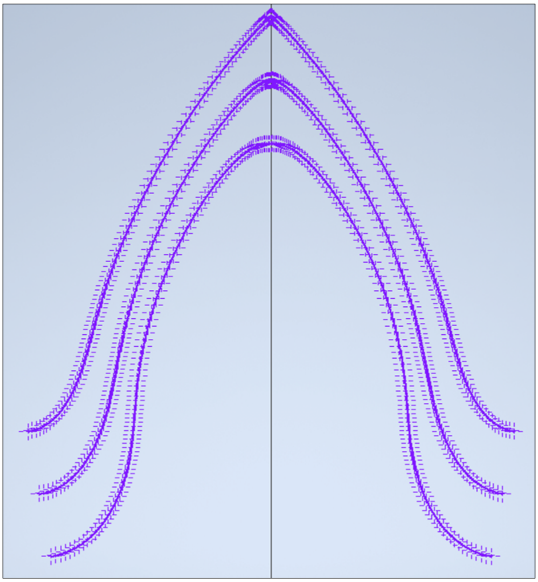

Fig. 10. *Profile shifted involute gear profiles.*

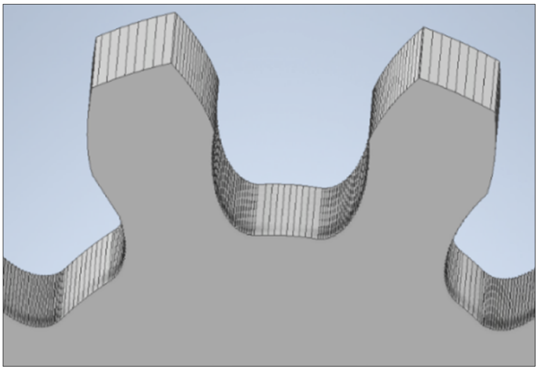

Fig. 11. *$Z=10$; Elementary involute gear.*

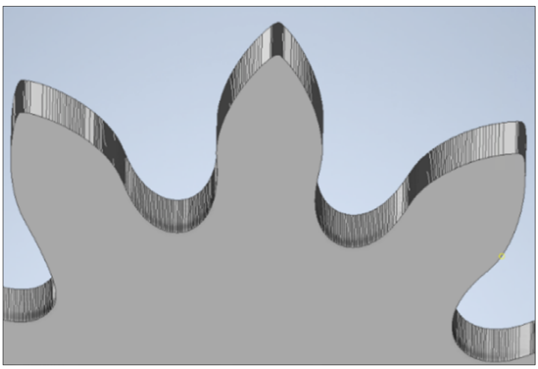

Fig. 12. *$Z=10$ fog; Cosine gear.*

Cosine gear root fillet shape is more advantageous than involute gears root fillet.

With smaller tooth number, the resulting cosine gear's tooth tip is sharpened.

Cosine profile allows elementary gears with tooth numbers smaller than $Z = 17$.

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