

A theoretical study of diameter constraints in circular microphone array beamforming

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Summary

Acoustic microphone arrays and beamforming techniques are widely used for the analysis of rotating noise sources, such as those in turbomachinery. In this study, a circular microphone array is employed to investigate these sources, with a focus on the fundamental constraints of the measurement setup. The interdependent relationship between array diameter and measurement distance is examined, defining a theoretical measurement range. Two key limitations – spatial resolution and spatial aliasing – are identified, and practical formulas are derived to estimate the array diameter range for a given distance. These principles are demonstrated through a simple test case.

1. Introduction

The use of microphone arrays in combination with beamforming is a widely applied technique for obtaining not only the amplitude and spectral characteristics of noise sources but also their spatial location. The number and arrangement of microphones have a significant impact on the results. Among the various array geometries, the circular (or ring) arrangement is frequently employed, as it allows installation around ducts or free jets without disturbing the airflow [1]. Circular arrays are particularly well suited for the investigation of rotating noise sources [2]. The present study aims to determine the theoretical limits of the key parameters governing the measurement setup. The obtained theoretical range is then demonstrated through a simplified test case.

2. Theoretical limits of the measurement range

The theoretical measurement range is investigated through a simplified turbomachinery test case, shown in Fig. 1. For any given turbomachinery, the rotor diameter is fixed. The most important parameters of the measurement setup

are the diameter of the microphone array and the distance between the rotor and the array in the axial direction. These two parameters are interdependent, i.e., variations in rotor-array distance result in changes in the range of array diameter, and vice versa.

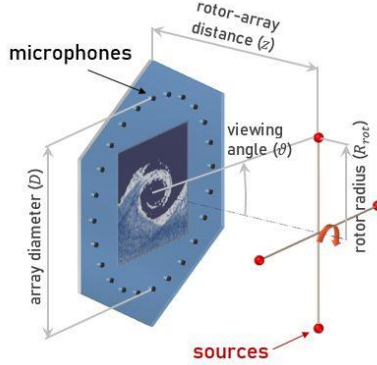


Figure 1. A simplified model of the turbomachinery measurements test case, investigated using a microphone array.

2.1 Limit resulting from the spatial resolution capabilities of a microphone array

Microphone arrays and beamforming techniques, like other imaging methods, are subject to limitations in spatial resolution. According to the Rayleigh criterion, the resolution ρ for on-axis incidence of spatially incoherent waves can be expressed as [3]:

$$\rho = \alpha_R \frac{z}{D} \lambda, \quad (1)$$

where z denotes the rotor-array distance, D is the array diameter, and λ is the wavelength of the incoming wave. The array geometry is accounted for by the shape factor α_R , which takes a value of approximately 1.22 for a circular array. This formulation is applicable to the test case illustrated in Fig. 1, providing a lower limit for the microphone array diameter range.

2.2 Limit resulting from spatial aliasing on beamforming maps

Due to the discrete sampling of incoming acoustic waves, the issue of spatial aliasing arises. According to the Nyquist criterion, spatial aliasing can be avoided if the spacing between adjacent microphones does not exceed $\lambda/2$ [4]. By narrowing the field of view ϑ from the full angular range of $|\pm\pi/2|$ to the

region encompassing the actual noise sources $|\pm \vartheta_s|$ (see Fig. 1), aliasing can be avoided under the condition:

$$|\Delta x \sin(\vartheta_s)| < \lambda/2 \quad (2)$$

where Δx is the distance between two neighboring microphones. By expressing Δx and ϑ_s in terms of the array diameter D , the rotor-array distance z , and the radius of the sources' path R_{rot} , this criterion can be adapted to the test case under consideration. It thereby provides an upper limit on the microphone array diameter range for a fixed number of microphones.

2.3 Determination of the theoretical measurement range of the microphone array diameter with respect to the rotor-array distance

After substitutions and expressions from Eq. (1) and (2), the array diameter D has the following theoretical range:

$$\frac{\alpha_{R_{rot}} \frac{z}{R_{rot}}}{2 \sin\left(\frac{\pi}{N}\right)} \lambda < D < \frac{\sqrt{\left(\frac{z}{R_{rot}}\right)^2 + 1}}{2 \sin\left(\frac{\pi}{M}\right)} \lambda \quad (3)$$

where N is the number of evenly distributed sources, and M is the number of microphones. The theoretical measurement range is visually presented in Fig. 2.

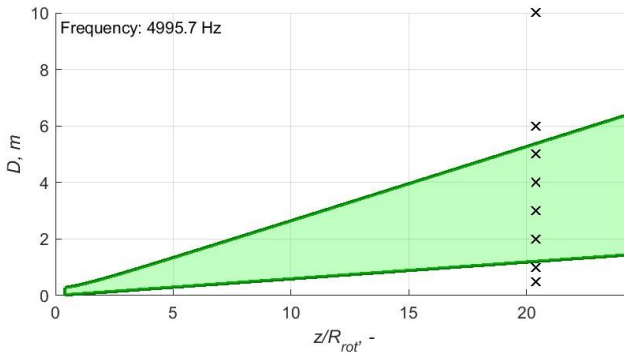


Figure 2. The theoretical measurement range of a circular array of 24 microphones, for a frequency of 4995.7 Hz

The green area shows the ideal range of the array diameter with respect to the rotor-array distance. Although the diameter value is frequency-dependent, the characteristics of the diagram remain invariant with respect to frequency. The measurement distance is normalized by the rotor radius R_{rot} . A lower limit has

also been shown for the measurement distance based on the constraint that the microphones would be in the acoustic far-field of the sources, that is [5]:

$$kz \gg 1 \quad (4)$$

where $k = 2\pi/\lambda$ is the wavenumber.

3. Demonstration of the theoretical measurement range

Fig. 3 presents a series of synthetic beamforming results that demonstrate the effectiveness of the derived formulas. The configuration of the test case is shown in Fig. 1. Four incoherent simple point sources (marked by black squares in the images) emit white noise while rotating around the center of the image at a radius of 0.245 m and a rotational speed of 50 rot/s . The trajectory of the sources is indicated by a black circle in the images. The propagation of the sound is modelled analytically by the convected wave equation [6]. The microphone array, consisting of 24 microphones, is positioned axially aligned with the rotational axis at a distance of 5 m from the rotational plane. For each case, a 30 s recording has been taken at a sampling rate of 44100 Hz . Signal processing has been performed using the Virtually Rotating Array (VRA) beamforming algorithm [2], which compensates for the rotational motion of the sources, causing them to appear stationary in the beamforming output. The beamforming code has been implemented in-house and has been employed in other studies conducted by our research group [1,7]. The recorded signals have been processed and analysed for multiple frequency bins, but here only a single frequency bin, centered at 4995.7 Hz , is presented. The beamforming results have been computed on a 121×121 point grid, uniformly distributed over an area of $1\text{ m} \times 1\text{ m}$. Beamforming results are considered accurate if they exhibit high amplitudes near the true source locations and low amplitudes elsewhere. In this investigation, the array diameter has been systematically varied. The selected diameters, indicated by black $\times$ -marks in Fig. 2, include two values below and two above the theoretical optimal range, with four values within it. In Fig. 3, the corresponding array diameters are noted in the top-left corner of each image. In order to provide a clearer representation of both the peak amplitudes and the sidelobe structure, the results are presented over a broad dynamic range of 15 dB .

Examining the results at $D = 0.5\text{ m}$ and $D = 1\text{ m}$ reveals the effects of spatial resolution. In these cases, the noise sources are either not resolved or only partially separated, making them difficult to distinguish. This observation aligns with the theoretical lower limit of 1.21 m estimated by the derived formula. In

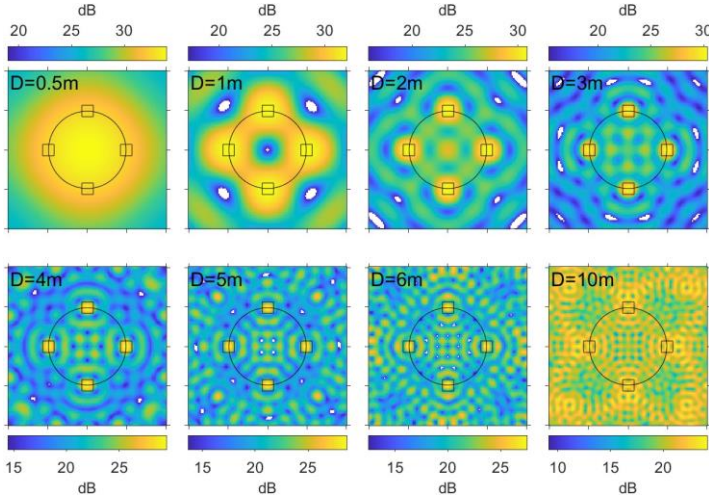


Figure 3. Beamforming results of four rotating point sources emitting white noise, evaluated at 4995.7 Hz.

contrast, the subsequent four beamforming maps – corresponding to $D = 2\text{ m}$, 3 m , 4 m , and 5 m – successfully localize the noise sources and exhibit a mainlobe-to-sidelobe ratio of at least several dB in the vicinity of each source. However, at $D = 5\text{ m}$, high sidelobe levels are observed in the four corners of the map. While these sidelobes are somewhat removed from the region of interest around the rotor, they are undesirable. This effect becomes more pronounced at $D = 6\text{ m}$, consistent with the theoretical upper limit of 5.38 m . The degradation continues at $D = 10\text{ m}$, where the beamforming output no longer displays distinct peaks at the true source locations, further confirming the validity of the derived theoretical limits.

4. Conclusions

In this study, the limits of circular microphone arrays for measuring rotating noise sources have been examined. For a simplified turbomachinery test case, the interdependent limits of microphone array diameter and rotor-array distance have been derived theoretically, based on the constraints imposed by spatial resolution and spatial aliasing. The theoretical estimations have then been validated through beamforming results obtained across a wide range of array diameters at a given measurement distance. The findings confirm that the range identified as optimal through examining the beamforming maps closely aligns with the range estimated by the derived formulas. This agreement supports the

practical applicability of the theoretical measurement range for guiding the design of measurement setups employing circular microphone arrays for rotating noise sources.

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