CONTRIBUTED TALK

TRANSIENT CHAOS REVEALS HARDNESS STRUCTURE OF HAMILTONIAN PATHS NEAR THE PHASE TRANSITION

László Mihály

laszlo.mihaly@ubbcluj.ro

Babeș-Bolyai University, Cluj-Napoca, Romania

Joint work with Mária Ercsey-Ravasz (Babeș-Bolyai University, Cluj-Napoca, Romania, Transylvanian Institute of Neuroscience, Cluj-Napoca, Romania), Botond Molnár (Babeș-Bolyai University, Cluj-Napoca, Romania, Transylvanian Institute of Neuroscience, Cluj-Napoca, Romania)

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ABSTRACT

The Hamiltonian path problem is an NP-complete problem whose computational hardness is strongly influenced by the underlying graph structure. Due to its NP-completeness, it can be transformed into a Boolean satisfiability (SAT) problem in polynomial time [1]. In this work, we construct such SAT formulations for random graph instances and analyze their solution dynamics.

We investigate the fine structure of hardness in Erdős–Rényi random graphs across different system sizes (15, 20, 25, 30, 35 node networks) and densities (from 10% to fully connected networks), with particular focus on the low-density regime near the phase transition for the appearance of Hamiltonian paths, as characterized by the *Komlós–Szemerédi bound* [2].

The resulting SAT instances are solved using a continuous-time dynamical system (CTDS) [3], where Boolean variables are mapped to continuous states $s_i \in [-1, 1]$. Clause satisfaction is encoded through functions $K_m(\mathbf{s})$, and the dynamics is governed by an energy function $V(\mathbf{s}) = \sum_m K_m^2(\mathbf{s})$, whose minima correspond to solutions. The system evolves through transiently chaotic trajectories (figure 1) before converging to a solution. Following the hardness definition introduced in [4], we quantify complexity via the escape rate κ of trajectories, defined as $\eta = -\frac{\log_{10} \kappa}{\log_{10} N}$, where N is the number of SAT variables.

Our results indicate that hardness reaches a maximum in the low-density regimes close to the transition where Hamiltonian paths begin to emerge, suggesting a structural change in the solution space around this region. Moreover, each density is associated with a characteristic η value, and these values align across different graph sizes, indicating a universal scaling behavior of hardness with respect to density.

This framework enables instance-level characterization of networks, linking graph topology to dynamical search complexity, suggesting that Hamiltonian path hardness may serve as a quantitative

descriptor for complex systems. These results support the applicability of escape-rate-based hardness measures for combinatorial optimization problems and their relevance for studying phase transitions in constraint satisfaction problems [5].

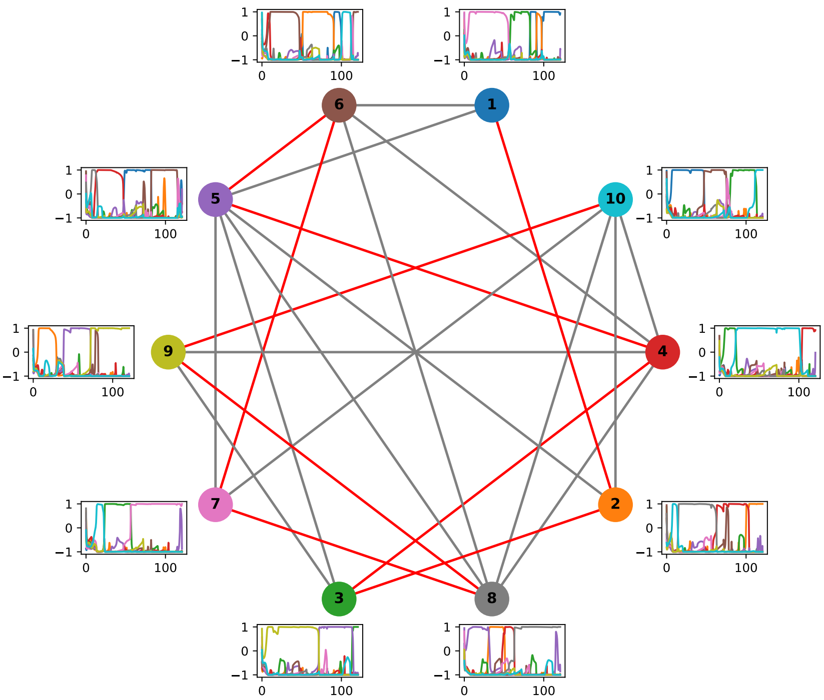

Figure 1: A small graph together with the CTDS dynamics of the associated SAT variables. Insets show the time evolution of selected variables, illustrating transiently chaotic behavior before convergence to a solution.

References

[1] Zhou, N.-F. (2020). In pursuit of an efficient SAT encoding for the Hamiltonian cycle problem. In: *Principles and Practice of Constraint Programming (CP 2020)*, pp. 585–602. Springer.

[2] Komlós, J., Szemerédi, E. (1973). Hamilton cycles in random graphs. In: *Infinite and Finite Sets*.

[3] Ercsey-Ravasz, M., Toroczkai, Z. (2011). Optimization hardness as transient chaos in an analog approach to constraint satisfaction. *Nature Physics* 7(12): 966–970.

[4] Varga, M., Sumi, R., Toroczkai, Z., Ercsey-Ravasz, M. (2016). Order-to-chaos phase transition in random SAT. *Physical Review E* 93: 052211.

[5] Monasson, R., Zecchina, R., Kirkpatrick, S., Selman, B., Troyansky, L. (1999). Determining computational complexity from characteristic 'phase transitions'. *Nature* 400: 133–137.