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Visco-hyperelastic characterization of polymeric foam materials

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Abstract

This article presents the visco-hyperelastic material modelling approach for open-cell polymer foams. The visco-hyperelastic material models combine the long-term hyperelastic and viscoelastic material models using finite-strain formulation. The long-term behavior is modelled using the Ogden-Hill's compressible hyperelastic model termed as "hyperfoam" in the commercial finite element software ABAQUS. The time-dependent material response is modelled by adopting the finite-strain viscoelastic formalism implemented in ABAQUS. From the visco-hyperelastic material model a closed-form stress solution for uniaxial extension is derived, which provides a more accurate parameter-fitting method to determine the visco-hyperelastic material parameters from the measurement data.

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1. Introduction

Polymer foams are widely applied cellular materials due to their mechanical behavior and high energy absorption properties [1]. The main field of application is packaging and impact protection, but polymer foams appear in everyday use like as ear-plugs and memory foams as well. Therefore, the complete understanding of the overall mechanical behaviour is particularly important. Polymer foams usually exhibit very large deformations in case of

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compression regime. In addition, the material is rate-dependent. Consequently, it is required to use material models which are applicable to describe these phenomena.

The behaviour of large elastic and viscoelastic materials can be described using the so-called visco-hyperelastic constitutive equation, which combines the hyperelastic and the viscoelastic material models. In this approach the time-dependent stress-relaxation phenomenon is modeled using the Prony-series representation, while for the long-term time-independent behaviour a hyperelastic material model is adopted, which can be derived from the corresponding strain energy function.

In this contribution we present the visco-hyperelastic model applied for open-cell polymer foams. Additionally, based on this material model, we derive the closed-form stress solution for uniaxial extension, which enables us to obtain material parameters directly from experimental data using a parameter-fitting algorithm. This method of parameter-fitting eliminates the error, which occurs in the traditional method, when the long-term response and the relaxation were fitted separately.

The outline of the paper is as follows: Section 1 presents the viscoelastic models applied in small and finite strain theories; In Section 2 the Ogden-Hill's compressible hyperfoam model and its solution for uniaxial extension are summarized; In Section 3 the different material parameter fitting methods and the closed-form stress solution presented; Section 4 summarizes the results.

2. Viscoelastic material models

2.1. Small-strain viscoelasticity in 1D

The most commonly used small-strain linear isotropic viscoelastic constitutive equation defines the stress response in 1D as a convolution (or hereditary) integral of the strain rate $\dot{\varepsilon}(t)$ and time-dependent elastic modulus $E(t)$ as

$$\sigma(t)=\int_0^t E(t-s)\dot{\varepsilon}(s)\mathrm{d}s \quad \text{with} \quad E(t)=E_\infty+\sum_{k=1}^{P}E_k\exp\left[-\frac{t}{\tau_k}\right], \tag{1}$$

where $E(t)$ is the elastic modulus expressed using Prony-series representation [2]. The formula can be rewritten in an alternative, but equivalent form, where the instantaneous elastic and the viscoelastic contributions are separated, thus

$$\sigma(t)=\sigma_0(t)-\sum_{k=1}^{P}\frac{e_k}{\tau_k}\int_0^t \sigma_0(t-s)\exp\left[-\frac{s}{\tau_k}\right]\mathrm{d}s\,. \tag{2}$$

In this formulation the instantaneous elastic response $\sigma_0(t)=E_0\varepsilon(t)$ calculated directly from the strain history $\varepsilon(t)$ and the instantaneous elastic modulus E_0, whereas the relative moduli (e_k) are obtained as $e_k=E_k/E_0$.

2.2. Finite-strain viscoelasticity

Since the deformation of open cell polymer foams show large strains and displacements, the small-strain theory cannot be applied. Therefore we have to rewrite the viscoelastic constitutive equation (Eq. 3) using finite-strain theory. A possible formulation of the finite-strain viscoelastic constitutive equation is provided by ABAQUS [3]. It should be noted that the formulation was updated in ABAQUS version 6.9, but the new formulation is not valid for the HYPERFOAM hyperelastic model, which is applied in our investigations (see Section 3). Therefore for our calculation the constitutive equation given ABAQUS version 6.7 was adopted. For compressible materials the instantaneous response can be split into two parts as

$$\boldsymbol{\tau}_0(t)=\boldsymbol{\tau}_0^D(\bar{\mathbf{F}}(t))+\boldsymbol{\tau}_0^H(J(t))\,, \tag{3}$$

where $\boldsymbol{\tau}_0^D$ and $\boldsymbol{\tau}_0^H$ are the deviatoric and the hydrostatic parts of the instantaneous Kirchhoff stress tensor $\boldsymbol{\tau}_0$. The distortional deformation gradient $\bar{\mathbf{F}}$ is related to the total deformation gradient $\mathbf{F}$ by $\bar{\mathbf{F}} = J^{-1/3}\mathbf{F}$, where $J = \det\mathbf{F}$ defines the volume ratio. ABAQUS employs the following equations for the deviatoric and hydrostatic contributions.

$$\boldsymbol{\tau}^D(t) = \boldsymbol{\tau}_0^D(t) - \mathrm{SYMM}\left[\sum_{k=1}^{PG}\frac{g_k}{\tau_k^G}\int_0^t \mathbf{F}_t^{-1}(t-s)\boldsymbol{\tau}_0^D(t-s)\mathbf{F}_t(t-s)\exp\left[-\frac{s}{\tau_k^G}\right]\mathrm{d}s\right], \tag{4}$$

$$\boldsymbol{\tau}^H(t) = \boldsymbol{\tau}_0^H(t) - \sum_{k=1}^{PK}\frac{k_k}{\tau_k^K}\int_0^t \boldsymbol{\tau}_0^H(t-s)\exp\left[-\frac{s}{\tau_k^K}\right]\mathrm{d}s, \tag{5}$$

where $\mathbf{F}_t(t-s)$ is the deformation gradient at time instant $t-s$ relative to state at t, which can be obtained as $\mathbf{F}_t(t-s) = \mathbf{F}(t-s)\mathbf{F}(t)$. Similarly to the small-strain theory, the relaxation moduli are expressed in Prony-series representation as

$$G(t) = G_0\left(g_\infty + \sum_{k=1}^{PG} g_k \exp\left[-\frac{t}{\tau_k^G}\right]\right) \text{ and } K(t) = K_0\left(k_\infty + \sum_{k=1}^{PK} k_k \exp\left[-\frac{t}{\tau_k^K}\right]\right), \tag{6}$$

where g_k and k_k are the relative shear and bulk moduli, respectively, while g_∞ and k_∞ are the long term relative moduli. The moduli can be related to each other as

$$1 = g_\infty + \sum_{k=1}^{PG} g_k = k_\infty + \sum_{k=1}^{PK} k_k \ . \tag{7}$$

In our calculations we use the same number of parameters, which means that $PG = PK = P$ and we assume also that the corresponding relative and bulk moduli values are identical, thus $g_k = k_k$. This approach is suggested by ABAQUS. Similarly, the relaxation times $\tau_k = \tau_k^G = \tau_k^K$ are also the same.

3. Hyperelastic material model

In order to obtain the stress response using the large-strain viscoelastic model (Eqs. 3-5), the instantaneous stress $\boldsymbol{\tau}_0(t)$ response should be given in the form of a hyperelastic constitutive equation. Since the volumetric compression of polymeric foam materials is significant, hyperelastic models proposed for nearly-incompressible materials (such as rubber) cannot be used for these materials. In the literature the number of available compressible hyperelastic models is limited. The most widely used hyperelastic model for foams is based on the work of Ogden and Hill [3,4], where the hyperelastic strain energy function is defined in the term of the principal stretches $(\lambda_1, \lambda_2, \lambda_3)$.

3.1. Ogden-Hill's compressible hyperelastic model

The Ogden-Hill's compressible hyperelastic model is implemented in the most commonly used finite element softwares, but named differently, because the introduction of the model can be related to three different authors. Firstly Ogden [4] introduced a strain energy function corresponding to a compressible hyperelastic material model, where the volumetric compression was defined as a separate function. After that, Hill [5] provided a possible form for the volumetric compression part. In the work of Storakers [6] the author investigated the behavior of Hill's model, and combined the two formulations and introduced new parameters to simplify the model.

In the commercial finite element software ABAQUS [3], the above introduced model is termed as HYPERFOAM model, which obtained in a slightly different form. In our calculations this formulation will be used.

$$W = \sum_{i=1}^{N}\frac{2\mu_i}{\alpha_i^2}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3 + \frac{1}{\beta_i}\left[J^{-\alpha_i\beta_i} - 1\right]\right). \tag{8}$$

3.2. Stress solution for uniaxial extension

Since the formulation of the hyperelastic constitutive equation is given, the stress responses can be derived [7]. In case of hyperelastic models, when the strain energy is given as the function of the principal stretches, the principal Kirchhoff stress τ_k can be obtained as

$$\tau_k = \sum_{j=1}^{N} \frac{2\mu_i}{\alpha_i} \left(\lambda_k^{\alpha_i} - J^{-\alpha_i \beta_i} \right) \tag{9}$$

In case of uniaxial extension the deformation gradient becomes

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda_T & 0 \\ 0 & 0 & \lambda_T \end{bmatrix}, \tag{10}$$

where λ denotes the longitudinal stretch, and λ_T the transverse stretches. We also assume that the transverse stretches are identical. Therefore the Kirchhoff stress solutions become

$$\tau_1 = \sum_{j=1}^{N} \frac{2\mu_i}{\alpha_i} \left(\lambda^{\alpha_i} - (\lambda \lambda_T^2)^{-\alpha_i \beta_i} \right) \text{ and } \tau_2 = \tau_3 = \sum_{j=1}^{N} \frac{2\mu_i}{\alpha_i} \left(\lambda_T^{\alpha_i} - (\lambda \lambda_T^2)^{-\alpha_i \beta_i} \right) = 0 \tag{11}$$

For open-cell polymer foams, like memory foams and earplugs, the transversal strains can be neglected, therefore the Poisson-ratio is considered to be zero $\nu_i \approx 0$ [8]. This approximation yields that $\beta_i \approx 0$ in the hyperelastic model. In the visco-hyperelastic material model the strain history is the input, therefore the time function $\lambda(t)$ must be given for uniaxial extension. Based on the assumptions above, and substituting the strain history back into (Eq. 12), the instantaneous Kirchhoff stress tensor becomes

$$\boldsymbol{\tau}_0 = \begin{bmatrix} \tau_0(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{ with } \tau_0(t) = \sum_{j=1}^{N} \frac{2\mu_i}{\alpha_i} \left[\left(\lambda(t) \right)^{\alpha_i} - 1 \right]. \tag{12}$$

4. Visco-hyperelastic material parameters

As we derived the instantaneous hyperelastic stress solution for compressible materials, this solution (Eq. 13) should be substituted back into the finite-strain viscoelastic material model (Eqs. 3-5). The material parameters included in the visco-hyperelastic model are: $2N$ $(\alpha_1, \alpha_2 ... \alpha_N; \mu_1, \mu_2 ... \mu_N)$ parameters for the instantaneous hyperelastic response; $2P$ $(g_1, g_2 ... g_P; \tau_1, \tau_2 ... \tau_P)$ parameters for the time-dependent response (Prony-series). It means that $2(N+P)$ parameters should be fitted to our measurement data in order to describe the visco-hyperelastic material behavior.

4.1. Separated fitting of parameters

The usually adopted algorithm to find the material parameters for a particular material is to separate the parameter-fitting of the long-term behavior and the stress relaxation. In this approach, it is assumed that the stress relaxation test is performed using step loading. However, it is well-known that in real measurements ramp loading is used instead of step loading, which involves error in the stress relaxation data (see. Fig. 1/ab.). [9-10]

The long-term behavior is determined from cyclic compression-relaxation loading (see Fig. 1/c), where the data points corresponding to the long-term behavior appears between the uploading and downloading curves [8]. The data points could be determined using interpolation from the corresponding stress minima and maxima, usually as the mean value. It also induces errors in the parameter fitting process. As a result, the fitted material parameters cannot describe accurately the overall visco-hyperelastic behavior, the solution will be inaccurate.

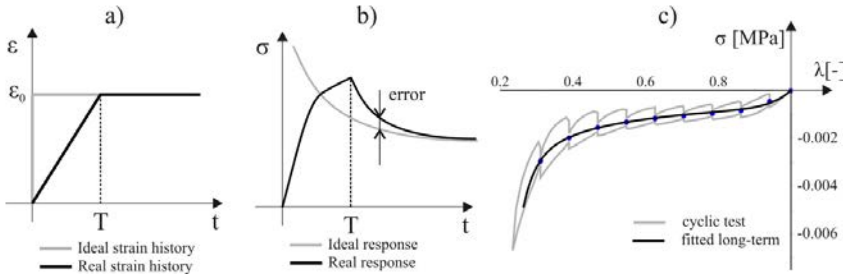

Fig. 1. (a) Ideal and real strain history; (b) Ideal and real response; (c) Long-term behavior fitted to cyclic test

4.2. Solution of the hereditary integral

In order to accurately fit the material model it is essential to obtain the stress solution for the ramp loading. Then, the entire material model can be fitted to the experimental data without separating the long-term behavior and the relaxation behavior. In case of the ramp test, the solution is divided into two parts: uploading (1) and relaxation (2). The time-history of the stretch becomes $\lambda_1(t)=1+\dot{\varepsilon}t$ and $\lambda_2(t)=1+\dot{\varepsilon}T$, where $\dot{\varepsilon}$ is the strain rate in the uploading phase, whereas and T denotes the time of uploading. These two parameters $(\dot{\varepsilon},T)$ characterize the ramp test. Therefore, after solving the hereditary integrals the closed-form solutions can be expressed explicitly as [11]:

- for the first part - uploading:

$$\tau_1(t)=\tau_0(t)-\sum_{k=1}^{P} g_k\left(\sum_{i=1}^{N}\frac{2\mu_i}{\alpha_i}\eta_{ik}\right), \tag{13}$$

$$\eta_{ik}=\exp\left[\frac{-t}{\tau_k}\right]-1-\exp\left[-\frac{1+\dot{\varepsilon}t}{\tau_k}\right]\left(\frac{-1}{\tau_k\dot{\varepsilon}}\right)^{-\alpha_i}\left(\Gamma\left[1+\alpha_i,\frac{-1}{\tau_k\dot{\varepsilon}}\right]-\Gamma\left[1+\alpha_i,-\frac{1+t\dot{\varepsilon}}{\tau_k\dot{\varepsilon}}\right]\right). \tag{14}$$

- for the second part – relaxation:

$$\tau_2(t)=\tau_0(T)\left(1-\sum_{k=1}^{P} g_k\left(1-\exp\left[-\frac{t-T}{\tau_k}\right]\right)\right)-\sum_{k=1}^{P} g_k\left(\sum_{i=1}^{N}\frac{2\mu_i}{\alpha_i}\vartheta_{ik}\right), \tag{15}$$

$$\vartheta_{ik}=\exp\left[\frac{-t}{\tau_k}\right]-\exp\left[\frac{T-t}{\tau_k}\right]-\exp\left[-\frac{1+\dot{\varepsilon}t}{\tau_k}\right]\left(\frac{-1}{\tau_k\dot{\varepsilon}}\right)^{-\alpha_i}\left(\Gamma\left[1+\alpha_i,\frac{-1}{\tau_k\dot{\varepsilon}}\right]-\Gamma\left[1+\alpha_i,-\frac{1+T\dot{\varepsilon}}{\tau_k\dot{\varepsilon}}\right]\right) \tag{16}$$

In the above formulae $\Gamma[a,x]$ represents the incomplete gamma function [12]. The analytical solution is used to obtain the material parameters from the time-stress data corresponding to the ramp-history loading. Using this method, the parameters can be fitted directly to the measured data without separating the long-term solution and the relaxation behavior. Therefore the fitted parameters will be more accurate, especially in the relaxation region.

5. Results

In order to illustrate the performance of the novel solution, different parameter-fitting methods were compared in case of a commercial open-cell polymer foam material applied in memory foam layers of mattresses. We applied a third order viscoelastic $(P=3)$ and a second-order hyperelastic model $(N=2)$. The resulted material parameters are presented in Table 1. It should be noted, that the separated and the closed-form method uses the long-term and the instantaneous hyperelastic material parameters, respectively. After the parameters were determined, the uniaxial compression test was simulated in ABAQUS [3], using the previously determined parameters. The Cauchy stress solutions are presented in Fig 2.

The results show, that the closed-form fitting gives significantly better solutions than the usually adopted method, where the long-term response and the time-dependent solutions are separated. It should be noted that, in the classical method, the error resulting from the ramp history may be reduced by modifying the experimental data according to the method proposed by Zapas and Phillips [13], for instance.

Table 1. Material parameters in case of different fitting methods

Fitting method	g_1	g_2	g_3	τ_1	τ_2	τ_3	μ_1	μ_2	α_1	α_2
Separated	0.01586	0.61213	0.03739	242.424	2.3221	20.6702	1.417 E-7	0.00669	-7.2397	18.3649
Closed-form	0.84226	0.03547	0.05311	0.16031	1.92388	0.84929	0.000239	0.02235	-2.1990	3.4435

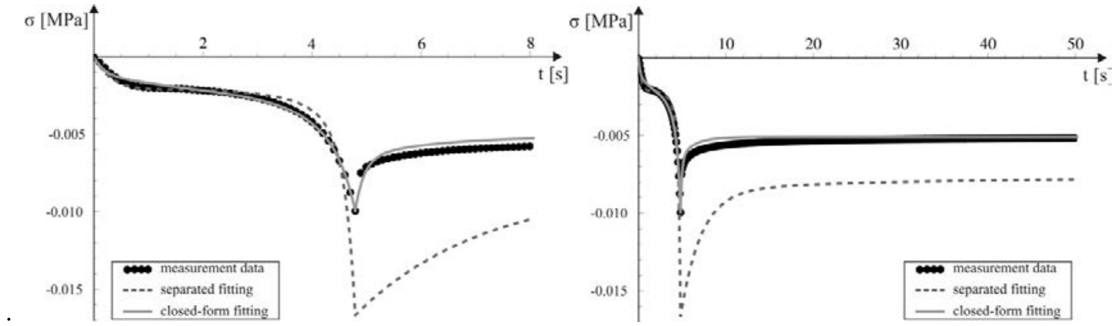

Fig. 2. Comparison of parameter fitting methods

6. Conclusion

As the results demonstrate, a closed-form solution for open-cell polymer foams was developed using finite-strain viscoelastic constitutive equation with HYPERFOAM hyperelastic material model. This solution makes the material parameter fitting process more accurate therefore it improves the modelling of open-cell polymer foams.

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