Műszaki Tudományos Közlemények vol. 19. (2023) 37–43.
DOI: Hungarian: https://doi.org/10.33895/mtk-2023.19.06
English: https://doi.org/10.33894/mtk-2023.19.06

SYNTHESIS AND CAD-EXAMINATION OF A CYLINDRICAL GEAR PAIR DERIVED FROM A COSINE PROFILED RACK

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Abstract

The problematic of gear profiles has again come to the fore during the power demands imposed by technical development. Most of nowadays gears have involute profile due to its numerous ad-vantages. The most important is that that axis distance variation does not affect the transmission ratio. According to **[1]**, a cosine gear drive has a lower slip coefficient and the contact and bending stresses by charge are significantly lower compared to an involute gear. This paper presents a study regarding the generation synthesis of a cosine profiled gear pair, using a single cosine profiled rack, in a CAD environment. The profile of the generating rack and the mathematical computations of the equations of gearing were performed in a Mathcad environment. The tool models were created in Autodesk Inventor. Then it was implemented in Autodesk AutoCAD and using its special programming language (Auto lisp), a simultaneous subtraction-rolling method was used to obtain a realistic simulation of the meshing of the cut gear with the rack. Implementing the generated gears in Autodesk Inventor environment, tests were performed in order to compare the cosine profile of the generated gears with the profile of the involute gears. In the near future, we intend to exploit the results obtained and compare the transmission quality of the involute gears with the cosine gears.

Keywords*: cosine profile, involute, CAD, Mathcad, Autolisp, cosine profiled rack, gear.*

1. Introduction

In terms of tooth profile, gears can be classified as involute, bolt, cycloid or circle arc shaped. The most widely used profile is the involute profile due to its numerous advantages among which the following must be emphasized: the transmission ratio is not affected by the variation of the axis distance, and the manufacturing technology is simple, robust and at reasonable costs. Certainly, the involute gear has also drawbacks such as the limited load capacity and the susceptibility to interference and undercut. Thanks to the appearance of numerical controlled machine tools, the area of special tooth profiles has been vigorously extended.

The use of cycloid profiled gears in today's machinery is significantly uncommon – the area of implementation being confined to the mechanical clock industry and some special gear pump constructions. It must be mentioned here that cycloidal gears have the advantage of zero relative slipping between the tooth flanks which results in neglectable mechanical wear, lubrication is not a prerequisite in any circumstances of exploitation, and the contact ratio is certainly greater than for the involute, bolt or circular arc teethed gears.

Bolt gears are the oldest type of gears. This type of gear has made a major comeback in clock mechanisms, cycloidal drives and lifting equipment. Cyclo-drives consist of a wheel having the teeth as circular section equiangular disposed rods and a cycloid profiled counter gear.

There exists also the possibility of designing the spur gears with circular arc profiled teeth. Their field of application comprises concrete mills, compressors, elevation systems and others. In the US machinery the endowment of plastic gears with circular arc profiled teeth is considered the most appropriate. These present certain advantages due to their rigidity, resulting in high load transmission capacity and silent functioning by a corresponding lubrication. The efficiency of the transmission is superior to non-grinded involute gear transmissions [2].

2. The cosine-rack generated gear pair

2.1. The elements of the involute rack

For easier further application, the basic profile is defined for the module $m=1$ mm. The standard profile angle is set at $\alpha=20^\circ$. To increase the rigidity of the dedendum zone, the root fillet is set to $\rho_{of}=0.38\cdot m$ (**Figure 1**). The definition of the cosine rack is based on the values of the standard involute rack, excepting the profile angle and the linear flank form [2].

2.2. The design of the cosine generating rack

The equations of the cosine rack profiles are written to the frame $O_{Si}\,X_{Si}\,Y_{Si}$, $i\in\{1, 2\}$.

Respecting Litvin's criterion, a gear pair's generating racks must match like a template and counter-template. Thus, the blue rack generates part 1, while the green rack part 2 of the gear pair. (**Figure 2**). Their common profile is the ABCDEFG cosine curve, shown in red. The equation of the profile is sought in the form:

$$y_{S1}(x)=a\cos(bx), \tag{1}$$

where

$a=a_s=b_s=(ho^*+co^*)m$,

and $h_0^*=1$, $c_0^*=0.25$.

The implementation of the cosine curve needs the pitch of the rack ($p=\pi\, m$) which must be synchronized with the period of the function. Using the periodicity of the cosine function it can be written, that:

$$y_{S1}(x)=y_{S1}(x+p) \Rightarrow a\,\cos(bx)=a\cos b(x+p) \tag{2}$$

After transforming the sum above in a product, the result is:

$$\sin\left(bx+\frac{pb}{2}\right)\sin\left(\frac{bp}{2}\right)=0 \tag{3}$$

Equation (3) must be true for any x, thus:

$$\sin\left(\frac{bp}{2}\right)=0 \tag{4}$$

The solution of the trigonometric equation in canonic form (4) is:

$$\frac{pb}{2}=(-1)^k\arcsin(0)+k\pi, k\epsilon\mathbb{Z} \tag{5}$$

So we get

$$b=\frac{2k\pi}{p}=\frac{2k\pi}{\pi m}=\frac{2k}{m} \tag{6}$$

The next possible value results in for k = 1, thus:

$$b=\frac{2}{m} \tag{7}$$

Although the profile angle of the cosine rack is variable, its nominal value is defined on the pitch line. This choice allows the comparison with a standard involute rack of, $\alpha_0=20°$. Based on **Figure 1**, the cosine rack profile angle is defined by the pitch line-point tangent of the cosine curve and the perpendicular to the x axis.

Exploiting the geometric sense of the derivative, one gets

$$\mathrm{tg}\varphi=\mathrm{ctg}\,\alpha_c=\frac{\mathrm{d}\,y}{\mathrm{d}\,x}\Big|_{\left(x=-\frac{\pi m}{4}\right)}=-ba\sin\,bx \tag{8}$$

After performing the computation we obtain:

$$\alpha_c=\frac{\pi}{2}+\mathrm{arctg}(-2.5)\approx 21^\circ\,48'$$

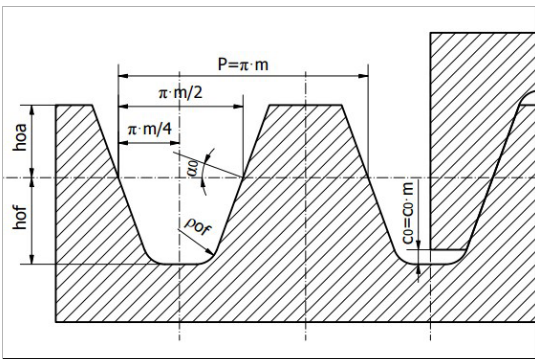

Figure 1. *Standard basic profile.*

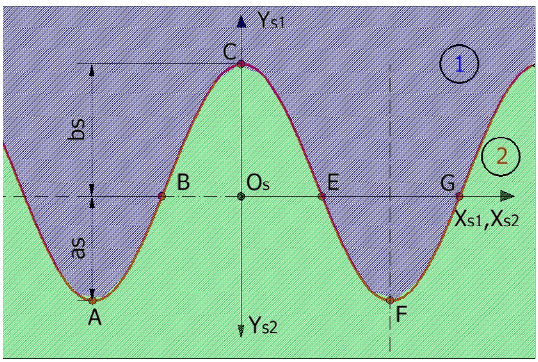

Figure 2. *The common profile of the rack pair.*

The symmetry axis of the tooth gap of the element 1 generating rack is axis y_S1 as can be seen in **Figure 2**. Thus, the explicit equation of the rack 1 results in:

$$y_{S1}(x_{S1}) = a\cos\left(\frac{2}{m}x_{S1}\right) \tag{9}$$

The axis y_{S2} is the symmetry axis of the tooth of the element 2 generating rack. Let us observe the template-counter template principle in the build-up of the two racks. Thus, the equation of rack 2 results in:

$$y_{S2}(x_{S2}) = -a\cos\left(\frac{2}{m}x_{S2}\right) \tag{10}$$

If admitting the translation of a half pitch, to overlap the symmetry axis of the tooth gap with the axis y_{S2} it must accept

$$x_{S2} = x_{S2}{}^{*} + \frac{\pi m}{2}.$$

As following $y_{S2}{}^{*}(x_{S2}{}^{*}) = a\cos\left(\frac{2}{m}x_{S2}{}^{*}\right)$, thus one gets the equation of the profile of rack 1. By this calculus it was proven that the principle of the pattern- counter pattern persists and the two racks not only present the same profile, but they are congruent, exactly as in the case of involute racks.

2.3. The synthesis of a cosine gear pair

The generating principle of the cosine gear pair by conjugate generating racks is presented in **figure 3**. On the left are presented the used frames in their initial positions while on the right, in a position that corresponds to an arbitrary moment of the generation process. The indices of the axes have the following significance: zero is the index of the fixed frame, 1 and 2 the indices of the generated wheels while indices s1 and s2 are the indices of the corresponding rack.

The transformation of coordinates between the involved frames is given by the following matrix equations:

$$\begin{aligned}
\mathbf{r}_1 &= \mathbf{M}_{10}\,\mathbf{M}_{0s1}\,\mathbf{r}_{s1}\\
\mathbf{r}_2 &= \mathbf{M}_{20}\,\mathbf{M}_{0s2}\,\mathbf{r}_{s2}\\
\mathbf{M}_{1s1} &= \mathbf{M}_{10}\,\mathbf{M}_{0s1}\\
\mathbf{M}_{2s2} &= \mathbf{M}_{20}\,\mathbf{M}_{0s2}
\end{aligned} \tag{11}$$

In this paper the detailed form of the matrices is omitted; although, if performing the computing, the structural identity of $\mathbf{M}_{10}$ and $\mathbf{M}_{20}$ is obvious.

2.4. The equations of gearing

The equations of gearing are written for wheels 1 and 2 which are contacting their racks. The relative velocities result from the position vectors and angular velocities shown in **figure 4**. **[7]**

The expressions of the relative velocities become:

$$\begin{aligned}
&\mathbf{v}_{s1}^{(1,s1)} = \mathbf{v}_{s1}^{(1)} - \mathbf{v}_{s1}^{(s1)} =\\
&\boldsymbol{\omega}_{01}^{(1)} \times \mathbf{r}_{s1} + \mathbf{A}_1 \times \boldsymbol{\omega}_{01}^{(1)} - \omega_1 r_1\,\mathbf{i}_{s1}\\
&\mathbf{v}_{s2}^{(2,s2)} = \mathbf{v}_{s2}^{(2)} - \mathbf{v}_{s2}^{(s2)} =\\
&\boldsymbol{\omega}_{02}^{(2)} \times \mathbf{r}_{s2} + \mathbf{A}_2 \times \boldsymbol{\omega}_{02}^{(2)} - \omega_2 r_2\,\mathbf{i}_{s2}
\end{aligned} \tag{12}$$

2.5. The generation of conjugate profiles with a common rack

Due to the fact that the validity of the template-counter template principle was demonstrated in subsection 2.2, a cosine gear pair can be machined with a unique rack, admitting $\xi = 0$ profile shifting. Thus, the equation of the rack coincides with those given in subsection 2.2.

The matrix transformation equations used for the meshing are as follows:

$$\begin{aligned}
\mathbf{M}_{1s} &= \mathbf{M}_{10}\,\mathbf{M}_{0s}\\
\mathbf{M}_{1s} &= \mathbf{M}_{10}\,\mathbf{M}_{0s}
\end{aligned} \tag{13}$$

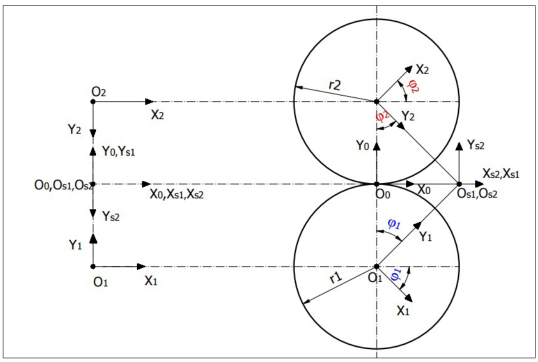

Figure 3. *Generating geometry of a gear pair.*

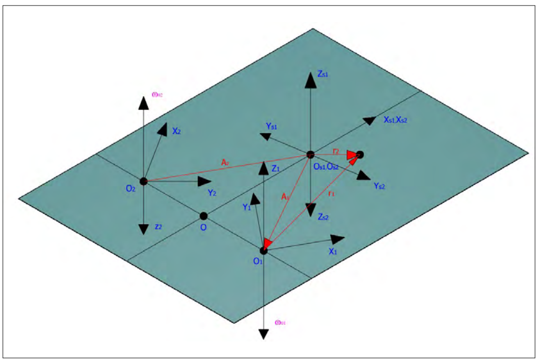

Figure 4. *The scheme of coupling involving two different racks*

Angular velocity vectors and their moments related to the origin O_s are as follows:

$$\boldsymbol{\omega}_{O1s}^{(1)}=\begin{bmatrix}0\\0\\-1\end{bmatrix};\ \mathbf{A}_{1s}=\begin{bmatrix}-r_1\varphi_1\\-r_1\\0\end{bmatrix};$$
$$\boldsymbol{\omega}_{O2s}^{(2)}=\begin{bmatrix}0\\0\\i_{21}\end{bmatrix};\ \mathbf{A}_{2s}=\begin{bmatrix}-r_2\varphi_2\\r_2\\0\end{bmatrix}; \tag{14}$$

If substituting column vectors (14) in velocity expressions (12) one gets the relative velocity column vectors related to the frame of the rack:

$$\mathbf{v}_s^{(s1)}=\begin{bmatrix}-y_S\\x_S-r_1\varphi_1\\0\end{bmatrix}$$
$$\mathbf{v}_s^{(s2)}=\begin{bmatrix}y_S\\x_S+r_2\varphi_2 i_{21}\\0\end{bmatrix} \tag{15}$$

In this model it is to remark that both profiles are simultaneously in contact which means that if M is the contact point, then $\mathbf{n}_1=\mathbf{n}_2$, thus, the equations of gearing become:

$$\mathbf{v}_s^{(s1)}\cdot\mathbf{n}^{(1)}=0$$
$$\mathbf{v}_s^{(s2)}\cdot\mathbf{n}^{(2)}=0 \tag{16}$$

The cosine profile's normal vector is written using the parametric form (9) (**Figure 5**):

$$\mathbf{n}(u)=\begin{bmatrix}\frac{2a}{m}\sin\left(\frac{2}{m}u\right)\\1\\0\end{bmatrix} \tag{17}$$

To get the first gearing equation expression (15) and (17) are replaced in equation (16):

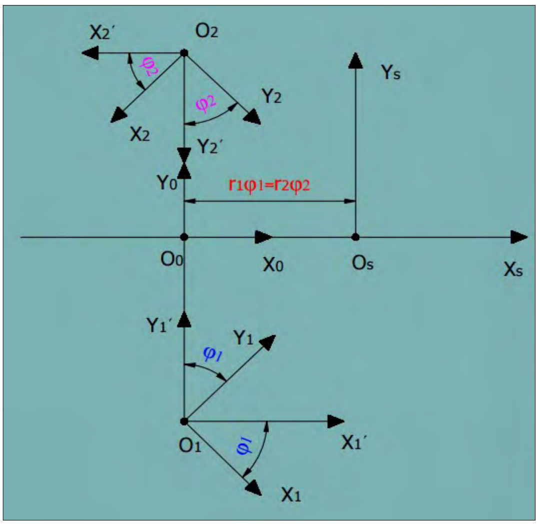

Figure 5. *Machining with a unique rack.*

$$-y_S\frac{2a}{m}\sin\left(\frac{2}{m}u\right)+(x_S-r_1\varphi_1)\cdot 1=0 \tag{18}$$

This leads to the following solution:

$$\varphi_1(u)=\frac{2}{mz_1}\left(x_S-y_S\frac{2a}{m}\sin\left(\frac{2}{m}u\right)\right) \tag{19}$$

Similarly, the second gearing equation is:

$$y_S\frac{2a}{m}\sin\left(\frac{2}{m}u\right)-(x_S-r_2\varphi_2)\cdot 1=0, \tag{20}$$

whose solution is presented in the following expression:

$$y_S\frac{2a}{m}\sin\left(\frac{2}{m}u\right)-(x_S-r_2\varphi_2)\cdot 1=0, \tag{21}$$

Implementing equations above in the Mathcad environment, the resulting left (red) and the right (blue) tooth gap profile's shapes are drawn in **figure 6**.

Transposing the coordinates of the contact points gained with the function of gearing $\varphi=\varphi(u)$ in the fixed frame using the matrix equations $\underline{\mathbf{r}}_{0i}=\mathbf{M}_{0si}\,\underline{\mathbf{r}}_{si}$, $i\in\{1;2\}$, the lines of actions are obtained. These are represented in **figure 7**.

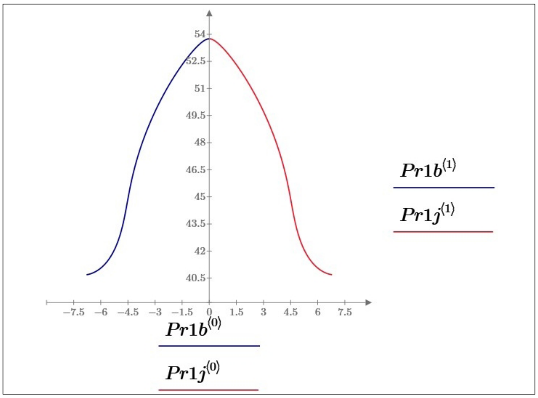

Figure 6. *Tooth gap profiles.*

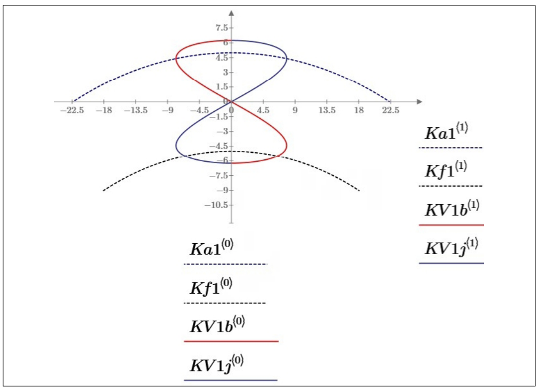

Figure 7. *Lines of actions for the left and right tooth profiles.*

The real segments of action are comprised of the addendum circle and the inner circle arcs (radius of the inner circle is considered here greater that the dedendum circle radius by $c_0^* m$.

3. Simulation of the gearing

In order to simulate the gearing and meshing process, first the solid model of the generating rack must be built up.

Model initial data was considered as follows:

$m = 5$ mm; $h_0 = 1$; $c_0 = 0.25$; $z_1 = 19$; $z_2 = 27$; $b = 2/5$

$$a = (h_0 + c_0)\,m = 6.25 \text{ [mm]} \tag{22}$$

Using the deduced equations with initial data in the Mathcad environment, the generated cosine profile is shown in **figure 8**.

The 3D rack model was realized in Autodesk Inventor environment, using the coordinates of the profile points (**figure 9**).The simulation of manufacturing was performed in the Autodesk AutoCad environment because the Inventor environment cannot handle 3D solid object subtraction. Using the AutoCad programming space, an Autolisp **[3–5]** program was realized which simultaneously preforms the rolling and subtraction.

4. Comparative study

Investigations were carried out on the gear pair realized with the common rack. An involute gear pair defined by similar initial data (number of teeth, module, axis distance) was also modelled.

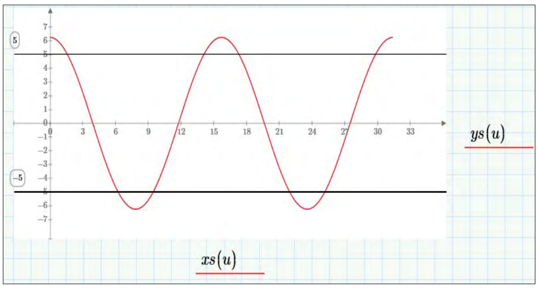

Figure 8. *Generating rack's cosine profile.*

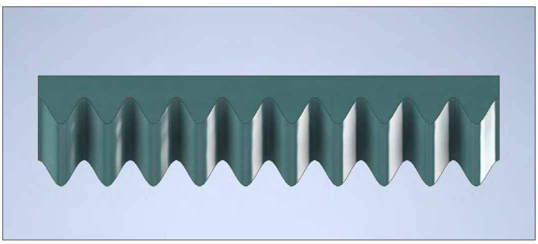

Figure 9. *Solid model of the generating rack.*

In order to obtain valid data, the profile shifting coefficient was set to $\xi = 0$. To increase the profile precision of the cosine rack, the maximum possible number of profile points (500 points) was considered, and spline fitting functions were defined by them. Using the Autolisp programming environment the solid models of cosine gears with $z_1 = 19$ and $z_2 = 27$ teeth were obtained. The precision value for the subtraction was set at 0.2 mm, due to the very high hardware demand of this operation (a complex operation consisting of step by step subtraction and surface regeneration). **Figure 12** shows the linear traces drawn by the generating rack on the tooth surface **[3]**.

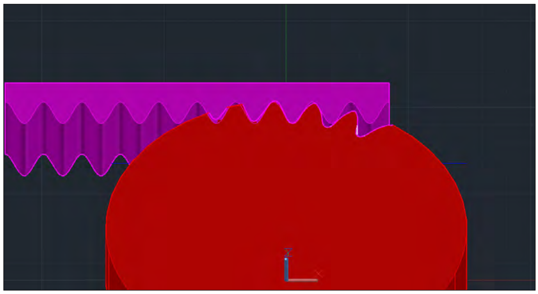

Figure 10. *The subtracting operation through the rolling process.*

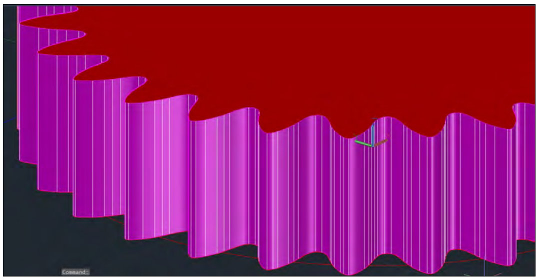

Figure 11. *Gear generated in the AutoCad environment*

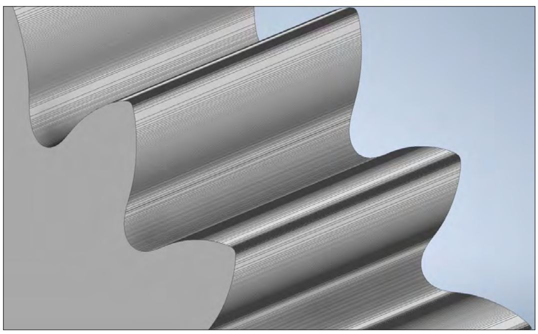

Figure 12. *Traces of the generating rack on the tooth surface.*

4.1. Graphical validation of the mathematical model

Firstly, the matching of the mathematically deduced and the 3D modelled profiles was examined.

Figure 13 shows a perfect match of the profile deduced with the use of the equation of gearing in the Mathcad environment with the profile realized by CAD-simulation. Thus, the validation of the analytical formulae defining the mathematical model has been confirmed.

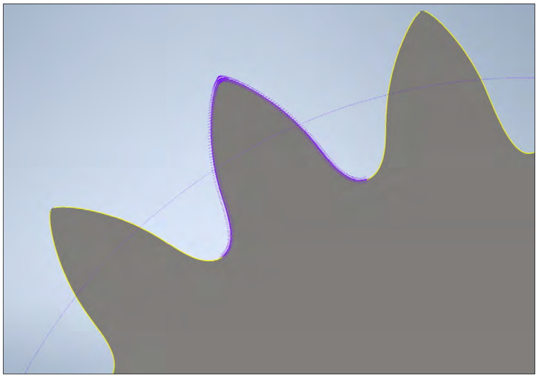

Figure 13. *Proofing of the mathematical model*

4.2. The influence of tooth number on tooth profile form

As is well known, involute profile curvature depends on the tooth number, due to the variation of the basic circle radius. This conclusion remains valid for the cosine gears too, as can be seen in figure 14.

It shows the profile of the tooth of $z = 19$ gear (magenta) set on the symmetry axis of the tooth of $z = 27$ gear (yellow), to realize a visual geometric comparison. If looking on the profiles from the pitch circle out to the addendum circle, one can observe that profiles defining the $z = 19$ tooth approach eachother faster at the addendum , thus, the tooth tip sharpens faster. Now if looking on the profiles from the pitch circle to the root circle, the observed tendency is the decreasing of the dedendum width teeth number. (see the magenta profiles) That decrease causes the weakening of the tooth root.

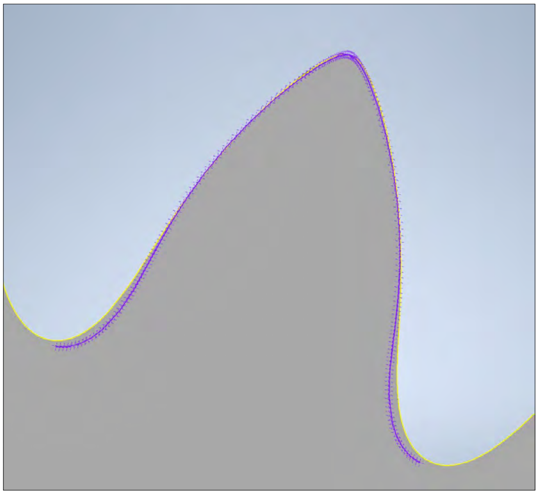

Figure 14. *The influence of tooth number on the shape of the profile*

4.3. Comparison of involute and cosine profiles

The involute profiles of the involute gear pair were drawn in the Mathcad environment. They consist of an involute arc and a root fillet. The last is a looped involute drawn by the tip of the generating edge of the involute rack (if considering a simplified rectilinear rack tooth). The profiles were overlapped using the axis y as symmetry axis of the tooth together with their pitch circle points. The calculus of the involute profile can be found in a very large number of publications thus it is omitted here.

Figures 16 and 17 show the profile comparison in the CAD environment. In both cases the involute tooth root fillet curve produces a weaker tooth root than the cosine tooth root. If analyzing the tip geometry, it is obvious that cosine gear tip width in both cases is smaller than involute tip width.

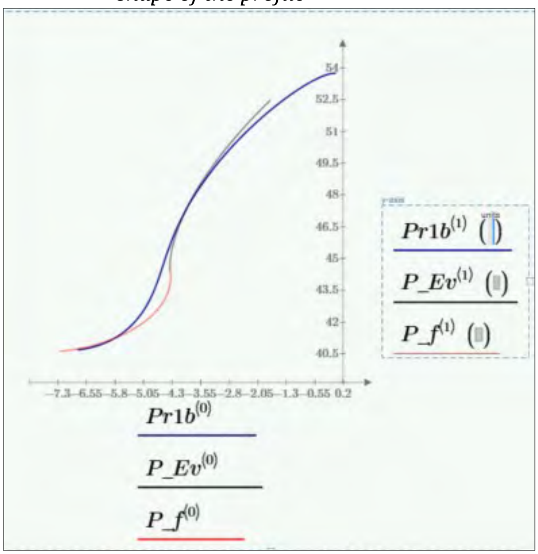

Figure 15. *The cosine and involute profiles.*

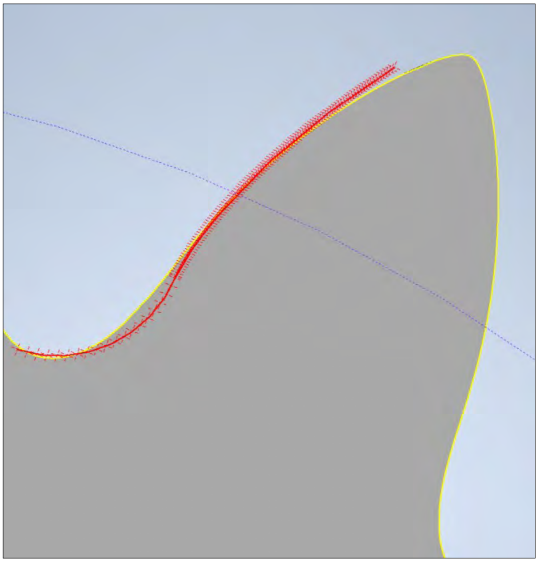

Figure 16. *Case of Z=19.*

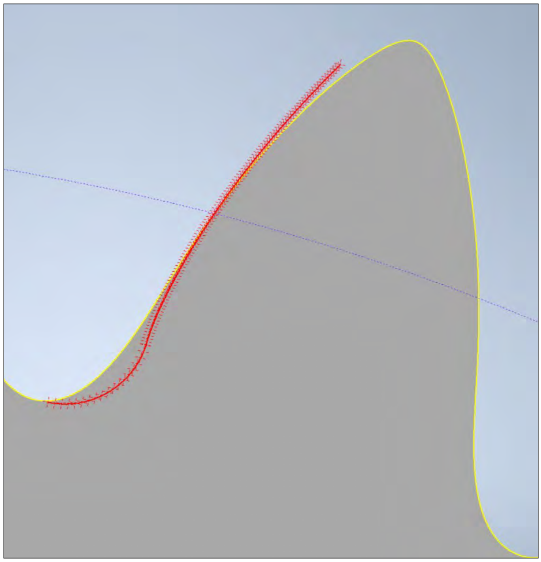

Figure 17. *Case of Z=27.*

5. Conclusions

The synthesis of a cosine gear pair requires identical generating racks thus their cutting with a single cosine planning comb is possible.

Tooth number has a significant influence on the tooth profile shape.

Root width increases with the number of teeth. The cosine gear tooth root's rigidity is always higher than that of the involute tooth. This will probably lead to a more extended lifetime of the gear pair.

At small tooth numbers, cosine gears are susceptible to tip sharpening.

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