



Bending resistance of welded hybrid slender I-section beams

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ABSTRACT

Hybrid steel beams, in which plates of different steel grades are combined – with higher-strength steel applied in the flanges – offer an efficient solution for increasing load-carrying capacity while optimizing material use and cost efficiency. The research thoroughly investigates the bending behavior of I-section hybrid beams across a wide range of geometric configurations. The primary objective of the paper is to address the lack of analytical methods in current literature for determining the resistance of cross-sections featuring Class 4 flanges and webs. The applicability of existing design proposals is rigorously assessed through detailed numerical analyses. Numerical simulations are performed, encompassing a broad spectrum of slenderness and hybrid ratios. The simulations are based on a sophisticated numerical model that has been validated with previous laboratory experimental data (from the literature). The numerical results obtained are then used to evaluate the accuracy of various analytical formulas for the bending resistance calculation. Furthermore, the study specifies the necessary partial factor (γ_{M^*}) for the different formulas. Ultimately, this research aims to analyze and describe the behavior of slender beams under bending, which could lead to more reliable and economical design solutions for hybrid steel structures in the future.

Notations

$b_{uf,eff}$ Effective width of the compressed flange [mm].
 f_{af} For cross-section class 4, the tensile stress developed in the outer edge line of the flange of the effective (not doubly symmetric) section; for classes 1–3, it corresponds to the yield strength of the flange [MPa].
 f_{aw} For cross-section class 4, the tensile stress developed in the outer edge line of the effective (not doubly symmetric) section web; for classes 1–3, it corresponds to the yield strength of the web [MPa].
 f_{yf} Yield strength of the flange [MPa].
 f_{yw} Yield strength of the web [MPa].
 f_{1w} Compressive stress occurring in the web at the end of the linear stress distribution according to Fig. 13 in the case of Class 4 web with partially plastic stress distribution [MPa]

$$f_{1w} = \min\left(\frac{f_{yf}}{\gamma_{G,eff} + t_{fu}} \cdot (h_{w,1,eff} - h_a) : f_{yw}\right)$$

 f_{2w} Compressive stress in the web at the line closer to the pressure of the missing web section, according to Fig. 13 [MPa] $f_{2w} = \min\left(\frac{f_{yf}}{\gamma_{G,eff} + t_{fu}} \cdot (h_{w,1,eff} + h_{out} - h_a) : f_{yw}\right)$

f_{3w} Compressive stress in the compressed edge line of the web, according to Fig. 13 [MPa] $f_{3w} = \min\left(\frac{f_{yf}}{\gamma_{G,eff} + t_{fu}} \cdot \gamma_{G,eff} : f_{yw}\right)$
 h_a Length of the tensioned web section [mm].
 h_b Length of the compressed web section without the plastic part [mm].
 h_c length of compressed web section, linear stressdistribution, or $\max(h_{w,1,eff} - h_a)$ [mm].
 h_{out} length of the missing compressed web section for class 4 [mm].
 $h_{f,l}$ distance between the center line of the lower flange and the center of gravity of the effective cross-section [mm]. $h_{f,l} = h_w - \gamma_{G,eff} + \frac{t_{fu}}{2}$
 $h_{f,u}$ distance between the center line of the upper flange and the center of gravity of the effective cross-section [mm] $h_{f,u} = \gamma_{G,eff} + \frac{t_{fu}}{2}$
 $h_{w,pl,u}$ length of the compressed, plastic web section without the missing web section, [mm] $h_{w,pl,u} = \max(\gamma_{G,eff} - h_b - h_{out}; 0)$
 $h_{w,1,eff}$ length of the lower tensioned and compressed web section up to the missing web section [mm], according to Fig. 13.

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$h_{w,u,eff}$	length of the upper compressed web section up to the missing web portion [mm], according to Fig. 13.
$M_{R,pl}$	plastic bending resistance [kNm] according to EN1993-1-1 [8].
$M_{R,eff,el}$	elastic bending resistance with effective flange and web [kNm] according to EN1993-1-1 [8].
$M_{R,eff,mod}$	proposed elastic-plastic bending resistance with effective flange and web [kNm]
$M_{R,eff,simpl}$	simplified elastic-plastic bending resistance with effective flange and web [kNm] $M_{R,eff,simpl} = M_{R,el,f,mod} + M_{R,w,simpl}$
$M_{R,EC}$	bending resistance [kNm] according to EN1993-1-1 [8] for homogen sections.
$M_{R,VeljKovic}$	bending resistance [kNm] according to Veljkovic [7].
$M_{R,Zhu}$	bending resistance [kNm] according to Zhu et al. [11].
$M_{R,num,G}$	bending resistance [kNm] according to GMNI analysis using Gardner material model detailed in Section 4.3.
$M_{R,num,RO}$	bending resistance [kNm] according to numerical calculations using Ramberg-Ostgood material model detailed in Section 4.3.
t_{fl}	Thickness of the lower flange [mm].
t_{fu}	Thickness of the upper flange [mm].
t_w	Thickness of the web [mm].
W_{el}	elastic section modulus about the major axis [mm ³].
W_{pl}	plastic section modulus about the major axis [mm ³].
$W_{el,f}$	elastic section modulus associated with the flanges according to Zhu et al. [11] [mm ³].
$W_{el,w}$	elastic section modulus associated with the elastic portion of the web according to Zhu et al. [11] [mm ³].
$W_{pl,w}$	plastic section modulus associated with the plastic portion of the web according to Zhu et al. [11] [mm ³].
$y_{G,eff}$	For cross-section class 4, the distance of the effective (not doubly symmetric) section's center of gravity from the lower line of the compressed flange [mm].

1. Introduction

Efficiency and economy are paramount in structural design, representing a fundamental challenge for the construction industry. In recent decades, numerous innovations, initially perceived as unconventional, have become integral to everyday design practice. Steel hybrid beams, enabled by advancements in manufacturing capabilities and the emergence of high-strength steel, exemplify this evolution, offering significant opportunities for improved material efficiency and sustainability.

High-strength steels (HSS), defined by a nominal yield strength of 460 MPa or greater, are increasingly common in civil engineering practice [1,2]. Alongside homogeneous HSS beams, hybrid structures that combine normal strength steel (NSS) with HSS are gaining prevalence. These hybrid beams are particularly advantageous because they strategically position each material to maximize its unique strength benefits. For instance, in traditional steel I-beams, flanges primarily resist bending moments while webs handle shear forces. Hybrid beams capitalize on this by employing higher-grade steel in the flanges while retaining normal strength steel in the web. This arrangement ensures optimal bending resistance and adequate shear capacity.

Compared to normal strength steel (NSS), high-strength steel (HSS) enables the design of structural elements with smaller cross-sections, leading to significant weight reduction and more slender structures. In addition to material savings, the reduced structural weight leads to lower demands on supporting elements, such as columns and foundations, due to the smaller loads transmitted to them [3]. Furthermore, lighter structures can contribute to reduced transportation demands, shorter construction times, and overall cost savings. Additional benefits include reduced welding and painting requirements, contributing to lower CO₂ emissions and energy consumption [4,5]. The strategic use of HSS in NSS-HSS beams, achieved through optimal material placement,

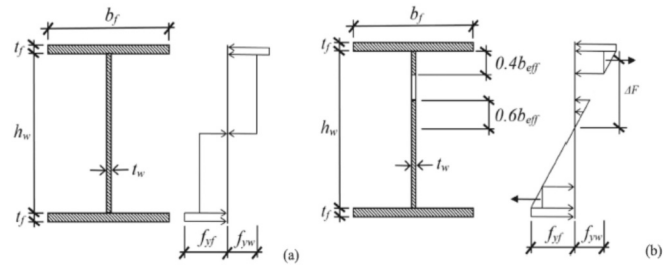


Fig. 1. Stress distribution in hybrid beams under bending (a) for cross-section classes 1 and 2 (b) for cross-section class 4 [7].

provides an economical and highly efficient solution [6,7].

To advance the adoption of sustainable and innovative steel structures, a profound understanding of their physical behavior, as well as the development and validation of accurate and safe manual calculation methods, is crucial. Consequently, this study employs detailed numerical parametric analysis to investigate the bending behavior of hybrid beams across a wide range of slenderness ratios and material combinations. The numerical results are compared with existing literature to draw comprehensive conclusions. Importantly, a novel manual calculation method is proposed for Class 4 cross-section beams, addressing the current knowledge gap in international literature. The reliability and accuracy of various design formulas are thoroughly evaluated, and the necessary partial factor for their application is determined.

2. Literature review

Currently, the use of hybrid beams is still quite limited in civil engineering practice, as EN 1993-1-1 [8] does not include a design approach for hybrid beams. The concept of hybrid steel beams dates back to the 1940s [6,7], although little research was conducted on the subject until the early 2000s, and during this period their application and study was largely limited to the United States. With the development of new high-strength structural steels (HSS) in the 2000s, and especially since the 2010s, interest in hybrid beams has grown, resulting in a significant increase in research activities, particularly in Asia and Europe [9-17]. One of the pioneering studies on bending strength was conducted by Veljkovic and Johansson [7], who observed that the difference in yield strength between the flanges and the web significantly affects bending strength. They concluded that the web partially flows before the flanges reach the stress level corresponding to the yield point. The consequences of this phenomenon differ for different cross-section classes, which has led to the development of separate design models for each cross-section classes. For sections classified as Class 1 or 2, complete plasticization of the cross-section was assumed when calculating the bending resistance, as shown in Fig. 1.

In Class 3, elastic resistance is calculated in the case of homogeneous sections. While in hybrid beams, the web (due to its lower yield strength) may partially yield before the stress in the outer edge of the flange reaches the yield point, and the significance of this effect depends on the cross-section class. For Class 3 this partially plastic behavior can be incorporated into the calculation of the bending resistance. For Class 4 cross-sections, a separate stress distribution model must be used that takes into account the partial plasticity of the web, even with local buckling. The elastic bending resistance is given by eqs. (1)–(4):

$$M_{Rk,el,VeljKovic} = (W_{eff} - \Delta W) \cdot f_{yf} \quad (1)$$

where, if $h_w/t_w > 124\epsilon$

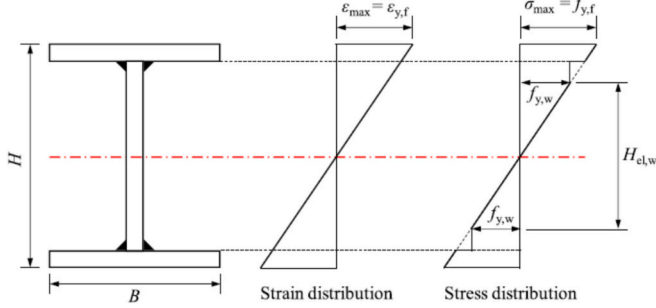
$$W_{eff} = W \cdot \left[1 - 0.1 \frac{A_w}{A_f} \left(1 - 124\epsilon \frac{t_w}{h_w} \right) \right] \quad (2)$$

otherwise, W is the elastic cross-sectional modulus relative to the centre

Table 1

Cross-section class limits according to EN 1993-1-1 [8] and Zhu et al. [11].

	1	2	3
$b_f/t_f/\epsilon_f$	9 (Zhu et al.: 8)	10	14
$h_w/t_w/\epsilon_w$	72 (Zhu et al.: 60)	83	121

**Fig. 2.** Stress distribution in hybrid beams in elastic calculations [11].

line of the flange.

$$W_{eff} = W = \frac{I_y}{y_G + \frac{t_f}{2}} \quad (3)$$

$$\Delta W = \frac{h_w A_w \cdot \left(1 - \frac{f_{yw}}{f_{yf}}\right)^2 \left(2 + \frac{f_{yw}}{f_{yf}}\right)}{12} \quad (4)$$

where I_y is the inertia, y_G is the distance of the centre of gravity from the inner edge of the flange and t_f is the thickness of the flange. However, Veljkovic and Johansson [7] provided the formula only for cases where the flange is not of cross-sectional class 4 and the section is doubly symmetrical.

Based on further research by Lateef et al. [18], a slenderness ratio limit of $h_w/t_w = 120$ was recommended for Class 4 sections. Zhu et al. [11] conducted experimental and numerical investigations on the bending strength of Class 1–3 beams, covering both homogeneous high-strength steel (HSS) and hybrid beams. Their work included an extensive parametric study using numerical simulations, which confirmed that the existing slenderness limits specified in EN 1993-1-1 [8] for class 2 and 3 sections are appropriate for classifying the behavior of compressed flanges and web members subjected to bending in homogeneous HSS and hybrid welded I-sections. To comply with the Eurocode requirements for rotation capacity, stricter limits were proposed for class 1 members, recommending a value of $b_f/t_f/\epsilon_f = 8$ for flanges subjected to compression and a value of $h_w/t_w/\epsilon_w = 60$ for internal webs subjected to bending. Table 1 summarizes the limits for flanges and webs according to EN 1993-1-1 [8] and the modification by Zhu et al. [11].

Zhu et al. [11] developed an analytical formula (Eq. (5)) for the elastic bending load capacity calculation of hybrid beams. The elastic and plastic cross-sectional moduli in the formula are summarized in Eqs. (6)–(9).

$$M_{el} = W_{elf} \cdot f_{yf} + W_{elw} \cdot f_{yw} + W_{plw} \cdot f_{yw} \quad (5)$$

$$W_{elf} = \frac{B \cdot t_f^3 / 3 + B \cdot t_f \cdot (H - t_f)^2}{H} \quad (6)$$

$$W_{elw} = \frac{1}{6} \cdot t_w \cdot H_{elw}^2 \quad (7)$$

$$W_{plw} = 2 \cdot t_w \cdot \left(\frac{H}{2} - \frac{H_{elw}}{2} - t_f\right) \cdot \left(t_f - \frac{H}{2} + H_{elw}\right) \quad (8)$$

Table 2

Parameter range examined.

Property [unit]	Range
Yield strength of the flanges [MPa]	235–960
Yield strength of the web [MPa]	235–960
Web thickness t_w [mm]	4–35
Web height h_w [mm]	300–1000
Flange width b_f [mm]	300
Flange thickness t_f [mm]	6–45
Length a [mm]	1500

$$H_{elw} = \frac{f_{yw}/E_w}{f_{yf}/E_f} \cdot H \quad (9)$$

The stress distribution used to develop the analytical model is shown in Fig. 2.

Zhu et al. [11] also developed a numerical model consisting of 4-node quadrilateral shell elements and performed numerical research program using GMNI analysis. A total of 400 virtual tests were examined within the parametric study. The flange width was kept constant (100 mm), while the height of the test specimens varied between 100 and 300 mm. The thickness of the flange plates varied between 3.4 and 17 mm, while the thickness of the web plate was always 0.6 times the thickness of the flange. Five material combinations were used: homogeneous S355, homogeneous S460, homogeneous S690, hybrid S690-S460, and hybrid S690-S355. The laboratory tests they conducted are used to validate the numerical model.

International literature, therefore, examines and analyzes hybrid beams in detail, and research on them is advanced in many respects. However, there is a complete lack of research on Class 4 beams, and traditional formulas are not applicable to them. This paper aims to fill this gap.

3. Research strategy

3.1. Research process, characteristics examined

A series of detailed numerical investigations are performed to verify the analytical-based design formulas and analyze the behavior of hybrid beams, and the results are evaluated. A geometrically and materially nonlinear imperfect model (GMNIA) is used to describe the complex nonlinear behavior of hybrid beams. This approach allows for a more accurate estimation of bending resistance compared to simplified analytical or linear methods, as it considers the interaction of geometric errors, material yield strength, post-bending effects, and structural instability. Despite certain approximations in the modelling of material behavior and geometric errors, GMNIA has consistently proven its effectiveness in accurately predicting bending resistance, making it a reliable tool for structural stability analysis. A parametric study is performed using a validated finite element model, and a wide range of beam configurations is analysed. The current parametric study covers (i) different cross-section classes (web and flange cross-section classes 1–4) and (ii) different hybrid ratios ($\phi_h = f_{yf}/f_{yw} = 1.3$ –4.1). Homogeneous sections are also examined as a reference for each cross-section class. The parameter range examined is summarized in Table 2, the length and width of the beams are kept constant. A total of 1290 numerical simulations are carried out, of which 890 relate to hybrid beams and the rest 400 studied homogeneous cross-sections. This strategy made the direct comparison of hybrid and non-hybrid beams possible. The results of the numerical parametric study are compared to the available design formulas from literature, and a new one is also developed. Statistical evaluations are used to check the accuracy of each method. Finally for each design method the accurate γ_{M^*} is determined to check its adequacy according to the Eurocode-based safety requirements.

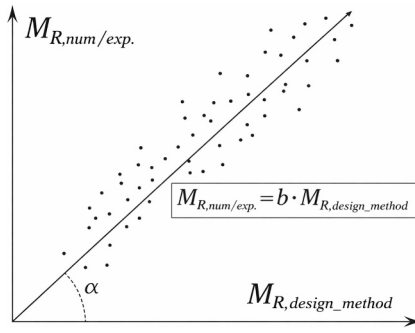


Fig. 3. Comparison of experimental and analytical bending resistances, [31].

3.2. Statistical evaluation of resistance models (EC1990 annex D chapter 8)

The determination of the partial factor is carried out according to EN1990 Annex D Chapter 8 [34] in the following steps.

1. Development of the design model – different design models are evaluated and compared.
2. Comparison of experimental and theoretical values, as shown in Fig. 3. In the current study, the numerical results (virtual experiments) are considered as accurate experimental values and the analytical-based design resistances are considered as theoretical values. The vertical axis shows the numerically calculated bending resistances ($M_{R,num,G}$). The horizontal axis presents the bending resistances calculated by different analytical design methods ($M_{R,design_method}$).

4. Estimation of the mean value correction factor b

If the resistance function produced values identical to the experimental or numerical results, all points would lie on a straight line with a slope of 45° ($b = 1.00$). In practice, the points exhibit scatter, which reflects the variability and errors of the individual results. Typically, the data are distributed around a line whose slope differs from 45° . The parameter b accounts for this systematic deviation.

$$b = \frac{\sum M_{R,num} \cdot M_{R,design_method}}{\sum M_{R,design_method}^2} \quad (10)$$

5. Estimation of the coefficient of variation of the errors

If the gradient of the mean line (b) is known, errors of all results can be calculated and statistically evaluated, as detailed in the following:

$$\text{error term for each value : } \delta_i = \frac{M_{R,num,i}}{b \cdot M_{R,design_method,i}} \quad (11)$$

$$\text{logarithm of the error terms : } \Delta_i = \ln(\delta_i) \quad (12)$$

$$\text{mean value of logarithm of the error terms : } \bar{\Delta} = \frac{1}{n} \sum_{i=1}^n \Delta_i \quad (13)$$

$$\text{standard deviation of the error terms : } s_{\Delta}^2 = \frac{1}{n-1} \sum_{i=1}^n (\Delta_i - \bar{\Delta})^2 \quad (14)$$

$$\text{coefficient of variation of the error terms : } V_{\delta} = \sqrt{\exp(s_{\Delta}^2) - 1} \quad (15)$$

6. Analysis of compatibility

It should be verified that the mean value, standard deviation, and coefficient of variation remain approximately constant over the entire

Table 3

Coefficients of variation of the basic variables (geometric and material properties) based on JCSS [19] and EN1993-1-1 [8].

	f_y	h_w	t_w	b_f	t_f	f_u
V_x	0.05	0.009	0.05	0.009	0.05	0.04

analysed parameter range. This ensures that the resistance model provides uniform accuracy throughout the range. If significant differences are observed, the parameter with the greatest influence on the scatter should be identified and examined, and the test results may be divided into subsets according to this parameter.

7. Determination of the coefficients of variation (V_{xi}) of the basic variables

The investigated resistance model depends on different variables. The coefficient of variation of the basic variables in the resistance functions should be determined.

According to the proposal of JCSS [19] and EN1993-1-1 [8], the coefficients of variation (V_{xi}) of the basic variables are chosen as shown in Table 3, assuming a more conservative larger CoV (where the suggestion is not consistent).

8. Determination of the characteristic and design value of the resistance model

From the product of coefficient of variations given in point 4 and 6 the coefficient of variation of the whole resistance model can be calculated by Eq. (16).

$$V_r^2 = (V_{\delta}^2 + 1) \cdot \left[\prod_{j=1}^n (V_{xi}^2 + 1) \right] - 1 \quad (16)$$

Alternatively, for small values of V_{δ}^2 and V_{xi}^2 the approximation of Eq. (17) can be used. In the paper this method is used.

$$V_r^2 = V_{\delta}^2 + \sum_{i=1}^j V_{xi}^2 \quad (17)$$

If a large number of tests ($n > 100$) are available, the characteristic and design resistances can be obtained from Eqs. (18)–(20).

$$M_{R,k} = b \cdot M_{R,design_method} \cdot e^{-k_{\infty} \cdot Q - 0.5 \cdot Q^2} \quad (18)$$

$$M_{R,d} = b \cdot M_{R,design_method} \cdot e^{-k_{d,\infty} \cdot Q - 0.5 \cdot Q^2} \quad (19)$$

where

$$Q = \sqrt{\ln(V_r^2 + 1)} \quad (20)$$

k_{∞} is the value of k_n for $n \rightarrow \infty$ ($k_{\infty} = 1.64$),

$k_{d,\infty}$ is the value of $k_{d,n}$ for $n \rightarrow \infty$ ($k_{d,\infty} = 3.04$).

The partial factor can be determined according to Eq. (21).

$$\gamma_{M1} = \frac{M_{R,k}}{M_{R,d}} = \frac{b \cdot M_{R,design_method} \cdot e^{-k_{\infty} \cdot Q - 0.5 \cdot Q^2}}{b \cdot M_{R,design_method} \cdot e^{-k_{d,\infty} \cdot Q - 0.5 \cdot Q^2}} \quad (21)$$

The partial factors γ_M according to EN1993-1-1 [8] are derived from γ_{M^*} (i.e. from the ratio of the nominal and design resistance values, not from the ratio of the characteristic and design resistance values). According to Kövesdi et al. [20], the partial factor γ_{M^*} required for design can be calculated as the product of the theoretical partial factor γ_M and a correction factor (Δk) using Eqs. (22)–(23).

$$\gamma_{M^*} = \Delta k \cdot \gamma_M \quad (22)$$

where:

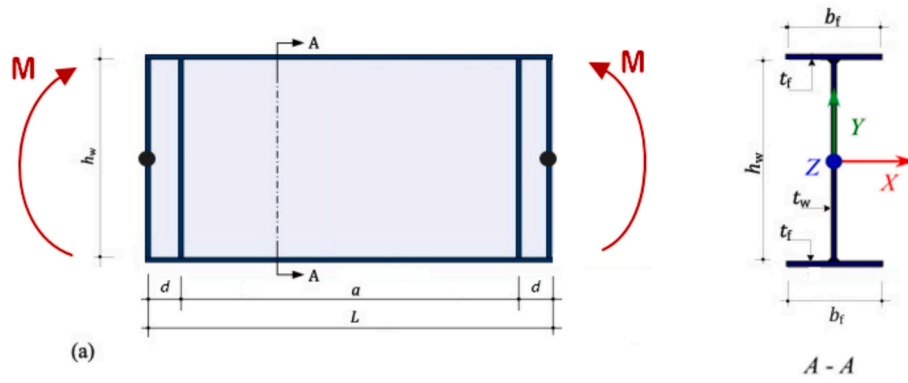


Fig. 4. The type of beam investigated, its loading, and the markings.

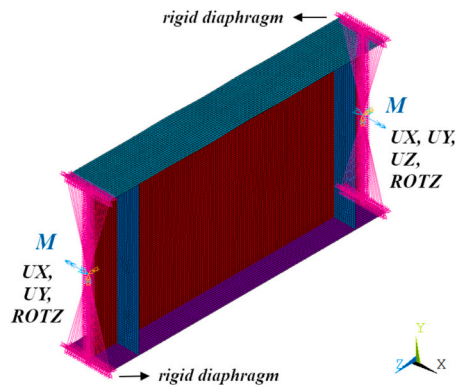


Fig. 5. Numerical model with the finite element mesh, boundary and load conditions.

$$\Delta k = \frac{r_{nom}}{r_k} = \frac{\exp(-2Q_{fy} - 0.5Q_{fy}^2)}{b \bullet \exp(-k_{\infty}Q - 0.5Q^2)} = \frac{0.867}{b \bullet \exp(-1.64Q - 0.5Q^2)} \quad (23)$$

where Q_{fy} is equal to 0.07 based on JCSS recommendations [19].

The modification factor takes into account that, in most cases, the parameters are used at their nominal values to determine the characteristic values, rather than at the lower quantile of 5%.

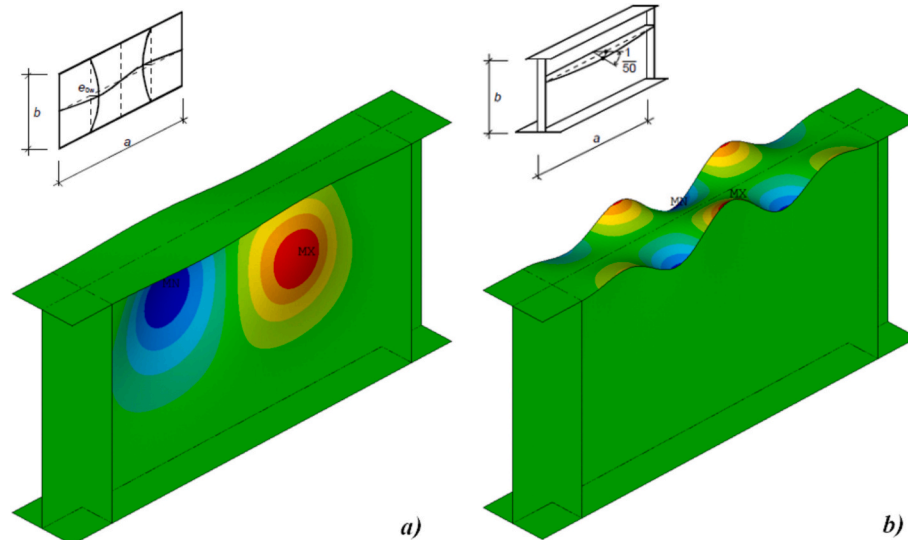


Fig. 6. Applied imperfection shapes (a) for web, (b) for flanges.

9. Numerical model development

9.1. Description of the numerical model and experiments used for validation

The experimental results used to validate the FE-model was published by Zhu et al. [11]. In the experiment, three-point bending tests were performed on homogeneous S690 and hybrid S690-S355 beams. Three different sections and a total of six experiments were performed. The support points were located 3000 mm apart, rotation was allowed, and the load was applied at the midspan. Diaphragms were placed in the cross-sections of the force applications to prevent the web plates from buckling. Lateral supports against rotation were placed at intervals of $\sim L/7$ and were designed to support the spine plates. In order to determine the exact strength of the specimens, 2–2 test specimens were measured, and the structural dimensions were accurately measured before the experiment so that these values could be used during the model validation.

Based on the laboratory tests, a numerical model is developed using Ansys 19.2 finite element software [21]. The model is verified by checking the mathematical and geometric adequacy of the finite elements and validated by comparing it with the test results. The numerical model used in the paper is a simply supported beam, which is subjected to concentrated bending moments (with opposite signs) at both ends. The beam is supported against displacement and torsion. The end cross-section nodes are joining to one node (this is subject to concentrated

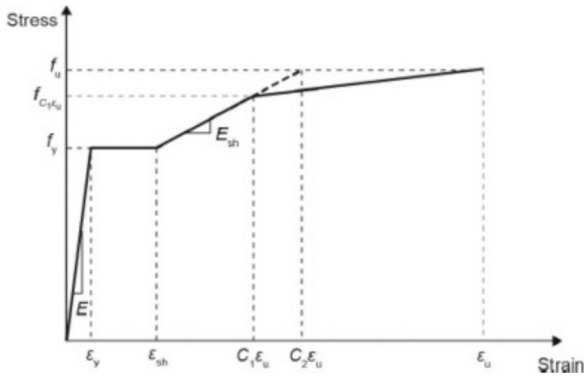


Fig. 7. Material model for steels with yield plateau acc. to EN 1993-1-14 [22].

Table 4

Material properties (strain values belong to the Gardner's material model) [8].

	E	f_y	f_u	f_u/f_y	ϵ_u	ϵ_{sh}
	[MPa]	[MPa]	[MPa]	[—]	[—]	[—]
S235	210,000	235	360	1.53	0.21	0.015
S355	210,000	355	490	1.38	0.17	0.017
S460	210,000	460	540	1.17	0.09	0.030
S690	210,000	690	770	1.12	0.06	0.030
S960	210,000	960	1115	1.16	0.08	0.030

bending moment), they behave like a rigid diaphragm. Fig. 4 shows the geometry and the applied notations of the beam's geometry.

In the parametric study, the span length is 1500 mm and each test specimen has a diaphragm at a distance of $0.2 \cdot h_w$ from the end of the support, and the plate thickness is $h_w/25$. The developed model is a full shell model using four-node thin shell elements (Shell181), as shown in Fig. 5, together with the finite element mesh, boundary and load conditions. The bending resistances are determined by geometrically and materially nonlinear imperfect analysis.

9.2. Applied imperfections

Equivalent geometric imperfections are used in the numerical model. Fig. 6 shows the imperfection shapes for the web and flange, which are applied with amplitudes of $h_w/200$ and $b_f/2 \cdot 1/50$ (leading to $b_f/100$), respectively, in accordance with the standardized values given in EN 1993-1-14 [23] and EN 1993-1-5 [22].

9.3. Material model

It is well known that high-strength steels exhibit mechanical behavior different from that of normal-strength steel girders. In particular, the linear behavior of the stress-strain relationship ends before the yield strength is reached, and a distinct yield plateau is typically absent [24]. However, several recent studies have shown, that depending on the manufacturing process, the stress-strain curve of HSS steel grades can be similar to that of conventional steel grades [25–28]. Therefore, this study considers two material models for nonlinear analysis of HSS members. The first is the multilinear material model proposed by Gardner et al. [29,30], which is shown in Fig. 7. This multilinear material model can be defined by three basic parameters: elastic modulus (E), yield strength (f_y), and tensile strength (f_u). In addition, we also examine the case of high-strength steels with a Ramberg-Osgood-type material model and compare the two sets of results.

As a second modelling approach for HSS materials, the numerical simulations adopt a Ramberg-Osgood-type constitutive model [23], which represents nonlinear elastic-plastic behavior including strain hardening. The formulation defined by Eq. (24) is applied up to a stress

Table 5

Data of test specimens used for validation.

Name	a [mm]	h_w [mm]	t_w [mm]	$b_{f,u}$ [mm]	$t_{f,u}$ [mm]	f_{yf} [mm]	f_{yw} [mm]
T1	3000	99.51	8.45	64.42	8.40	782.5	782.5
T2	3000	99.70	8.42	62.22	8.40	782.5	782.5
T3	3000	119.85	8.43	79.59	8.41	782.5	782.5
T4	3000	119.87	8.44	79.42	8.44	782.5	782.5
T5	3000	119.92	8.08	79.71	8.40	782.5	404.1
T6	3000	119.91	8.18	79.61	8.37	782.5	404.1

level of f_y . Beyond this point, the response of HSS grades is assumed to be linear, characterised by a reduced modulus of $E/1000$ up to a maximum strain of $\epsilon = 10\%$ according to Somodi [32]. The yield strength (f_y) is taken as the nominal yield strength of the corresponding steel grade, while a constant Young's modulus of $E = 210$ GPa is used throughout the numerical analyses. Material properties for HSS are shown in Table 4.

$$\epsilon = \frac{\sigma}{E} + 0.002 \left(\frac{\sigma}{f_y} \right)^n \quad (24)$$

9.4. Validation of the numerical model

Table 5 summarizes the basic data of the modelled experimental test specimens based on the experimental research program of Zhu et al. [11].

The validation results for the two material models are summarized in Table 6. The numerical results and experimental load capacities show good agreement (difference $< 3\%$). The Gardner's material model shows a smaller deviation on average, but in the case of hybrid beams, it is larger than by using Ramberg-Osgood-type material model.

Fig. 8 shows the failure mode and longitudinal stress diagram for one case of the numerical model (plate buckling).

10. Analysis of results and design recommendations for hybrid welded I-section beams

10.1. Influence of material model and hybrid ratio

To accurately evaluate the numerical results, the test specimens are classified into groups according to Table 7. It uses the EN1993-1-1 [8] cross-sectional classification procedure and Zhu et al.'s proposal [11] for modifying the limit value between cross-section class 1 and 2 (flange: 8 instead of 9, web: 60 instead of 72). However, the cross-section classification differs from the proposal by Zhu et al. [11] in that the classification of the web is performed using ϵ_f (similar to Veljkovic [7]). Thus, the same web belongs to a higher class in the case of high hybridity than in the case of a homogeneous or low hybridity section.

For category "A", all literature recommendations calculate with plastic bending resistance. The proposals by Zhu et al. [11] and Veljkovic [7] both apply to categories "B" and "C", but not to cross-sectional class 4. Table 8 and Fig. 9 show that there is no visible difference in the standard deviation or accuracy of the results when the steel grade is changed, and that modifying the hybrid ratio also has no significant effect. The Table and diagram only contain results that are within the validity range, i.e. not including cases for Class 4 flange.

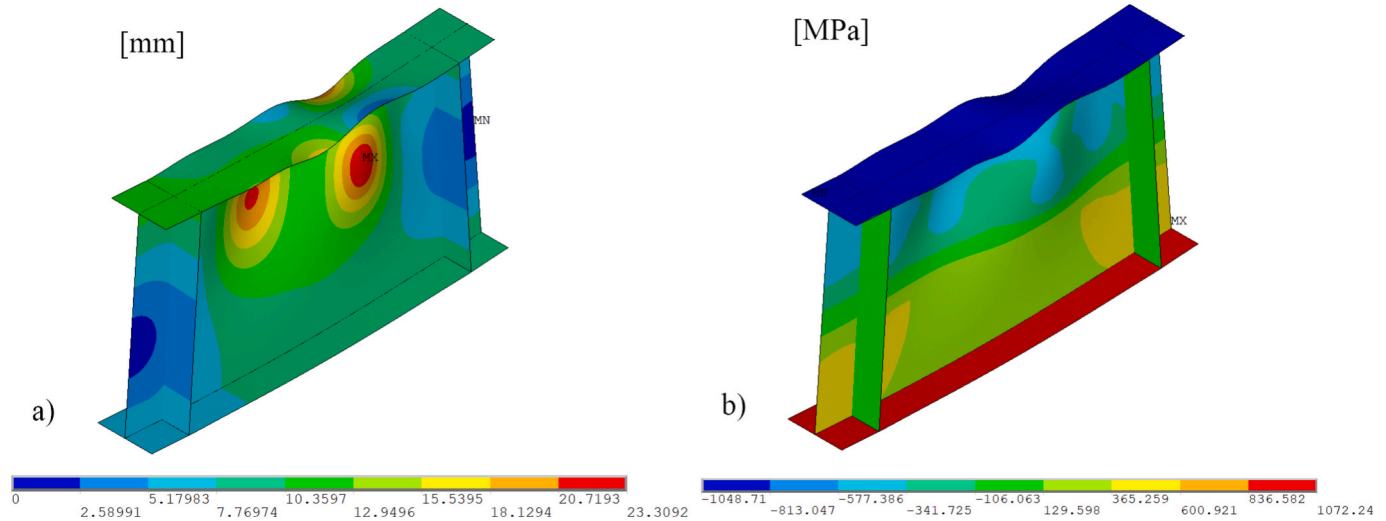
Fig. 10 shows the comparison of the numerical load-bearing values calculated using the two material models, in the horizontal axis λ is determined by manual calculation by using M_R bending resistance from GMNIA, M_{cr} from linear buckling analysis (LBA) and their quotient under the root.

For non-slender beams, where plastic deformation is more significant, the Gardner's material model gives lower resistance than the Ramberg-Osgood model. This can be explained by the fact that the yield plateau of the Gardner's material model shows a lower stress level. The

Table 6

Validation of the numerical model.

Name	R_{exp} [kN]	$R_{Gardner}$ [kN]	R_{RO} [kN]	$R_{Gardner}/R_{exp}$	Δ	R_{RO}/R_{exp} [kN]	Δ
T1	69.48	68.25	67.44	0.982	−1.8%	0.971	−2.9%
T2	67.50	67.40	65.66	0.998	−0.2%	0.973	−2.7%
T3	100.96	99.99	98.79	0.990	−1.0%	0.978	−2.2%
T4	100.91	100.22	98.79	0.993	−0.7%	0.979	−2.1%
T5	91.36	89.79	91.32	0.983	−1.7%	1.000	0.0%
T6	90.78	88.55	91.64	0.975	−2.5%	1.009	0.9%

**Fig. 8.** Failure mode (a) deformed beam, (b) longitudinal stress.**Table 7**

Cross-sections categories analysed in the current study.

Categories	Flange – Web Class
“A” – plastic section	1–1; 1–2; 2–1; 2–2
“B” – plastic flanges, class 3 web	1–3; 2–3
“C” – class 3 flange, web not class 4	3–1; 3–2; 3–3
“D” – class 4 flange, web not class 4	4–1; 4–2; 4–3
“E” – class 4 web	1–4; 2–4; 3–4; 4–4

results presented in the following are determined using Gardner et al.'s material model.

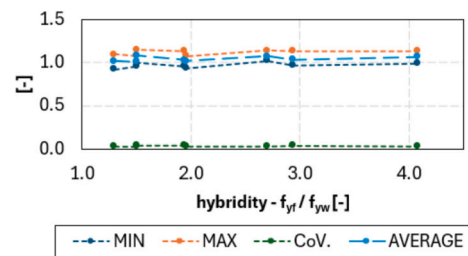
10.2. Comparison of results with previous literature recommendations

The results of the GMNI analysis performed on hybrid supports are compared with the results of Veljkovic and Zhu's analytical calculation method. Figs. 11 and 12 show the results for cross-sectional categories and in total. On the diagram, empty circles indicate results that fall outside the validity range of the formula (Veljkovic's formula: Class 4 flanges, Zhu's formula: Class 4 sections). Veljkovic's formula reproduces the results of the virtual experiment quite accurately, with little deviation, within the cross-sectional range limited by him. With the exception of flanges having class 4 sections, it provides a reliable solution for all

cases of hybridity examined.

Zhu et al.'s recommendation shows greater dispersion than Veljkovic's formula, but in the cross-sectional range in which it is valid, it is practically always on the safe side. Moreover, it is on the safe side for all hybrid ratios examined, even for partially plastic (flange or web: class 1–2) and partially class 4 cross-sectional members.

For the formula of Zhu et al., Table 9 summarizes the basic statistical evaluation of the results by steel grade. The table only contains results that are within the validity range, i.e., it does not include cases for Class 4 sections.

**Fig. 9.** Comparison of the numerical and analytical (design formula of Veljkovic) results depending on the hybrid ratio.**Table 8**

Statistical evaluation of the ratio of bending resistances obtained using the numerical and Veljkovic's formula for each steel grade.

	S460-S235	S690-S235	S960-S235	S460-S355	S690-S355	S960-S355	S690-S460	S960-S460
Min.	0.93	0.96	0.99	0.92	0.96	1.02	0.96	1.00
Max.	1.08	1.13	1.13	1.10	1.13	1.14	1.08	1.15
CoV.	0.03	0.04	0.03	0.03	0.04	0.03	0.03	0.04
Avg.	1.02	1.04	1.07	1.02	1.03	1.07	1.01	1.08

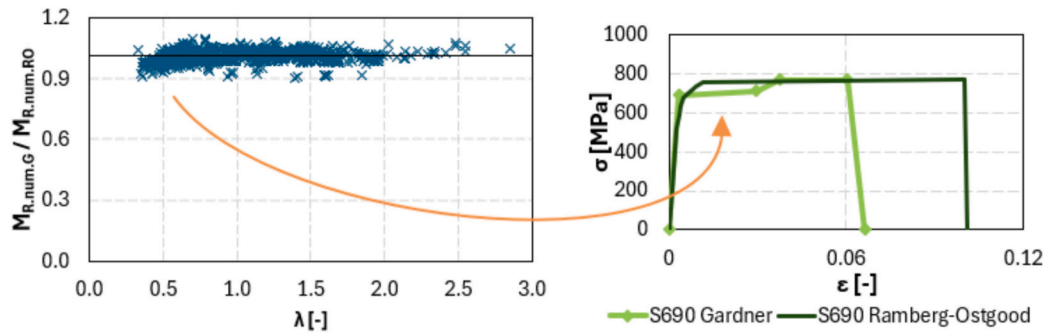


Fig. 10. The effect of Gardner's and Ramberg-Osgood-type material models on load-bearing capacity.

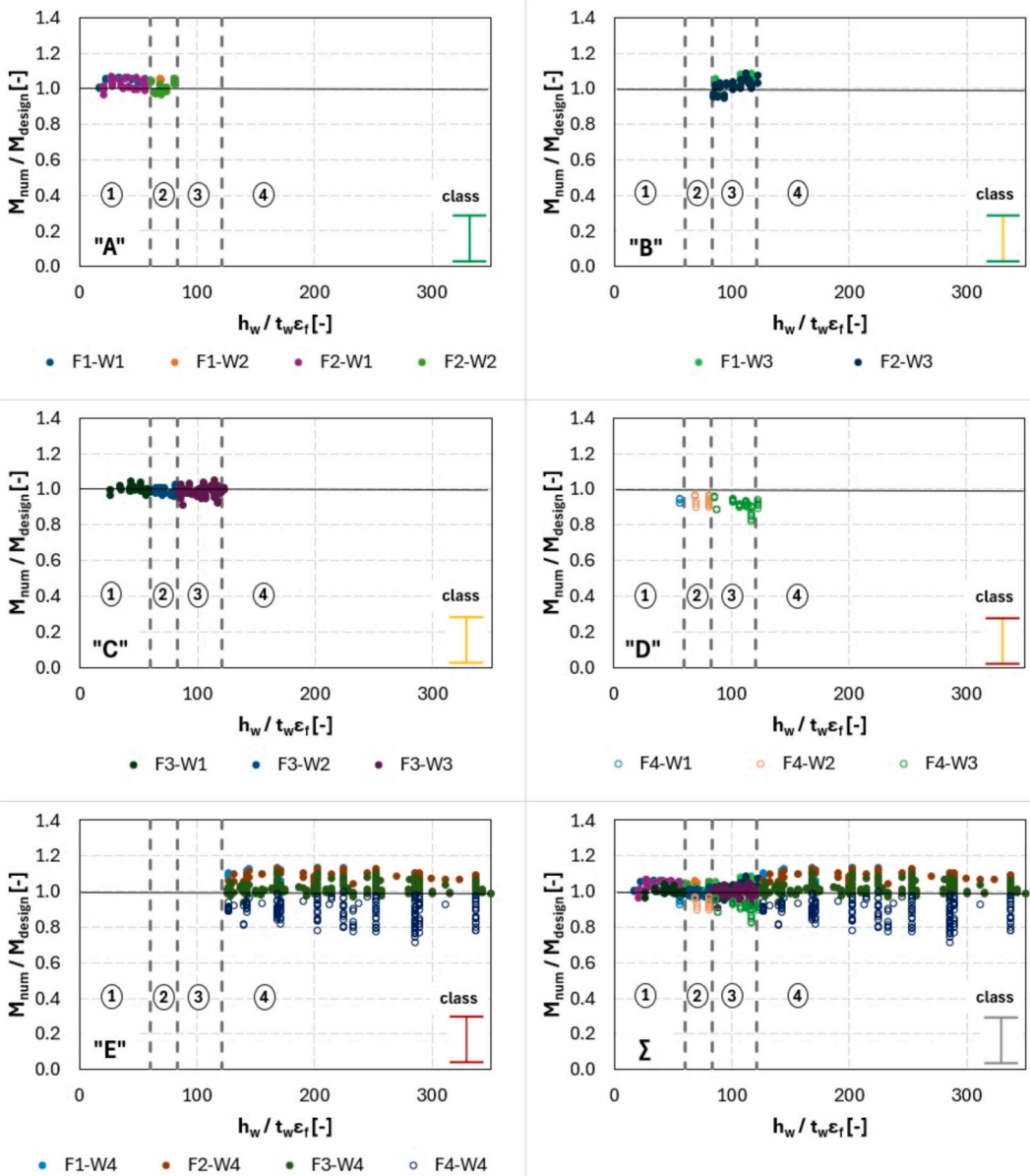


Fig. 11. Comparison of the numerical results and the resistances based on the proposal of Veljkovic [7].

Zhu et al.'s formula is significantly more conservative than Veljkovic's formula, with the average ratios being higher in the former, while in the latter they are around 1 (but never less than 1). Chapter 6 provides a separate evaluation of the different categories.

10.3. Proposal for analytical-based calculation method for class 4 cross-sections

Zhu et al.'s formula is based on elastic-plastic stress distribution,

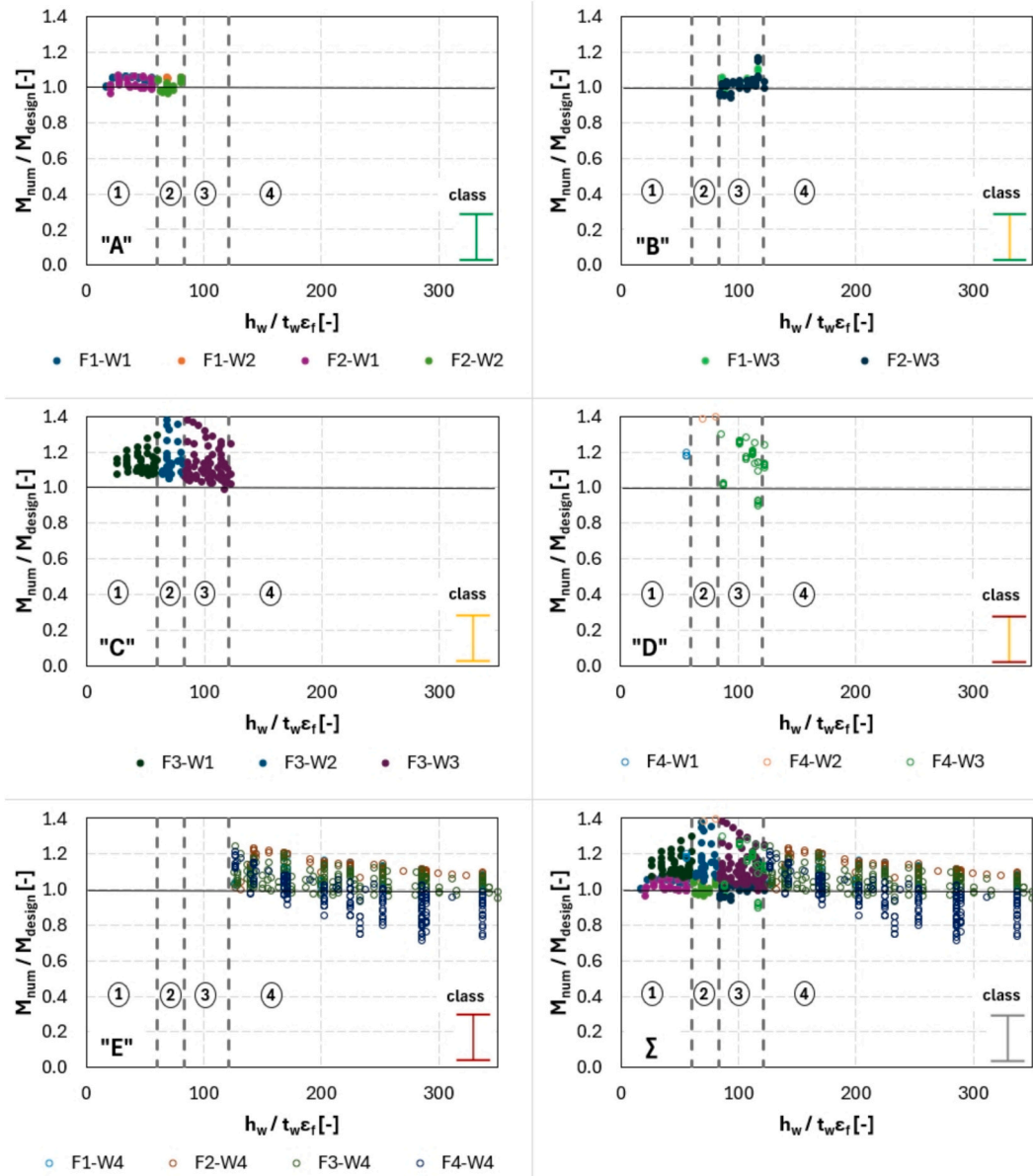


Fig. 12. Comparison of the numerical results and the resistances based on the proposal of Zhu et al. [11].

Table 9

Statistical evaluation of the ratio of bending resistances obtained using the numerical and Zhu's formula for each material grade

	S460-S235	S690-S235	S960-S235	S460-S355	S690-S355	S960-S355	S690-S460	S960-S460
Min.	0.97	1.01	1.03	0.95	1.00	1.05	0.98	1.02
Max.	1.34	1.42	1.48	1.18	1.39	1.44	1.20	1.29
CoV.	0.07	0.12	0.17	0.04	0.08	0.15	0.05	0.12
Avg.	1.07	1.14	1.12	1.04	1.12	1.11	1.10	1.11

which can also be traced in numerical models. Based on the same stress distribution, taking into account the effective cross-section, we can calculate the accurate bending resistance for hybrid beams for cross-section class 4. Fig. 13 shows the assumed stress distribution and the applied citations for a class 4 cross-section. It should be emphasized that the plastic ridge length marked with $h_{w,pl,u}$ does not include the missing web portion h_{out} (marked with a dashed line in the figure).

In the case of hybrid elastic cross-sections, resistance can be determined according to formula (25).

$$M_{R,eff,mod} = M_{R,elf,mod} + M_{R,w,mod} \quad (25)$$

where $M_{R,elf,mod}$ is the resistance of the flanges, taking into account that a stress lower than the yield strength develops on the tension side in the case of flanges of cross-section class 4, see Eq. (26).

$$M_{R,elf,mod} = \frac{\frac{1}{3} \cdot b_{fu,eff} \cdot t_{fu}^3 + b_{fu,eff} \cdot t_{fu} \cdot h_{fu}^2}{h_{fu}} \cdot f_{yf} + \frac{\frac{1}{3} \cdot b_{fl} \cdot t_{fl}^3 + b_{fl} \cdot t_{fl} \cdot h_{fl}^2}{h_{fl}} \cdot f_{yf} \quad (26)$$

and $M_{R,w,mod}$ is the resistance of the web taking into account both the

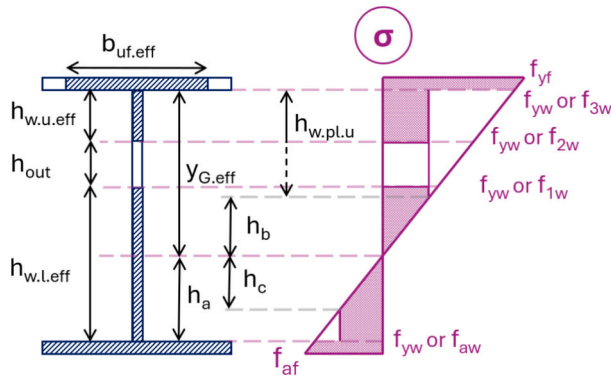


Fig. 13. Stress distribution and notations used in the calculation.

elastic, plastic and missing web portions, see Eq. (27).

$$M_{R,w,mod} = \vartheta_{w,1} \cdot f_{yw} + \vartheta_{w,2} \cdot f_{aw} + \vartheta_{w,3} \cdot f_{1w} + \vartheta_{w,4} \cdot f_{yw} + \vartheta_{w,5} \cdot f_{2w} + \vartheta_{w,6} \cdot (f_{3w} - f_{2w}) \quad (27)$$

where:

$$\vartheta_{w,1} = t_w \cdot (h_a - h_c) \cdot \frac{h_a + h_c}{2} \quad (28)$$

$$\vartheta_{w,2} = \frac{1}{2} \cdot t_w \cdot h_c^2 \cdot \frac{2}{3} \quad (29)$$

$$\vartheta_{w,3} = \frac{1}{2} \cdot f_{1w} \cdot t_w \cdot \min(h_{w,l,eff} - h_a, h_b)^2 \cdot \frac{2}{3} \quad (30)$$

$$\vartheta_{w,4} = t_w \cdot \max(h_{w,pl,u} - h_{w,u,eff}, 0) \cdot \left(\frac{h_{w,pl,u} - h_{w,u,eff}}{2} + h_b \right) \quad (31)$$

$$\vartheta_{w,5} = t_w \cdot h_{w,u,eff} \cdot \left(y_{G,eff} - \frac{h_{w,u,eff}}{2} \right) \quad (32)$$

$$\vartheta_{w,6} = \frac{1}{2} \cdot (f_{3w} - f_{2w}) \cdot t_w \cdot h_{w,u,eff} \cdot \left(y_{G,eff} - \frac{h_{w,u,eff}}{3} \right) \quad (33)$$

ϑ terms represent the area of each part of the stress diagram multiplied by the distance between their own center of gravity and the center of gravity of the effective cross-sectional area. In creating the formula, a specific aim is to make it applicable to elastic sections, both low and high hybrid ratios as well as to homogeneous sections. Different ϑ terms are non-zero in different cases. Fig. 14 shows two cases for class 4, where there are different numbers of non-zero terms.

By determining the effective cross-section, the reduction factor ρ may be taken in accordance with EN1993-1-5 [22]. However, it should be emphasized that when calculating the slenderness of the web plate, it is required to use ε_f as suggested by Veljkovic [7]. The presented plastic-elastic stress distribution in the web can be clearly observed in the results of the numerical model.

The following section presents some specific stress diagrams from the numerical model and the theoretical stress distribution assumed in the design calculation. Five diagrams are given for each category for two different hybrid ratios ("low" $\phi_h = 1.3$ and "high" hybridity $\phi_h = 2.7$). Table 10 summarizes the basic data of the sections and provides the cross-sectional classification based on the recommendations of EN1993-1-1 [8], Zhu et al. [11] and as considered in this paper.

Fig. 15 shows the longitudinal stress distribution along the web of a hybrid section using S960-S355 ($\phi_h = 2.7$) steel.

Fig. 16 shows the longitudinal stresses along the web in the same way, but with $\phi_h = 1.3$ (S460 and S355). Both Figs. 15 and 16 show that in the case of a web of class 4, the theoretical stress diagram does not

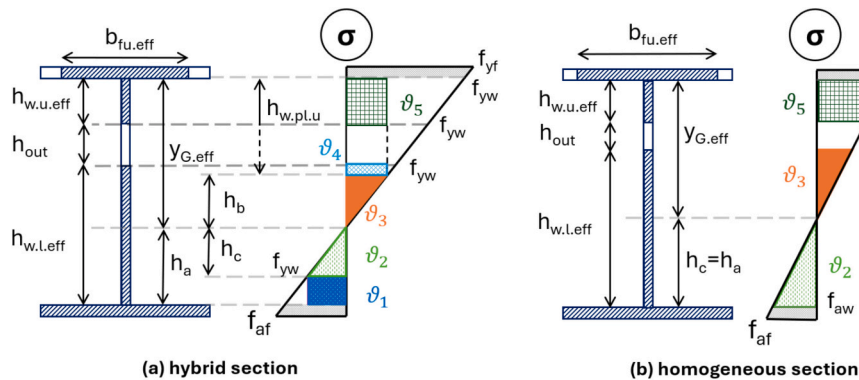


Fig. 14. Stress distribution and notations used in the calculation for hybrid and homogeneous cases (Class 4).

Table 10

Basic data and different classifications of sections presented with stress diagrams

Geometry			Material		Classification							
t_w [mm]	b_f [mm]	t_f [mm]	H/B [—]	f_{yf} [MPa]	f_{yw} [MPa]	flange (EC) [—]	web (EC) [—]	flange (Zhu) [—]	web (Zhu) [—]	flange (new) [—]	web (new) [—]	cat. [—]
25	300	36	3.6			1	1	1	1	1	2	"A"
18	300	33	3.6			1	1	2	2	2	3	"B"
25	300	25	3.5			3	1	3	1	3	2	"C"
25	300	16	3.4			4	1	4	1	4	2	"D"
6	300	16	3.4	960	355	4	4	4	4	4	4	"E"
25	300	25	3.5			1	1	1	1	1	1	"A"
12	300	25	3.5			1	3	2	3	2	3	"B"
12	300	16	3.4	460	355	3	3	3	3	3	3	"C"
16	300	12	3.4			4	2	4	2	4	3	"D"
7	300	25	3.5			1	4	2	4	2	4	"E"

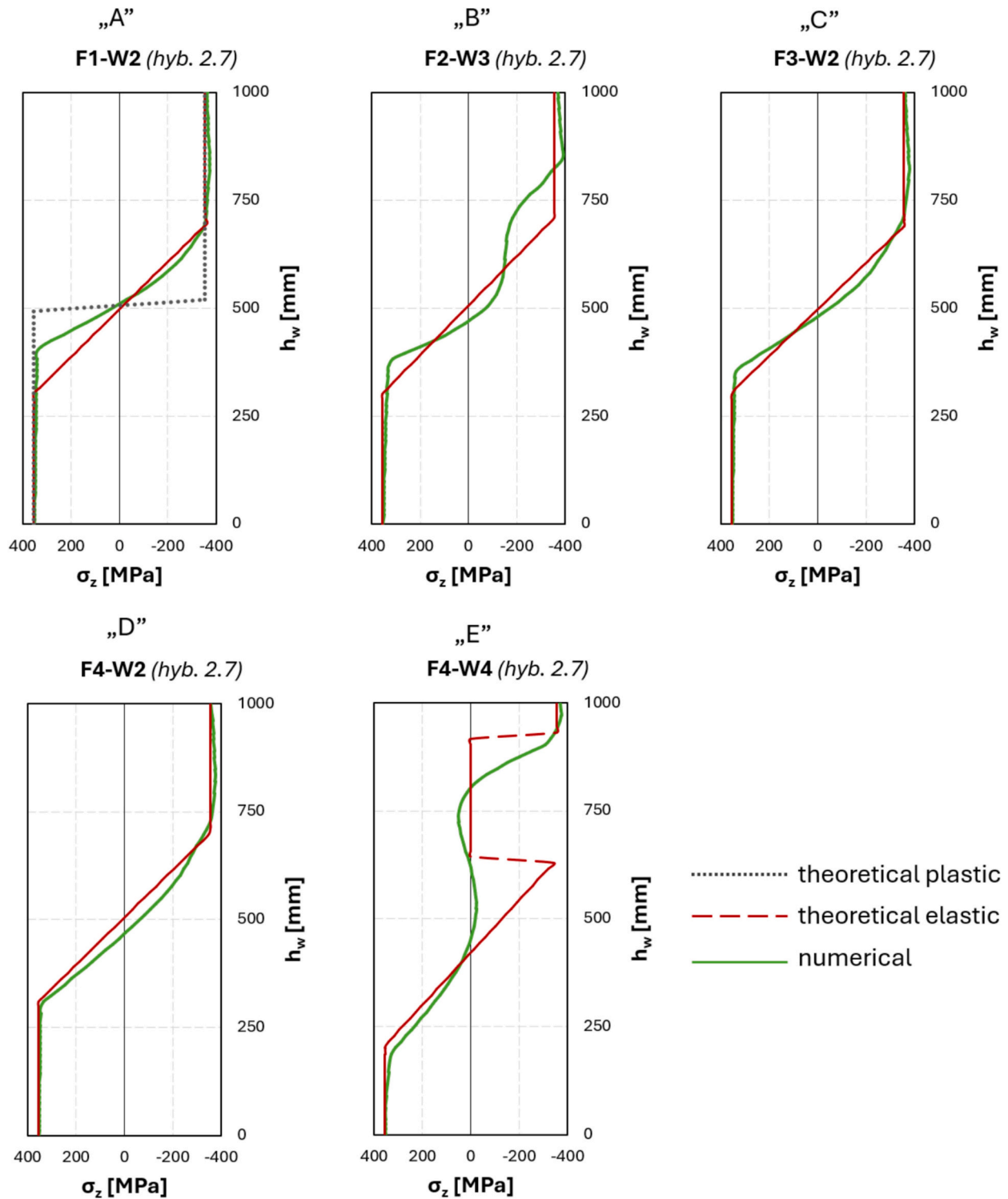


Fig. 15. Longitudinal stress distribution along the web according to the numerical model, $\phi_h = 2.7$.

exactly follow the distribution obtained with the numerical model, but with some “slicing” and shifting, the areas of the stress diagrams appear to be approximately the same.

The results obtained with the new formula are also compared with the results of numerical calculations for each category, as shown in Fig. 17. In the case of 1–2 cross-sectional classes, plastic resistance is taken into account, there is no change in this respect.

The statistical evaluation of the numerical results is summarized in Table 11.

According to Figs. 15 and 16, there would be an approximate bending resistance on the safety side if only the upper effective zone on the pressure side were taken into account. Thus, it can be calculated $M_{R,w,mod}$ in a simplified way (by omitting two terms), see Eq. (34). See the Notations Section and Eqs. (28)–(33) for the designations used.

$$M_{R,w,simpl} = \vartheta_{w.1} \cdot f_{yw} + \vartheta_{w.2} \cdot f_{aw} + \vartheta_{w.5} \cdot f_{2w} + \vartheta_{w.6} \cdot (f_{3w} - f_{2w}) \quad (34)$$

In the following, $M_{R,eff,simpl}$ denotes the simplified bending resistance applying this (the part calculated from the flanges remains unchanged).

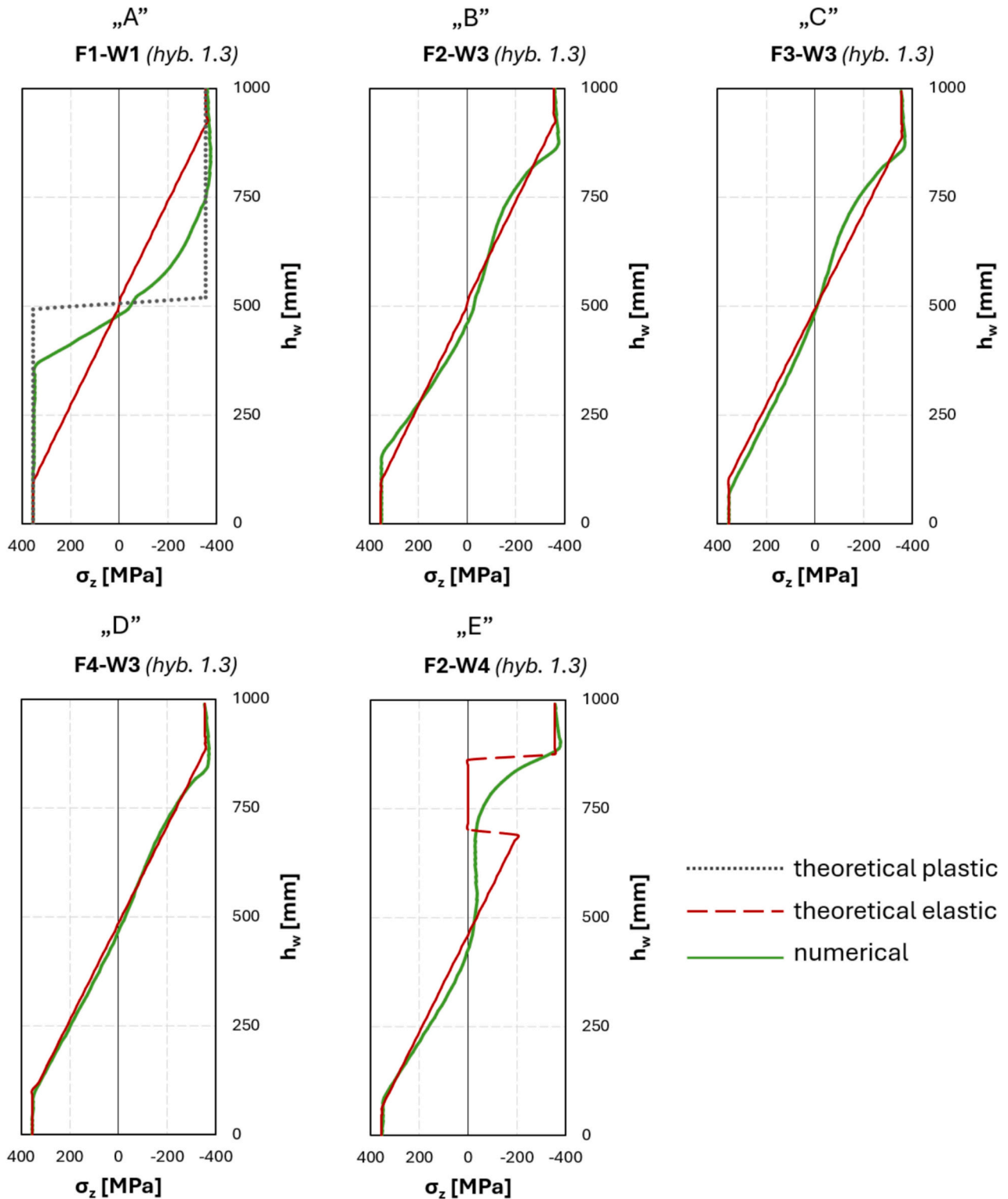


Fig. 16. Longitudinal stress distribution along the web according to the numerical model, $\phi_h = 1.3$.

The simplified formula is not mechanically completely justified, giving a smaller bending moment resistance by neglecting parts 3 and 4 shown in Fig. 14.

11. Partial factor calculation

The numerical results and their statistical evaluation are used to calculate the γ_{M^*} factor for each cross-sectional category. The partial

factor γ_{M^*} is calibrated in accordance with Annex D of EN 1990 (see Section 3.2 for a detailed description of the procedure). For the new proposal some key values of the calculation: the correction factor b the total variation coefficient V_r the Q parameter used in the lognormal transformation and correction factor Δk can be found in Table 12.

Table 13 summarizes the obtained γ_{M^*} factors for hybrid beams. The values are given for the entire database and for separate categories, with parentheses indicating cases where the formula in the literature is not

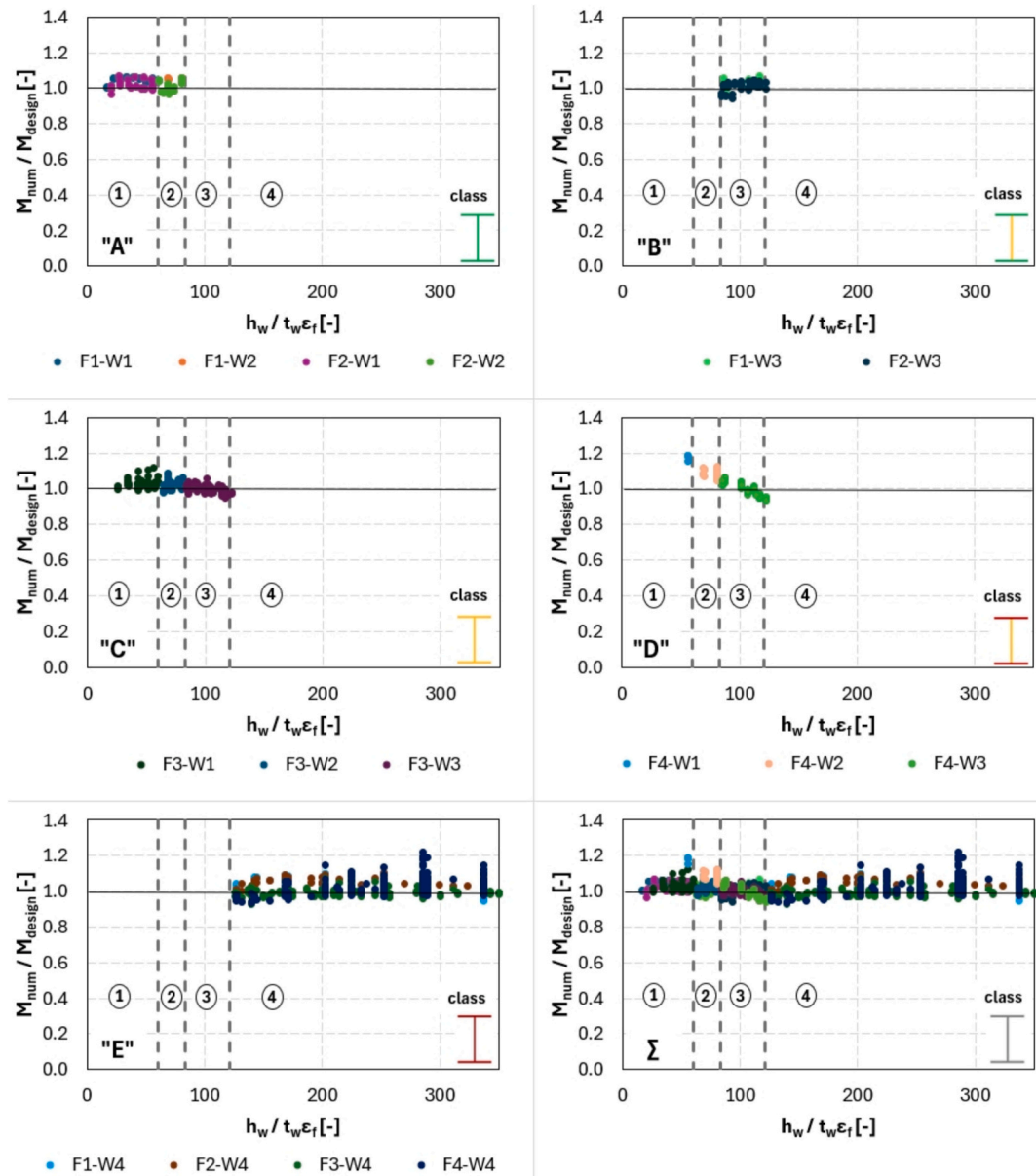


Fig. 17. Comparison of the numerical results and the resistances based on the new proposal.

Table 11

Statistical evaluation of the ratio of bending resistances obtained using the numerical and the new formula for each material grade.

	S460-S235	S690-S235	S960-S235	S460-S355	S690-S355	S960-S355	S690-S460	S960-S460
Min.	0.93	0.95	0.96	0.95	0.95	0.96	0.98	0.96
Max.	1.12	1.21	1.15	1.20	1.23	1.10	1.12	1.20
CoV.	0.03	0.04	0.03	0.05	0.05	0.04	0.04	0.04
Avg.	1.01	1.02	1.04	1.03	1.03	1.04	1.02	1.05

Table 12

Intermediate values during calculation of γ_{M^*} factors for the new proposal.

for the new proposal: $M_{R,eff,mod}$						
	"A"	"B"	"C"	"D"	"E"	Entire database
b	1.0483	1.0335	1.0320	1.0447	1.0471	1.0453
V _r	0.0999	0.1007	0.1002	0.1140	0.1059	0.1047
Q	0.0997	0.1004	0.0999	0.1136	0.1056	0.1044
Δk	0.9790	0.9943	0.9949	1.0066	0.9903	0.9900

valid and therefore the calculated factor is not relevant.

For categories „C”, a lower γ_{M^*} factor can be calculated using the formula of Zhu et al., but for cross-sections in class 4 and in terms of overall evaluation, the new design formula new proposed is more reliable. It can be seen that the simplified calculation further reduces the γ_{M^*} factor for cross-section class 4.

To verify the calculation, the evaluation is also performed for 400 homogeneous sections, see Table 14. In the case of homogeneous sections, partial plasticization of the web does not occur, of course, so the method takes into account the stress distribution specified in the current

Table 13
Calculated γ_{M^*} factors for hybrid beams.

	"A"	"B"	"C"	"D"	"E"	Entire database	wherever applicable
Veljkovic	1.13	1.14	1.18	(1.26)	(1.24)	(1.21)	1.12
Zhu	1.13	1.17	1.06	(1.16)	(1.22)	(1.24)	1.17
new proposal: $M_{R,eff,mod}$	1.13	1.14	1.14	1.18	1.15	1.15	1.15
simplified: $M_{R,eff,simpl}$	1.13	1.14	1.14	1.16	1.14	1.14	1.14

Table 14
Calculated γ_{M^*} factor for homogeneous sections.

	"A"	"B"	"C"	"D"	"E"	Entire database
new proposal	1.13	1.06	1.09	1.07	1.10	1.13

Eurocode standard regulations. The γ_{M^*} factor is practically the same for plastic cross-sections and is always below 1.1 for the other categories.

Based on the results, it appears that the proposed method reliably reproduces the results of virtual experiments. The calculations performed on homogeneous sections confirm the adequacy of the current specifications and the correctness of the partial factor calibration. For cross-section classes 1–2, the moment capacity of hybrid beams can be calculated using plastic resistance (as proposed in the international literature). For class 3 (flexible) cross-sections, the recommendations of Zhu et al. or Veljkovic can also be applied and are sufficiently accurate. The proposal presented here can be safely applied to both Class 3 and Class 4 hybrid cross-sections with a γ_{M^*} factor of 1.2 (since the maximum value was 1.18 when evaluating the categories, a conservative rounded-up value is used).

It is important to note that EN 1993–1-5 limits the yield ratio between the flange and the web to 2. The current research does not aim to argue against or to recommend the use of a higher hybrid ratio. It intends to describe the resulting stress distribution and provide a formula that allows for a good estimation of the load-bearing capacity for a wide range of hybrid ratios, even for slender sections. Further studies are needed in the future to comprehensively examine the effects on usability, plasticity, fatigue, and practical design constraints.

12. Conclusions

This study examines the bending of I-section hybrid (using different steel grades in flanges and webs) steel sections. Doubly symmetric beams are only examined across a wide range of slenderness ratios. The results of a validated virtual experiment series are compared with recommendations in the literature, and a new calculation method is proposed for beams in cross-section class 4. Effective plastic-elastic stress distribution is taken into account, and the effective width is calculated using the ϵ_f factor also for the web. The stresses determined from the numerical model and theoretical considerations are compared along the web. In addition to the theoretically sound formula, a simplification is also provided, which, based on parametric testing, is even more on the safe side. For the proposed method, γ_{M^*} factor is recommended by the value of 1.2 conservatively.

Based on the numerical analysis, the proposed γ_{M^*} factor can be applied to all hybrid ratios examined (up to S960). The proposed formula closely follows the behavior of hybrid steel beams in the examined parameter range. However, it should be emphasized that the combination of very high-strength flanges and very low-strength webs may need to be limited in the case of repetitive loading (fatigue). It is necessary to examine whether partial plasticization of the web is permissible and, depending on the design situation, to analyze whether plasticization occurs in the web at all in the serviceability limit state for a given hybrid ratio.

The formula of Zhu et al. [11] and Veljkovic [7] is confirmed, the proposals are well suited for determining the bending resistance of

hybrid beams of cross-section class 3. Based on the study, the material model used has only a minor effect on the given failure mode, and the hybrid ratio does not significantly affect the accuracy of the proposed calculation method.

The proposed formula is only recommended for double-symmetric cross-sections under pure bending. Future investigations are needed for non-symmetric hybrid beams, cases with large spans without lateral restraints, and cases with significant shear forces and moment-shear interaction.

CRedit authorship contribution statement

E. Bärnkopf: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation. **B. Kövesdi:** Writing – review & editing, Supervision, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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