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Letter

Boundary integrability from the fuzzy three sphere

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ABSTRACT

We consider $\mathfrak{so}_4$ invariant matrix product states (MPS) in the $\mathfrak{so}_6$ symmetric integrable spin chain and prove their integrability. These MPS appear as fuzzy three-sphere solutions of matrix models with Yang-Mills-type interactions, and in particular they correspond to scalar defect sectors of $\mathcal{N} = 4$ SYM. We find that the algebra formed by the fuzzy three-sphere generators naturally leads to a boundary reflection algebra and hence a solution to the boundary Yang-Baxter equation for every representation of the fuzzy three-sphere. This allows us to find closed formula for the overlaps of Bethe states of $\mathfrak{so}_6$ symmetric chains with the fuzzy three-sphere MPS for arbitrary bond dimensions.

1. Introduction

Exact results for real time evolution of interacting quantum systems are of increasing importance due to the growing interest in simulations in quantum computers and in analog optical traps [1–4]. On a practical level one would like to have exactly solvable quantum models which are simple enough to test simulations, yet rich enough to have non-trivial dynamics. Integrable models in $1+1$ dimensions are of course ideal for such a task, however despite their inherent solvability, the computations of real time observables is complicated by the inherent complexity of their many body wavefunctions. Recent progress towards this goal has been achieved via the identification of certain integrable states for which overlaps with energy eigenstates take a simple and universal form [5]. By now there is a compelling picture relating these integrable quenches to integrability in the presence of boundaries in a rotated channel and the universal form of the overlap formulas have been established in very general settings [6–10].

One particularly interesting class of such states are those which arise from matrix product states (MPS). Many of the known examples of integrable MPS come from studies of defect set-ups in the context of the AdS/CFT correspondence [11–18]. There the states have a natural interpretation as boundary states for the dual string description of the large N gauge theory, and the overlaps of the integrable MPS encode rich information about non-perturbative physics in the string theory. One of the prospects of said program is the determination of finite coupling OPE data in superconformal theories such as $\mathcal{N} = 4$ SYM and ABJM [19].

The duality between integrable spin systems describing the dynamics of single trace operators in the gauge theory and strings moving in anti-de-Sitter spaces opens an avenue for generating large classes of boundary states for spin systems inspired by known results in string theory. A common effect in string theories is that boundary conditions for open strings carry non-commutative data which is often geometric. This is well understood for instance in Yang-Mills matrix models in which brane configurations are often described by non commutative geometries [20]. Noncommutative geometries such as fuzzy spheres of various dimensions generically appear as classical configurations of matrix theories [21–23], such as conformal defect set-ups of $\mathcal{N} = 4$ SYM. These configurations can often be associated with integrable MPS making them interesting independently of their stringy origins. This motivated the conjecture that in fact all fuzzy spheres can be associated with an integrable MPS which led to the identification of novel integrable MPS based on odd dimension fuzzy spheres [15].

In this letter we establish the integrability of a class of $\mathfrak{so}_4$ symmetric MPS based on the fuzzy three-sphere for $\mathfrak{so}_n$ symmetric integrable chains. For applications to $\mathcal{N} = 4$ SYM we will keep in mind the case of $n = 6$. Such MPS is the simplest integrable quench based on odd dimensional fuzzy spheres and it demonstrates the inherent difference of $\mathfrak{so}_m$ invariant MPS for odd m relative to those of even m. The algebra generated by the fuzzy S^3 MPS is enough to guarantee the existence of a K-matrix giving a representation of the boundary reflection algebra for every rank for which the MPS exists. With this information we establish an overlap formula for the $\mathfrak{so}_6$ symmetric chain.

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We begin in Section 2 by presenting the fuzzy S^3 algebra. In Section 3 we review the integrability framework for computing overlaps of integrable MPS in the $SO(6)$ symmetric spin chain. In Section 4 we present the matrix for the fuzzy S^3 MPS and the corresponding Cartan generators of the reflection algebra. With this information we present the overlap formula in Section 5. Finally, in Section 6, we conclude.

2. Fuzzy S^3 algebra

We begin by presenting the fuzzy S^3 algebra. An $SO(4)$ invariant construction of the fuzzy three-sphere was presented in [24]. This method can be used to construct matrices satisfying the matrix equation $\sum_{i=1}^4 X_i^2 = r_n \mathbb{I}$, where the rank of the matrices is given by $r_n = \frac{(n+1)(n+3)}{2}$. The meaning of n will become clear momentarily. Instead of reviewing their explicit construction we instead summarize the results found in [25] which presents an algebra generated from the X_i. The algebra presented by Ramgoolam can be summarized by the following set of relations:

$$\{X_j, \mathcal{V}\} = [F_{j,k}, \mathcal{V}] = 0, \quad \mathcal{V}^2 = 1, \tag{1}$$

$$\sum_{k=1}^4 X_k^2 = \frac{(n+1)(n+3)}{2}\mathbb{I}, \tag{2}$$

$$[X_j, X_k] = F_{jk} + (n+2)F_{jk}^-, \tag{3}$$

$$[F_{jk}, X_l] = \delta_{kl}X_j - \delta_{jl}X_k. \tag{4}$$

$$[F_{ij}, F_{kl}] = \delta_{jk}F_{il} - \delta_{il}F_{kj} - \delta_{ik}F_{jl} + \delta_{jl}F_{ki}. \tag{5}$$

The F_{ij} act as $SO(4)$ generators when acting the X_i, while $\mathcal{V}$ acts as a chirality operator. The F_{ij}^- are not independent and they are defined by

$$F_{ij}^- = \frac{1}{2}\sum_{kl} \epsilon_{ijkl} F_{kl}\mathcal{V}. \tag{6}$$

Since the sum of the square of the coordinate matrices X_i is central in the algebra,

$$\left[\sum_{k=1}^4 X_k^2, X_j\right] = 0, \tag{7}$$

we have some additional relations that are useful for determining the K-matrix:

$$\sum_{j=1}^4 X_j F_{jk} = \frac{3}{2}X_k, \tag{8}$$

$$\sum_{j=1}^4 X_j F_{jk}^- = \left(\frac{n+2}{2}\right)X_k. \tag{9}$$

Representations of this algebra act on a reducible representation of $SO(4)$ $\mathcal{R}_+ \oplus \mathcal{R}_- = \left(\frac{n+1}{4}, \frac{n-1}{4}\right) \oplus \left(\frac{n-1}{4}, \frac{n+1}{4}\right)$ on which the generator $\mathcal{V}$ acts diagonally with eigenvalues ± 1. The matrices X_i are maps between the irreducible components $\mathcal{R}_+$ and $\mathcal{R}_-$ while the generators F_{ij} are symmetry generators.

3. Integrable SO(6) spin chains and their overlaps

The planar spectral problem of the $\mathcal{N}=4$ SYM is solved by an integrable super spin chain [26]. In particular, at the one-loop level the single trace operators in the scalar sector can be described by state of the $SO(6)$ symmetric spin chain. The Hamiltonian is [27]

$$H = \sum_{j=1}^J 1 - P_{j,j+1} + \frac{1}{2}K_{j,j+1}, \tag{10}$$

where P and the K are the permutation and trace operators acting on the tensor product of two copies of 6 dimensional vector space

$$P = \sum_{i,j=1}^6 e_{i,j} \otimes e_{j,i}, \quad K = \sum_{i,j=1}^6 e_{i,j} \otimes e_{i,j}, \tag{11}$$

where $e_{i,j}$ are the unit 6×6 matrices. The J is the length of the spin chain.

The Hamiltonian is integrable which means there exist a transfer matrix which generates infinte series of conserved charges. The transfer matrix is defined from the monodromy matrix

$$T_0(u) = R_{0,J}(u-1)\dots R_{0,1}(u-1), \tag{12}$$

where $R(u)$ is the $SO(6)$ symmetric R-matrix

$$R_{0,j}(u) = 1 + \frac{1}{u}P_{0,j} - \frac{1}{u+2}K_{0,j}. \tag{13}$$

These operators are written in the real basis of the scalar fields. We also need the form in the complex basis

$$\begin{aligned}
|-3\rangle &= Z = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_6), \quad |3\rangle = \bar{Z} = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_6),\\
|-2\rangle &= Y = \frac{1}{\sqrt{2}}(\phi_2 + i\phi_5), \quad |2\rangle = \bar{Y} = \frac{1}{\sqrt{2}}(\phi_2 - i\phi_5),\\
|-1\rangle &= X = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4), \quad |1\rangle = \bar{X} = \frac{1}{\sqrt{2}}(\phi_3 - i\phi_4).
\end{aligned} \tag{14}$$

In this new basis the permutation has the same form but the trace operator reads as

$$K = \sum_{i,j=-3}^3 e_{i,j} \otimes e_{-i,-j}, \tag{15}$$

where sum goes through the indexes $i,j = -3,-2,-1,1,2,3$. The dictionary between Bethe eigenstates $|\mathbf{u},\mathbf{v},\mathbf{w}\rangle$ and operators built from complex fields can be found for instance in [27].[1]

Let us define the MPS without the trace

$$\langle \psi_{\alpha,\beta}| = \sum_{i_1,\dots,i_J} (\omega_{i_J}\dots\omega_{i_1})_{\alpha,\beta}\langle i_1,\dots,i_J|, \tag{16}$$

where α,β are indexes in the bond space. The integrable MPS is defined with the KT-relation

$$\sum_{k,\gamma} K_{i,k}^{\alpha,\gamma}(u)\langle\psi_{\gamma,\beta}|T_{k,j}(u) = \sum_{k,\gamma}\langle\psi_{\alpha,\gamma}|T_{i,k}(-u)K_{k,j}^{\gamma,\beta}(u), \tag{17}$$

where the α,β,γ and i,j,k are indexes in the bond and the six dimensional space, respectively. Defining operators in the bond space as $\mathbf{K}_{i,j} = \sum_{\alpha,\beta} K_{i,j}^{\alpha,\beta}(u)e_{\alpha,\beta}^B$ and $\langle\Psi| = \sum_{\alpha,\beta}\langle\psi_{\alpha,\beta}|\otimes e_{\alpha,\beta}^B$ we obtain more compact form of the KT-equation

$$\mathbf{K}_0(u)\langle\Psi|T_0(u) = \langle\Psi|T_0(-u)\mathbf{K}_0(u). \tag{18}$$

The MPS overlap formula can be expressed from the components of the K-matrix in the complex basis [10,28]. Since our MPS has $SO(4)\times SO(2)$ symmetry, the Bethe vectors with non-vanish overlaps have achiral pair structure [29] for which $\mathbf{v} = -\mathbf{w}$ and $\mathbf{u} = -\mathbf{u}$. The overlaps can be obtained in the following way. At first, we define the nested K-matrix

$$\mathbf{K}_{i,j}^{(2)}(u) = \mathbf{K}_{i,j}(u) - \mathbf{K}_{i,-3}(u)\mathbf{K}_{3,-3}^{-1}(u)\mathbf{K}_{3,j}(u), \tag{19}$$

for $i,j = -2,-1,1,2$. After that, we calculate the G operators as

$$\begin{aligned}
\mathbf{G}^{(1)}(u) &= \mathbf{K}_{3,-3}(u),\\
\mathbf{G}^{(2)}(u) &= \mathbf{K}_{2,-2}^{(2)}(u),\\
\mathbf{G}^{(3)}(u) &= \mathbf{K}_{1,1}^{(2)}(u) - \mathbf{K}_{1,-2}^{(2)}(u)\left[\mathbf{K}_{2,-2}^{(2)}(u)\right]^{-1}\mathbf{K}_{1,-2}^{(2)}(u).
\end{aligned} \tag{20}$$

and the $\mathbf{B}$-operator as

$$\mathbf{B} = \langle\Psi|0\rangle. \tag{21}$$

These operators commute, i.e.,

$$\left[\mathbf{B}, \mathbf{G}^{(s)}(u)\right] = \left[\mathbf{G}^{(s)}(u), \mathbf{G}^{(r)}(v)\right] = 0. \tag{22}$$

[1] The three sets of Bethe roots are associated with nodes of the $SO(6)$ Dynkin diagram. The one associated with the middle node (momenta carrying excitations) is denoted by $\mathbf{u}$. The other two nodes correspond to the sets $\mathbf{v}$ and $\mathbf{w}$, respectively.

Next step is to define the F-operators

$$\begin{aligned}\mathbf{F}^{(1)}(u) &= \left[\mathbf{G}^{(1)}(u)\right]^{-1}\mathbf{G}^{(2)}(u),\\ \mathbf{F}^{(2)}(u) &= \left[\mathbf{G}^{(2)}(u)\right]^{-1}\mathbf{G}^{(3)}(u).\end{aligned} \tag{23}$$

We can diagonalize these operators simultaneously

$$\begin{aligned}\mathbf{B} &= \mathbf{A}\mathrm{diag}(\beta_1,\dots,\beta_d)\mathbf{A}^{-1},\\ \mathbf{F}^{(s)} &= \mathbf{A}\mathrm{diag}(\mathcal{F}^{(s)}_1,\dots,\mathcal{F}^{(s)}_d)\mathbf{A}^{-1}.\end{aligned} \tag{24}$$

The MPS overlap is given by

$$\frac{\langle \mathrm{MPS}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle}{\sqrt{\langle \mathbf{u},\mathbf{v},\mathbf{w}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle}} = \sum_{\ell=1}^{d}\beta_\ell\prod_{j=1}^{n_u/2}\tilde{\mathcal{F}}^{(1)}_\ell(u_j)\prod_{j=1}^{n_v}\tilde{\mathcal{F}}^{(2)}_\ell(v_j)\sqrt{\frac{\det G_+}{\det G_-}}, \tag{25}$$

where

$$\begin{aligned}\tilde{\mathcal{F}}^{(1)}_\ell(u) &= \mathcal{F}^{(1)}_\ell(iu+1)\sqrt{\frac{u^2}{u^2+1/4}},\\ \tilde{\mathcal{F}}^{(2)}_\ell(u) &= \mathcal{F}^{(2)}_\ell(iu)\frac{u}{u+i/2}.\end{aligned} \tag{26}$$

The Gaudin determinants with achiral pair structure enters in this expression. Explicit expressions for them in our conventions can be found in Ref. [9,10,29,30]

4. Fuzzy sphere K-matrix and the corresponding Cartan operators

When the MPS is built from the operators

$$\omega_j = \begin{cases} X_j, & j=1,2,3,4,\\ 0 & j=5,6,\end{cases} \tag{27}$$

the K-matrix in the real basis can be expressed as

$$K_{i,j} = \begin{cases} c_1\delta_{i,j} + X_iX_j + c_2X_jX_i + c_3F^-_{i,j}, & i,j=1,\dots,4,\\ c_4\delta_{i,j}, & i,j=5,6,\\ 0, & \text{otherwise},\end{cases} \tag{28}$$

where

$$\begin{aligned} c_1 &= \frac{u-1}{u-2}\frac{(4u^2-8u+n^2+4n+7)}{4(2u-1)},\\ c_2 &= -\frac{u-1}{u-2},\\ c_3 &= -\frac{2(n+2)(u-1)^2}{(u-2)(2u-1)},\\ c_4 &= -\frac{u-1}{u-2}\frac{(2u-n-3)(2u+n+1)}{4(2u-1)}.\end{aligned} \tag{29}$$

The F-operators can be expressed as with the Casimir operators of the subalgebras $\mathfrak{so}_2 \subset \mathfrak{so}_3 \subset \mathfrak{so}_4$:

$$C_k = \frac{1}{2}\sum_{i,j=5-k}^{4} F_{i,j}^2. \tag{30}$$

which generate commuting subalgbra in $\mathfrak{so}_4$. The F-operators can be expressed with the Casimirs as

$$\begin{aligned}\mathbf{F}^{(1)}(u) &= \frac{u(u-1)}{(u-\frac{1}{2})^2}\frac{(u-\frac{n+3}{2})(u+\frac{n+1}{2})\left[(u-\frac{1}{2})^2+C_2\right]}{((u-1)^2-\frac{1}{4}+C_3)(u^2-\frac{1}{4}+C_3)},\\ \mathbf{F}^{(2)}(u) &= \frac{u-1/2}{u}\frac{u^2-\frac{1}{4}+C_3}{(u+\frac{1}{2}-iF_{3,4})(u-\frac{1}{2}-iF_{3,4})}.\end{aligned} \tag{31}$$

where expressions in the denominator denote the invertation.

5. Overlap formula

For the final overlap formula we only need to diagonalize the F-operators which are expressed with the Casimirs. The Casimirs are diagonal in the Gelfand-Tsetlin basis [31]. In the GT-basis, every basis vector corresponds to a GT-pattern. For the representations $(\frac{n+1}{4},\frac{n-1}{4})$ or $(\frac{n-1}{4},\frac{n+1}{4})$, the possible GT-patterns are

$$|k,l\rangle \Longleftrightarrow \begin{matrix} \frac{n}{2} & & \pm\frac{1}{2}\\ & k & \\ & & l\end{matrix}$$

where k,l are half integers between $\frac{1}{2}\leq k\leq\frac{n}{2}$ and $k-\leq l\leq k$. The Casimirs act diagonally on these basis vectors

$$\begin{aligned} C_3|k,l\rangle &= -k(k+1)|k,l\rangle,\\ C_2|k,l\rangle &= -l^2|k,l\rangle,\\ F_{3,4}|k,l\rangle &= il|k,l\rangle.\end{aligned} \tag{32}$$

The pseudo-vacuum overlap is

$$\mathbf{B} = X_1^L. \tag{33}$$

and the X_1^2 can be expressed as

$$X_1^2 = \frac{1}{4} - C_3. \tag{34}$$

For a vector $|k,l\rangle$ the eigenvalues are

$$\begin{aligned}\beta_{k,l} &= (k+1/2)^L,\\ \mathcal{F}^{(1)}_{k,l}(iu+\frac{1}{2}) &= \frac{u^2+\frac{1}{4}}{u^2}\frac{(u^2+\frac{(n+2)^2}{4})(u^2+l^2)}{(u^2+k^2)(u^2+(k+1)^2)},\\ \mathcal{F}^{(2)}_{k,l}(iu) &= \frac{u+i/2}{u}\frac{u^2+(k+1/2)^2}{(u-i(l+\frac{1}{2}))(u-i(l-\frac{1}{2}))}.\end{aligned} \tag{35}$$

Substituting back to the general overlap formula (25), our overlap can be expressed as

$$\frac{\langle \mathrm{MPS}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle}{\sqrt{\langle \mathbf{u},\mathbf{v},\mathbf{w}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle}} = 2\sum_{k=\frac{1}{2}}^{\frac{n}{2}}\sum_{l=-k}^{k}\left(k+\frac{1}{2}\right)^L\times \tag{36}$$

$$\prod_{j=1}^{N_u/2}\frac{(u_j^2+\frac{(n+2)^2}{4})(u_j^2+l^2)}{(u_j^2+k^2)(u_j^2+(k+1)^2)}\sqrt{\frac{u_j^2+\frac{1}{4}}{u_j^2}}\times \tag{37}$$

$$\prod_{j=1}^{N_v}\frac{v_j^2+(k+1/2)^2}{(v_j-i(l+\frac{1}{2}))(v_j-i(l-\frac{1}{2}))}\times\sqrt{\frac{\det G_+}{\det G_-}}. \tag{38}$$

6. Conclusion

We studied integrable matrix product states with $\mathfrak{so}_4$ symmetry for and proved their integrability by constructing a corresponding K-matrix. Using this we established an overlap formula for the fuzzy S^3 MPS with the Bethe states of the $\mathfrak{so}_6$ symmetric XXX chain for arbitrary bond dimensions. These matrix product states are related to nonsupersymmetric defect configurations of $\mathcal{N}=4$ SYM of various codimensions, and the overlap formulas compute leading order one point functions of non-protected single trace operators in the planar limit. One important question is to understand to what extend these defects make sense as quantum theories, since the classical analysis on the field theory is under less control relative to supersymmetric defects. On the other hand integrability is a much more constraining symmetry than supersymmetry, and so it is not unreasonable to expect non-supersymmetric defects to survive in the strong coupling regime. An example of this is the non-supersymmetric D3-D7 interface described by a fuzzy S^4 MPS [32,33] in which agreement with holography were achieved in a certain double scaled limit [34]. An important step would be to understand the holographic dual of the fuzzy S^3 defect, for instance be revisiting the classification of integrable boundaries for the string sigma model. One hint comes from the fact for the defect to be integrable in the complete bosonic sector of the theory, the pair structure for the Bethe roots must be compatible. The pair structure is fixed by the residual symmetry and for the fuzzy S^3 MPS it singles out the symmetry breaking patterns $SO(2,4) \to SO(2,2) \times SO(2)$ and $SO(2,4) \to SO(2) \times SO(4)$. This suggests that there is a nonsupersymmetric $D5$ configuration wrapping $AdS_3 \times S^3$ which is potentially integrable which would be the holographic dual description of the fuzzy S^3 surface defect. Similar configurations were introduced in [35].

The analysis presented here hints at a systematic construction of $\mathfrak{so}_{2n}$ symmetric integrable MPS. In general their construction relies on projections from $\mathfrak{so}_{2n+1}$ symmetric MPS which are more straightforward to construct. This projection procedure complicates the analysis since the algebraic relations between generators are somewhat obscured. An alternative approach would be to assume some kind regularity condition on the K-matrix and so solve the KT-relation perturbatively by finding commutation relations order by order. Relatedly one might try to find all possible integrable MPS in other related systems, such as the alternating chains arising from the ABJM theory or of deformations of the XXX chain.

Data availability

No data was used for the research described in the article.

Declaration of competing interest

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