



Model factor calibration for FEM-based design of slender steel welded plated structures

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ABSTRACT

FEM-based design is getting widely used in the daily design practice of steel structures. A new Eurocode standard – prEN 1993-1-14 is under preparation to regulate its application. The direct resistance check, where numerical model determines the resistance of the structure needs improvement in terms of handling the uncertainties and ensuring the required safety. The FEM-based design should take into account all the uncertainties handled in traditional analytical calculations and the uncertainties of the numerical model, thus ensuring an equivalent level of safety as required by the Eurocode standards. Therefore, the value of the partial factor associated with the uncertainty of the calculation model should be reassessed and the uncertainty and reliability of the numerical models should be examined. A methodology to evaluate the uncertainties on the resistance side is developed and summarized in this paper, which proposes partial safety factors (γ_{Rd} , γ_m , γ_M and γ_{FE}) adapted to the failure probability objectives of the EN 1990 standard. The focus of the study is on a specific failure mode - bending resistance of steel box-section beams - using the results of laboratory experiments. The current paper presents the calculation of the model factor (γ_{FE}) for this specific failure mode by using a shell-type numerical model and also compares its application to Monte Carlo simulation-based reliability analysis results to prove the appropriate safety.

1. Introduction

In recent decades, there have been significant changes in the design of steel structures. The application of numerical model-based design is becoming more common in the civil engineering practice, which can be made by different methods. One option is the application of indirect resistance check (also called as analysis required by subsequent design check), when the numerical model is applied to calculate the System Response Quantities (SRQs) which can be further applied to calculate the resistance of the structure by using analytical design expressions. Another option is the application of the direct resistance check, when the numerical model is used to calculate the load carrying capacity of the structure.

The application of direct resistance check, where a geometrical and material nonlinear analysis using imperfections (GMNIA) is performed to calculate the load carrying capacity of the structure is becoming more common. This approach can be in many cases more efficient than traditional analytical design calculations and allows more accurate design, which is economically advantageous. However, the change in calculation method raises the question whether the resistance-side

partial safety factors given in the Eurocode standard are appropriate and safe to use in the design. Therefore, a review is needed for different failure modes and structure types when the FEM-based design is applied.

Currently, there are numerous open questions in the international literature regarding the meaning, significance and way of determination of the model factor (γ_{FE}) to be used in direct resistance check. This paper aims to explore the methods and aspects involved in determining the resistance side uncertainties and to determine the model factor for one specific failure mode, which is the bending resistance of welded steel box-section beams. Similar studies should be performed for different failure modes and numerical models to deeply investigate the accuracy and reliability of the direct resistance check and to determine the necessary value of the model factor. Theoretically, the model factor can be treated as a project dependent factor, which should be determined for each design case for each designer. This approach would serve as a significant handicap to the direct resistance check for steel structures. Therefore, a better and more usable approach would be to develop predefined values for the model factor, which can be applied by designers after a careful model validation (which professes the accuracy of the applied numerical model). The predefined values can be failure mode and numerical model level (beam element model, shell element

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Notations¹

R_{exp}	resistance value measured in laboratory tests, which $R_{test,known}$ in the study
$R_{num.m}$	resistance value determined by deterministic calculations using numerical model applying GMNI analysis and measured geometry, material properties and imperfection,
R_{det}	resistance value determined by deterministic calculations using a numerical model applying GMNI analysis and nominal values for geometry, material properties,
$R_{k.stoch}$	the lower 5 % percentile value of resistance obtained from stochastic calculations (numerical model, GMNIA), referred as characteristic value,
$R_{d.stoch}$	the lower 0.1 % percentile value of resistance obtained from stochastic calculation (numerical model, GMNIA), referred as design value,
$R_{d.1}$	the design value of the resistance determined according to 'Method 1' (the method described in Section 3.2)
$R_{d.2}$	the design value of the resistance determined according to 'Method 2' (the method described in Section 3.2)
γ_{FE}	model factor; it takes into account that different designers build different models for the same problem: different software, different finite element type, different mesh type, etc. There might be different models, therefore, its value should be determined for each model separately. The determination of the model factor is practically part of the validation process. By using a well-documented experimental campaign with known measured material properties, geometric dimensions, imperfections, it shows, how accurate the experimental load capacity can be obtained by the numerical model. The calculation can be performed by using Eq. (3) comparing R_{exp} and $R_{num.m}$; if k_n is chosen to determine the 5 % characteristic value according to EN1990 [1] Table D1. The calculated value is noted by $\gamma_{FE,k}$. If k_n is taken from Table D3 for the determination of the design value of the load carrying capacity, the calculated model factor is called $\gamma_{FE,d}$.
γ_{M^*}	resistance side uncertainty, calculated from the ratio of characteristic to design values according to Eq. (13).
γ_m	material uncertainty from the ratio of the nominal yield strength to its design value for a given material grade.
$\gamma_{Rd.method}$	uncertainty of the calculation method (imperfection definition method), the calculation is performed according to Eq. (3), $R_{k.stoch}/R_{det}$ ratios are calculated, and from these values the mean value and coefficient of variations are determined, $k_n = 3.04$ is used instead of $k_n = 1.64$;
$\gamma_{Rd.num}$	uncertainty of the resistance model, the calculation is performed according to Eq. (3), $R_{exp} / R_{num.m}$ and $R_{k.stoch}/R_{det}$ ratios are evaluated using the coefficient of variation and mean value calculated from these ratios; $k_n = 3.04$ is used instead of $k_n = 1.64$.

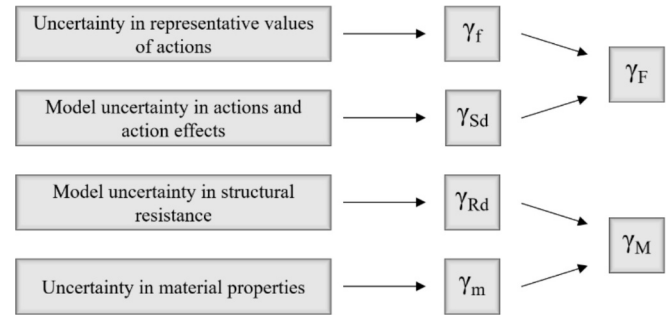


Fig. 1. Subdivision of partial safety factors applied in EN 1990 [1].

general definition of methodology is still missing from the international literature. Therefore, the current paper is one of the pioneering projects to determine the value of the model factor and to prove the accurate safety of its application.

2. Literature review

2.1. Uncertainties covered by the Eurocode standard

The Eurocode standard takes into account the uncertainties on the action and resistance sides separately by using partial factors. These are used to increase the value of the loadings to be taken into account and to reduce the value of the calculated resistances to ensure appropriate safety. The idea is that safety factors should be applied directly where uncertainties are present. On the action side, model uncertainties in actions and action effects (γ_{Sd}) and the uncertainty in the representative values of actions (γ_f) are taken into account. On the resistance side, the uncertainty in the structural resistance model (γ_{Rd}) and the uncertainty in the variables that essentially determine the resistance (γ_m) are taken into account. The product of these two values gives the partial factors of the action side (γ_F) and the resistance side (γ_M) accordingly, which ensures the necessary safety requirement of the structures based on the rules of EN 1990 according to Eqs. (1)–(2), as shown in Fig. 1.

$$\gamma_F = \gamma_{Sd} \cdot \gamma_f \quad (1)$$

$$\gamma_M = \gamma_{Rd} \cdot \gamma_m \quad (2)$$

In the present study, uncertainties on the resistance side are discussed. It is necessary to review whether the design using numerical models has changed the required partial factor γ_M so that the level of safety does not change compared to the application of analytical calculations. The design rules for the determination of the model factor given in EN 1993-1-14 [2] do not intend to modify this reliability approach and the safety level required by EN 1990. Therefore, EN 1993-1-14 does not give specific values for the partial safety factors (different from the given values in other Eurocode Parts) if numerical model-based design is applied. However, the uncertainties and their effects can be different if numerical models are used for the resistance calculation. Therefore, specific considerations are necessary for the determination and consideration of the model uncertainties, which depends on the applied numerical model and analysis type; the application of the model factor is closely linked to the applied analysis method and its value depends on the developed numerical model.

If the numerical model is employed for indirect verification methods (i.e. numerical design calculation and analysis requiring subsequent design check), the FE-model is used for the calculation of the system response quantities (SRQs – internal forces, deformations, etc.). In this case, the calculation is not that different from hand calculations, with the uncertainties considered in the value of γ_{Sd} (model uncertainties in actions and action effects). Therefore, there is no need to enhance the model uncertainties and to increase the safety level by applying an

¹ The notations associated with partial safety factors are the followings.

model, solid element model, etc.) dependent, however its general determination is still missing from the international literature. Even the

additional model factor. The model uncertainties are covered by the partial factors given in EN 1990 and EN 1993 (all Parts).

If the numerical model is used for direct verification methods (i.e. numerical design calculation and direct resistance check), the design uncertainties should be evaluated on the basis of the combination of the partial factors according to EN 1993 (all Parts) and the model factor (γ_{FE}). The reason for this is the following. In the case of a direct resistance check, the numerical model is intended to result in the characteristic value of the resistance. In this case, no analytical design resistance models are applied that could implicitly cover the uncertainties of the model. Therefore, irrespective of the analyzed failure mode and model level, the uncertainties of the numerical model and the resistance calculation method need to be additionally considered. This can be done through the application of the model factor (γ_{FE}). Its determination is based on a comparison of the numerical calculation (R_{check}) with the known test results ($R_{test,known}$) or established resistances ($R_{k,known}$) obtained by approved calculation methods. In this procedure, the ratio of test data or characteristic resistance to resistance values obtained from numerical analyses ($R_{k,known}/R_{check}$ or $R_{test,known}/R_{check}$) should be calculated for each case. The mean value (m_X) and the coefficient of variation (V_X) are then to be calculated for these analyzed ratios. By statistical evaluation the model factor can be derived according to Eq. (3). To derive the characteristic resistance from the results of the numerical analysis, Eq. (4) can be used, where R_{GMNIA} is the resistance obtained from the deterministic geometric and material nonlinear analysis (GMNIA).

$$\gamma_{FE} = \frac{1}{m_X \bullet (1 - k_n \bullet V_X)} \geq 1,0 \quad (3)$$

$$R_k = \frac{R_{GMNIA}}{\gamma_{FE}} \quad (4)$$

where:

m_X is the mean of the ratio of the measured (or known) to the calculated results for sample n ,

k_n the characteristic factor according to EN 1990 [1], Annex D, Table D.1 (the data series for the unknown V_X shall be used),

V_X is the coefficient of variation of the ratio of the measured (or known) to the calculated results for n samples.

Eq. (3) follows the evaluation process of EN 1990 Annex D specified for a single property by using the Gaussian distribution calculating the partial factors for the characteristic and design values. Alternatively, prEN 1993-1-14 provides an equation based on the evaluation process outlined in EN 1990 Annex D for resistance methods. This equation uses lognormal distribution function to calculate the partial factors for both characteristic and design values. The application of the lognormal distribution leads to smaller model factors, but in the practical application domain the two calculation methods lead to negligible differences, so both can be applied.

Individual determination of the model factor for each project, design situation and structure is a large work for designers and handicap for the daily engineering practice. It may be avoided by using a simplified validation process and predefined values for the model factors could be also used, which are determined from pre-calibration processes. The current paper aims to start this pre-calibration process and support the FEM-based design for daily application. The model factor and its verification process is model and failure mode specific. The model factor should be determined for each developed numerical model and analyzed failure mode. However, if there is no significant change in the analyzed failure mode and the numerical model is previously validated for problems with similar or slightly changed geometrical, loading or supporting conditions, the predefined model factor could be used. In practice this means, if designers are analyzing similar failure modes using the same or a similar numerical model, the model factor does not need to be determined for each individual case. Instead, it can be

Table 1

Relationship between β and P_f [1].

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.28	2.32	3.09	3.72	4.27	4.75	5.20

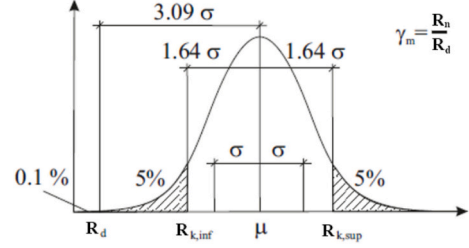


Fig. 2. The safety concept used in Eurocodes [3].

established during the model's first application, or a predefined value can be used if similar assumptions (e.g., FE model setup and analysis level) are applied as those used to determine the predefined value.

2.2. Previous calculations of the resistance side uncertainties

Recent research made by Schillo [3] presents the methodology for determining the partial factor γ_m according to EN 1990 [1] and defines its value for different steel grades. The safety concept of EN 1990 [1] is based on probabilistic methods. This means that the classification of a structure as safe or unsafe is determined by a predefined probability of failure. In general, the reliability Z of a unit (e.g. a structure) is the property that describes its ability to perform its intended function with a specified probability under given conditions and over a specified time period. The objective is that the probability of failure of a structure is acceptably low. The optimal safety is defined as the minimum of the complex costs, including all cost factors (material and moral damages) over the design lifetime of the structure. Reliability can be expressed as a complement to the probability of failure P_f (Eq. (5)).

$$Z = 1 - P_f \quad (5)$$

To quantify the probability of failure, a reliability index is introduced, which is correlated with P_f as shown in Eq. (6):

$$P_f = \phi(-\beta) \quad (6)$$

Here ϕ denotes the standard normal distribution function. The above equation allows for direct quantification, which can be found, e.g., in Schillo [3] and is illustrated in Table 1. For a 50-year period, the value of 3.8 represents a probability of failure of 1/10000.

To determine the design values, the β is multiplied by the sensitivity factor $\alpha_R = 0,8$ as given in [1]. This factor allows for the isolation of the load side effect (where α_E - the sensitivity factor for actions is 0,7 and the sum of the square roots of α_R and α_E would lead to 1). The design value can thus be calculated for e.g. lognormal distribution [1] according to Eq. (7).

$$R_d = \mu \bullet e^{\left(-\alpha_R \bullet \beta \bullet \frac{\sigma}{\mu}\right)} \quad (7)$$

Eq. (7) denotes the mean and the standard deviation. Knowing the mean value (μ) and the standard deviation (σ), the coefficient of variation (CoV, marked: V) can be derived according to Eq. (8). CoV is a standardized measure of the variance of a probability distribution.

$$V = \frac{\sigma}{\mu} \quad (8)$$

Substituting $\beta = 3,8$ and $\alpha_R = 0,8$, the equation simplifies to Eq. (9)

$$R_d = \mu \cdot e^{(-3.04 \cdot V_r)} \quad (9)$$

The background of Eqs. (7)–(9) is described in [4]. For the design, it means that the probability of failure should not exceed 1/1000 cases in 50 years. A graphical interpretation of the safety concept is illustrated in Fig. 2.

On the left-hand side, it can be seen that for a distance of ~ 2 standard deviations ($= 1.64 \cdot \sigma$) from the mean value μ , a lower 5 % percentile is reached which gives the characteristic value. Achieving a design value corresponding to a percentile of 0.1 % requires ~ 3 standard deviations ($= 3.09 \cdot \sigma$ or $0.8 \cdot 3.8 \cdot \sigma$). The characteristic values R_k can be determined by Eq. (10) for lognormal distributions:

$$R_k = \mu \cdot e^{(-1.64 \cdot V_r)} \quad (10)$$

Schillo [3] determined the partial factor γ_m from the ratio of the nominal yield strength to its design value for a given material grade. On the basis of Annex D of EN 1990 [1], the design value of the load carrying capacity based on a considerable amount of experimental results is determined and the corrected partial safety factor γ_m^* has been derived from the ratio of the nominal values to the design values according to Eq. (11).

$$\gamma_m^* = \frac{R_n}{R_d} \quad (11)$$

For the partial factor determination γ_m , the characteristic resistance R_k should be used instead of R_n in Eq. (11). According to Schillo [3] the values of γ_{mi} in EN 1993 are all derived from γ_m^* .

Rebello et al. [5] and da Silva et al. [7] determined the partial safety factors γ_m , γ_{rd} and γ_m for the lateral-torsional buckling failure mode of steel I-beams based on a large number of numerical simulations. The problem is approached in such a way that it is possible to completely separate the uncertainties associated with the material properties and the uncertainty associated with the resistance model in order to calibrate a new design procedure [6]. In the first part of the paper [5] the variability of the load, material and geometric properties are not considered to allow an unbiased comparison of different computational methods. The “experimental” resistance obtained in this study corresponds to the result obtained from the numerical (FEM) simulation and it can be compared with the “theoretical” resistance r_t obtained by applying the design model to the same situation. Since the nominal values of the material properties are used in the calculations, the variance associated with the uncertainties in the material properties is not present in the study, so that the calibration of γ_{rd} can be dealt with directly, assuming that the geometric variations are included in the design model. Part 2 of this study [7] presents a probabilistic assessment of the effect of variability in steel material properties on the deflection behaviour of steel beams and evaluates the partial factor γ_m required to achieve a resistance that meets Eurocode reliability targets.

Radwan [8] investigated the applicability of previously developed local and global imperfections to purely local, global and interaction buckling problems of welded box-section columns subjected to pure compression. The results of the numerical load carrying capacity calculated deterministically for the different imperfections proposed in the literature are compared with the 5 % lower percentile value determined from stochastic calculations, taking into account the probability distribution of material, geometric and imperfection parameters. Using the design specifications of the prEN 1993-1-14 [2], a factor is defined as a model factor for each case separately, which is $\gamma_{FE} = 1.0$ – 1.1 in general.

The model uncertainty for concrete structures has been widely studied in the last decades. The most up-to-date research results are given in [9–12]. Similar studies to those presented in the current paper for concrete structures have been conducted by V. Cervenka et al. [9] in 2018. Numerical simulations were performed by a nonlinear analysis with constitutive models for concrete, reinforcement and their interaction using the FE-program ATENA. The model uncertainty is defined by

Table 2

Model factors for model uncertainties [9].

Failure type	μ_Θ	V_Θ	γ_{rd}
Punching	0.971	0.076	1.16
Shear	0.984	0.067	1.13
Bending	1.072	0.052	1.01
All failure modes	0.979	0.081	1.16

Table 3

Tested relative slendernesses, upper flange sizes and the names given to the cross-sections.

Name	b_{fl} [mm]	t_{fl} [mm]	$\bar{\lambda}$ [–]
BOX-1	250	10	0.541
BOX-2	250	8	0.676
BOX-3	250	7	0.773
BOX-4	250	6	0.902
BOX-5	250	5	1.082
BOX-6	250	4	1.352
BOX-7	280	4	1.515
BOX-8	320	4	1.731

the ratio of the resistance found by the experiments and the simulation results obtained by the numerical model. In order to investigate the pure model uncertainty, the other random effects were reduced to the minimum. It means, the measured values of the material properties of the concrete and the reinforcing bars are applied in the numerical model, thus the variation of the material properties are not considered within the evaluation process. The model uncertainty is considered as a random entity, which can be described by the mean value, standard deviation and the coefficient of variation. The authors considered a log-normal statistical distribution function for the model uncertainty calculation and the model factor is calculated by Eq. (12)

$$\gamma_{rd,num} = \frac{\exp(\alpha_R \cdot \beta \cdot V_X)}{m_X} \geq 1, 0 \quad (12)$$

where the safety measure is described by the reliability index $\beta = 3.8$, the sensitivity factor $\alpha_R = 0.4 \times 0.8 = 0.32$, where 0.8 is a sensitivity factor for the resistance and 0.4 for the model uncertainty as a non-dominant entity. These safety parameters were suggested in the Model Code 2010 Section 4 Principles of structural design [12]. By applying these principles, the model factors are determined for solid element models using the ATENA software, and the values given in Table 2 are proposed as failure mode dependent model factors.

Similar study as shown in [9] for concrete structures is made in the current investigations for bending-type failure of steel structures. The final result is also compared to Monte Carlo simulation-based reliability analysis to prove the accuracy of the calculated model factor.

3. Research strategy and methods

3.1. Strategy and objectives

In this study, box-section beams under bending are investigated. In order to validate the numerical model and to determine the γ_{FE} factor, specimens from a series of laboratory experiments are tested. The laboratory investigations are presented in Section 3.2. A FE-model is developed for the further analysis, as described in Section 3.3. Subsequently, 8 specimens of different slenderness (varying the thickness and width of the top flange) are tested in stochastic and deterministic calculations, as summarized in Table 3. $\bar{\lambda}$ is determined by manual calculation by using the Eurocode terms.

In the deterministic calculations, wide ranges of relative slenderness values are considered, while the material quality for all specimens is S355 (unlike in laboratory tests, where hybrid and homogeneous

Table 4
Nominal dimensions of the test specimens.

Specimen code	Flanges [mm]			Webs [mm]		Length [mm]
	b_f	t_{f1}	t_{f2}	h_w	$t_{w1} = t_{w2}$	
BOX4-355		10	10		4	
BOX4-700		10	12		4	
BOX4-H1		10	10		4	
BOX4-H0		10	12		4	
BOX6-H0		10	12		6	
BOX8-H0	320	10	12	480	8	1000

sections, high strength steels are also tested). The height of the web is 455 mm, the thickness of the web is 20 mm for all specimens. The cross-sectional class varies between 1 and 4.

For both the deterministic and stochastic calculations, GMNI analysis are performed for each slenderness and the resistance values are determined separately. The design value of the resistance is determined according to the three methods as described in Section 3.4. The differences of resistance values $R_{d,1}$, $R_{d,2a}$ and $R_{d,2b}$ are then compared and discussed.

3.2. Laboratory experiments

The experiment used for the determination of the model factor was performed at the Department of Structural Engineering of the Budapest University of Technology and Economics in 2023. The implemented experimental research programme is summarized in detail by Somodi et al. [13]. A total of 14 large-scale hybrid girders were tested, six of them were subjected to pure bending and eight of them were subjected to interaction of bending and shear. This paper focuses exclusively on

the results of the purely bent specimens. The nominal values of the geometrical data of the specimens tested are summarized in Table 4 using the notations given in Fig. 3. The same geometry was used for each specimen, except that the thickness of the web and the lower flange thickness are varied.

The reference code for the specimens starts with BOX, followed by a one-digit number indicating the thickness of the webs in mm, followed by a dash indicating the material quality as follows:

- 355 - homogeneous section with S355 steel grade,
- 700 - homogeneous section with steel grade S700,
- H0 - hybrid section with S355 steel grade for the webs and S700 for the flanges,
- H1 - hybrid section, S355 webs, flanges S500 steel grade.

The height of the web is 480 mm, the width of the flanges is 320 mm, the internal spacing between the webs is 230 mm. The test specimen has a 30 mm thick end plate at each end.

Within the test set-up, the test specimens are connected to two beams with header plate bolted connections. These beams are called BOX-T in Fig. 4, where the general test setup is shown. The girders are subjected to 4-point bending (4P), which creates a constant bending moment zone in the middle of the test specimens (between the load introduction places, which part of the girder is investigated; after failure only the middle part of the girders is changed).

The load was applied using hydraulic jack. A load-distributing beam was placed beneath the hydraulic jacks to ensure uniform load distribution along the top edge of the loaded endplates. A force-measuring cell is positioned under the hydraulic jack to monitor the applied load during testing. In the case of M-type specimens, failure consistently

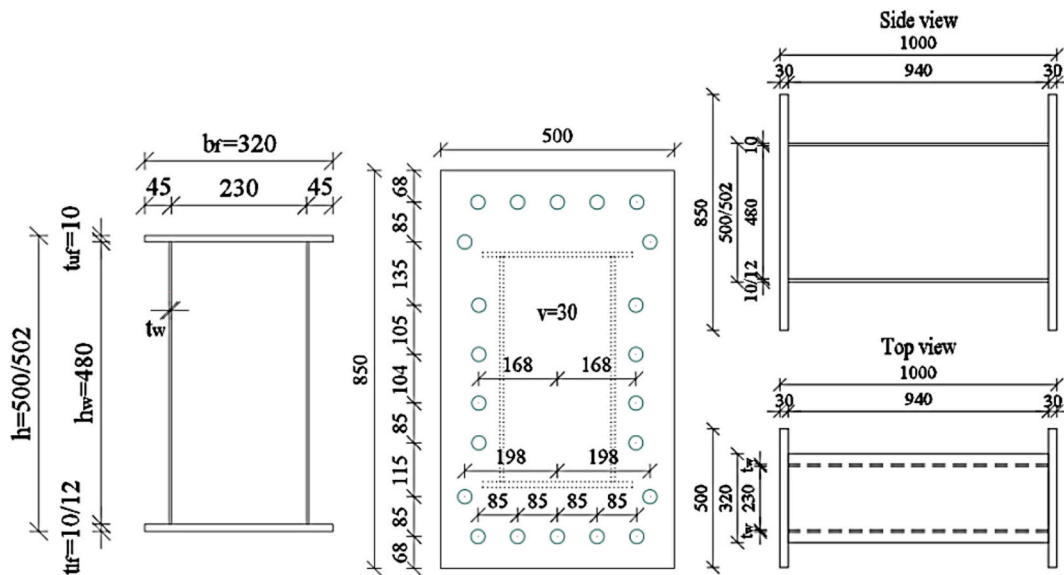


Fig. 3. Left: Cross-section of the test specimens; Middle: Geometry of the endplate; Right: Side and top views of the test specimens, dimensions in mm [13].

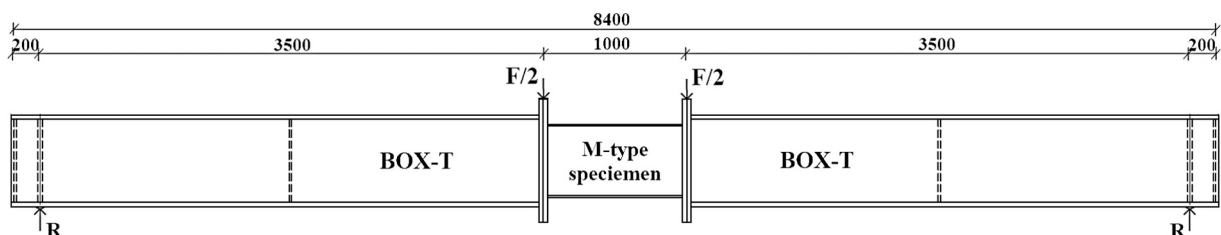


Fig. 4. General test setup.

Table 5
Load capacities obtained from laboratory experiments.

Name	M_{Rd} [kNm]
BOX4-355	849.205
BOX4-700	1260.683
BOX4-H1	1007.283
BOX4-H0	1231.353
BOX6-H0	1304.188
BOX8-H0	1431.098

Table 6
Maximum and minimum imperfections measured on a series of laboratory experiments.

b/...	Maximum		Minimum	
Imperfection	Web	Flange	Web	Flange
BOX4-355	140	530	560	845
BOX4-700	<u>535</u>	<u>750</u>	635	880
BOX4-H1	190	425	290	<u>625</u>
BOX4-H0	130	420	<u>145</u>	<u>600</u>
BOX6-H0	245	575	<u>1025</u>	<u>2130</u>
BOX8-H0	550	856	<u>735</u>	<u>1730</u>

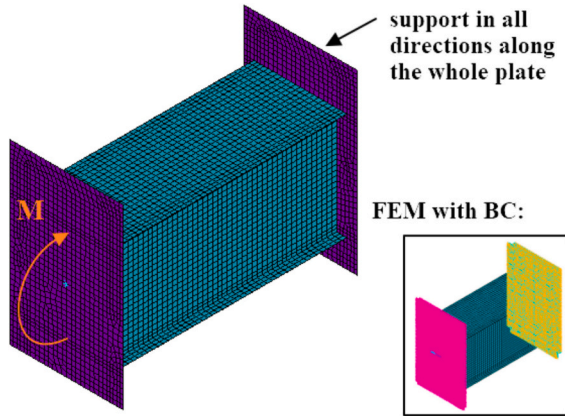


Fig. 5. Finite element model (mesh, boundary and loading conditions).

occurred due to local buckling of the top flange. The maximum load sustained by a specimen was always evident, as indicated by a clear peak on the F-d curve. The load capacities obtained in the laboratory experiments are shown in Table 5.

Prior to the experimental tests, each test specimen was subjected to detailed imperfection measurements to accurately determine the shape and geometric imperfections. After the tests, coupon tests were cut from the specimens and material tests are carried out to determine the material behaviour of the plates. Table 6 shows the minimum and maximum values of the measured imperfections and underlining indicates the one used for validation.

3.3. Numerical model development

3.3.1. Geometry of the numerical model

Based on the laboratory tests, a numerical model is developed using Ansys 19.2 finite element software [14]. The model is verified by checking the mathematical and geometrical adequacy of the finite elements and validated by comparing to the test results. The numerical model presented in this paper is the internal part of the global test specimens subjected to a concentrated bending moment at both ends. For the load application, the end cross-section nodes are coupled to a master node and all degrees of freedom are assigned to the coupling sets.

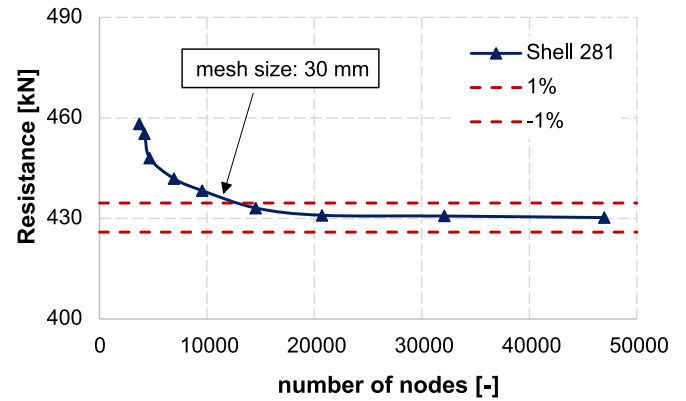


Fig. 6. Mesh sensitivity study and the applied mesh size for one geometry.

The master node is located in the centre of gravity of the end cross-sections. The developed model is a full shell model using eight-node thin shell elements as shown in Fig. 5 together with the finite element mesh, boundary and loading conditions.

3.3.2. Applied material model

The applied material model is based on the international recommendations and given in prEN 1993-1-14. For the nonlinear calculations, the multilinear material model developed by Gardner et al. [15,16] is used for normal strength steel (NSS). This material model is defined by three basic parameters – elastic modulus (E), yield strength (f_y) and tensile strength (f_u).

For high strength steel (HSS), the Ramberg-Osgood type material model is used in numerical simulations, which is a nonlinear elastic-plastic material model that applies strain hardening. Measurement data are used to estimate the parameter n in the Ramberg-Osgood type material model formula according to Eqs. (13)–(14), in the present study $n = 14$ is used based on previous material tests available to the authors on the steel materials under investigation [17].

$$\varepsilon = \frac{\sigma}{E} + 0.002 \cdot \left(\frac{\sigma}{f_y} \right)^n \quad (13)$$

$$n = \frac{\log\left(\frac{\varepsilon_1}{\varepsilon_2}\right)}{\log\left(\frac{\sigma_1}{\sigma_2}\right)} \quad (14)$$

All calculations assume an elastic modulus of 210 GPa with a Poisson's ratio of 0.3.

3.3.3. Applied imperfections

The effect of geometric imperfections and residual stresses are considered separately in both deterministic and stochastic analysis. Somodi [18] gives a formula for the magnitude of geometric imperfections based on measured data. Thus, the expected value of the geometric imperfections is calculated according to Eq. (15).

$$e_0 = \frac{b}{68 \cdot t} \quad (15)$$

The shape of the geometric imperfection is taken by the first eigen-mode magnified to the above given amplitude.

The residual stresses [2] at the web-flange junction are tension-type stresses equal to the yield strength. At the middle of the plates compressive residual stresses are applied with a maximum value of $0.13 \cdot f_y$ (the width of the tensioned and compressed parts is calculated by equilibrium conditions).

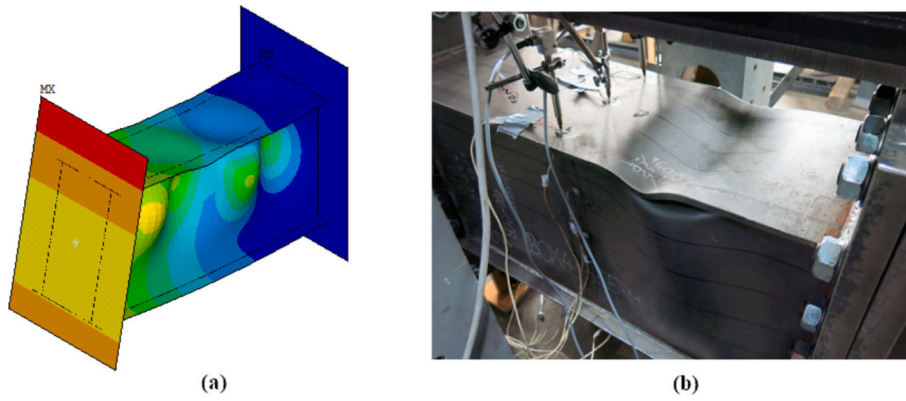


Fig. 7. Failure mode a) in the numerical model and b) in the laboratory for one specimen.

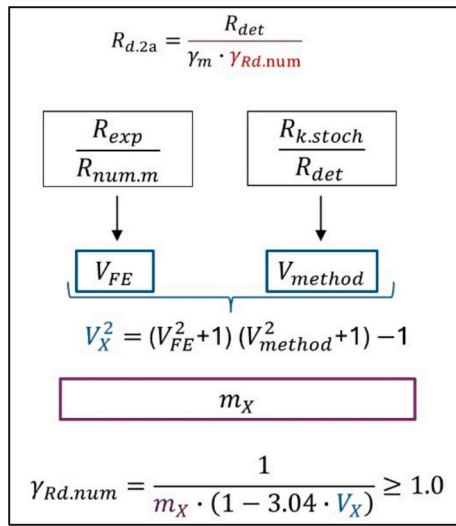


Fig. 8. Overview of 'Method 2a'.

3.3.4. Analyzed failure mode and resistances

For the FE model, a mesh sensitivity test is first performed on each test specimen to select the required mesh size (Fig. 6).

Local buckling of the top flange was the failure mode for all specimens. The numerical model reproduces the failure mode well, as illustrated in Fig. 7. In the validation, the load capacity values for the

virtual and laboratory experiments show good agreement with a difference of less than 5 %.

3.4. Description of 'method 1' and the two versions of 'method 2'

3.4.1. 'Method 1'

Stochastic analysis is performed in which the probability variables are the followings:

- geometric dimensions,
- material properties,
- amplitude of imperfections.

The partial factor γ_{M^*} is determined according to Eq. (16).

$$\gamma_{M^*} = R_{k,stoch,M} / R_{d,stoch,M} \quad (16)$$

Dividing the value of R_{det} by $\gamma_{FE,k}$ and γ_{M^*} , the design value of the resistance is obtained according to 'Method 1': $R_{d,1}$. Hence, it can be calculated according to Eq. (17).

$$R_{d,1} = \frac{R_{det}}{\gamma_{FE,k} \cdot \gamma_{M^*}} \quad (17)$$

3.4.2. 'Method 2'

In the case of 'Method 2', the partial factor on the resistance side is not treated as a single variable but is separated according to the different uncertainties. 'Method 2.a' and 'Method 2.b' versions are developed, their calculations are given by Eqs. (18)–(19).

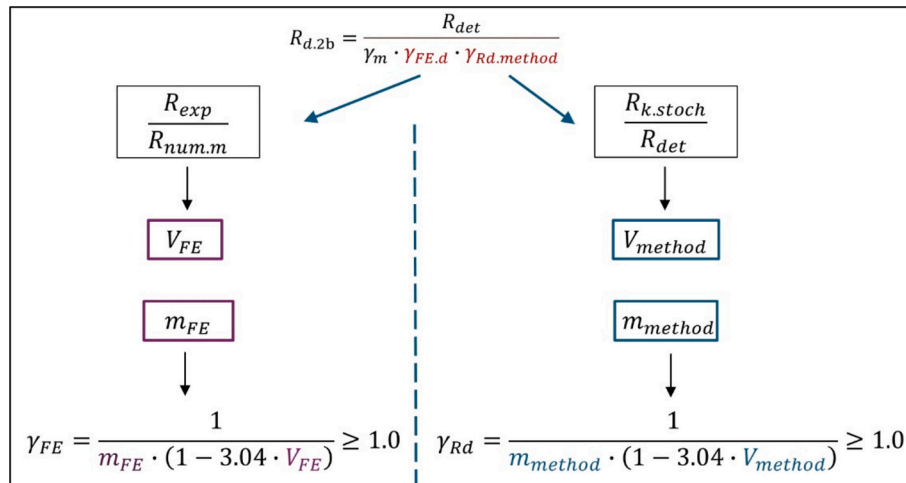


Fig. 9. Overview of 'Method 2b'.

Table 7
Determination of γ_{FE} based on laboratory tests.

Name	Ratio
BOX4-355	1.0227
BOX4-700	1.0445
BOX4-H1	1.0260
BOX4-H0	1.0198
BOX6-H0	0.9694
BOX8-H0	0.9691
average	1.0086
k_n	1.64
V_x	0.0317
γ_{FE}	1.05

Table 8
Probability variables for stochastic analysis.

Variable		Distribution type	Mean value	V_x
Yield strength	f_y	Lognormal	Nominal*1.2	0.05
Web height	h_w	Normal	Nominal	0.009
Web thickness	t_w	Normal	Nominal	0.05
Flange width	b_f	Normal	Nominal	0.009
Flange thickness	t_f	Normal	Nominal*0.98	0.5
Tensile strength	f_u	Lognormal	Nominal*1.15	0.04
Imperfection amplitude	e_0	Gamma	$b/68 t$	

$$R_{d.2a} = \frac{R_{det}}{\gamma_m \bullet \gamma_{Rd.num}} \quad (18)$$

$$R_{d.2b} = \frac{R_{det}}{\gamma_m \bullet \gamma_{FE.d} \bullet \gamma_{Rd.method}} \quad (19)$$

These two sub-variants are similar, differing only in whether the uncertainties in the calculation model and calculation method are considered separately. The difference between the two methods is illustrated in Figs. 8–9. In ‘Method 2a’, m_x is obtained as the average of both $R_{exp}/R_{num.m}$ and $R_{k.stoh}/R_{det}$ considered together (in this case it is the mean of 14 ratios - 6 from experimental tests and 8 from FEM study). On the other hand, in ‘Method 2b’, m_x is split into m_{FE} (average of 6 values) and m_{method} (averaged of 8 values).

4. Result discussion

4.1. Calculation results according to ‘method 1’

4.1.1. Determination of the model factor (γ_{FE})

In the determination of the model factor γ_{FE} , the load carrying capacity obtained from an accurate numerical model constructed with the measured geometry, material properties and imperfection amplitudes are compared to the load carrying capacities measured in the tests, as described in Section 3.2. Based on the six R_{exp}/R_{num} ratios, the γ_{FE} is determined using Eqs. (3)–(4) according to the design specifications of prEN 1993-1-14: 2020 [2]. The k_n parameter in the equation equals to 1.64. The result of the calculation leads to the value of $\gamma_{FE} = 1.05$. The details of the calculation process and the ratio for all the six specimens

Table 9
Load carrying capacities based on the deterministic calculations.

	M_R [kNm]
BOX-1	2509.50
BOX-2	2153.07
BOX-3	2007.29
BOX-4	1891.58
BOX-5	1759.33
BOX-6	1604.45
BOX-7	1597.92
BOX-8	1762.42

are given in Table 7.

This chapter presents the methodology of the model factor determination. The achieved value is valid and can be used for design purposes for bending-type failure mode of steel box sections, only if shell element models are applied with similar assumptions as used in the present paper.

4.1.2. Input parameters for stochastic analysis

After the determination of the model factor, Monte Carlo simulations are carried out using the eight test girders and the lower 5 % and 0.1 % percentile values of the FEM-based load carrying capacity as a function of relative slenderness are determined. In order to determine γ_M^* uncertainties related to geometry, material and imperfection are taken into account as described in Section 3.2. In the stochastic analysis, the input parameters of the numerical model are taken as probability variables, whose type of distribution, mean value and coefficient of variation are summarized in Table 8.

The expected values and standard deviations of the probability variables are always taken from Annex E of prEN 1993-1-1:2020 [19], except for the magnitude of the geometric imperfection, for which data are used based on a large number of geometrical measurement campaign made by the authors and taken from international literature. The expected value of the amplitude of the geometric imperfection is calculated using the formula presented in Section 3.3.3. based on [18]. Somodi [18] proposed to use the gamma distribution function for local buckling type imperfection of welded plated structures with parameters of $\alpha = 3.81$ and $\beta = 0.2625$ based on a large database of measurement data. For the other parameters, a normal distribution is used as recommended by JCSS [20] and a lognormal distribution for yield strength and tensile strength.

4.1.3. Details of the stochastic analysis

In the stochastic analysis, Latin Hypercube Sampling (LHS) method is employed due to its unique capability of selecting values at the distribution’s extremities with low probability. This attribute is particularly crucial in this context, as the goal is to identify the characteristic and design resistance to bending, which involves determining the lower 5 % and 0.1 % percentile. An additional benefit of LHS is that it prevents clustering of selected values during random generation, unlike the conventional Monte Carlo method, which can often result in points being closely grouped in some ranges while leaving other ranges empty, especially with small sample sizes.

The required number of samples (n) for a Monte Carlo simulation is dependent on the probability being sought. To accurately evaluate low probability ranges, if P is the desired probability, a minimum sample size ranging from $30/P$ to $100/P$ is necessary for reliable results. For the characteristic resistance ($P = 5 \%$) $n \approx 600$ – 2000 specimens should be tested to reduce the number of required samples, whereby the response surface method can be used. This method involves fitting a response surface to the Monte Carlo simulation results using various regression models. The runs use a quadratic (second order) regression model according to Eq. (20) where x is the vector of the variables, A is the quadratic coefficient matrix, k is the linear coefficient matrix and m is a constant.

$$W_{resp.surf} = \bar{x} \bullet A \bullet \bar{x}^T + \bar{k} \bullet \bar{x}^T + m \quad (20)$$

The number of probability variables determines how many samples are needed to correctly fit the response surface in the Monte Carlo simulation. In the stochastic analysis, we use seven probability variables with 21 coefficients in the second-degree polynomial and at least 80 samples (points) are required to produce the fit [20]. When running the new Monte Carlo simulation on the fitted response surface, the resistances are not determined by new runs, but by using the approximate function generated for the response surfaces, which made the calculation of the results much faster. Thus, in the first run $n_1 = 80$ separately

Table 10
Results of the Montae Carlo simulations – Method 1.

	BOX-1	BOX-2	BOX-3	BOX-4	BOX-5	BOX-6	BOX-7	BOX-8
$\bar{\lambda}$	0.541	0.676	0.773	0.902	1.082	1.352	1.515	1.731
$R_{k, \text{stoch}}$	2415.43	2131.06	2063.04	1875.37	1793.03	1644.79	1663.23	1757.06
$R_{d, \text{stoch}}$	2349.77	2012.70	1902.65	1757.04	1619.02	1522.38	1530.29	1578.21
R_{det}	2509.49	2153.07	2007.29	1891.58	1759.33	1604.44	1597.92	1762.42
γ_{FE}	1.05							
γ_{M^*}	1.03	1.06	1.09	1.07	1.10	1.08	1.09	1.11
$R_{d,1}$	2323.61	1939.71	1760.78	1688.91	1516.66	1419.86	1401.69	1506.34

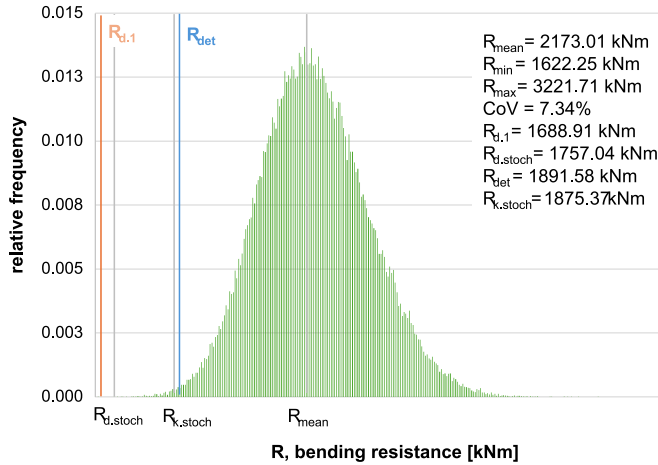


Fig. 10. Histogram in the case of BOX4 according to Table 10.

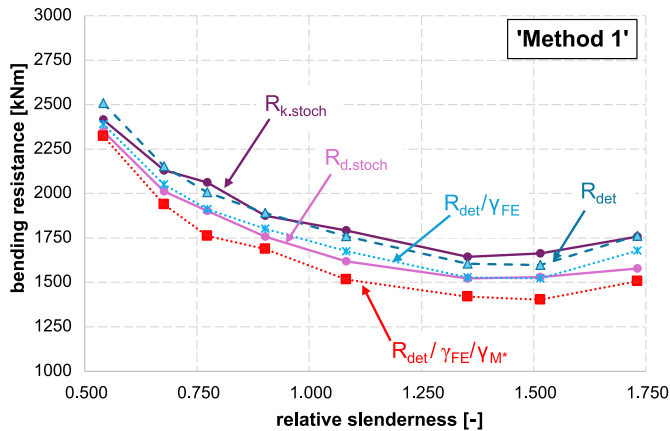


Fig. 11. Results of 'Method 1' depending on the local plate slenderness.

for each of the 8 tests, based on which unique response surfaces were obtained and in the second run $n_2 = 100,000$ load capacity values are obtained, which gives the sample number needed for the evaluation. (In total, $80 + 10,000$ samples were evaluated for each slenderness.)

Table 11
Results of 'Method 2a'.

	BOX-1	BOX-2	BOX-3	BOX-4	BOX-5	BOX-6	BOX-7	BOX-8
$\bar{\lambda}$	0.541	0.676	0.773	0.902	1.082	1.352	1.515	1.731
R_{det}	2509.49	2153.07	2007.29	1891.58	1759.33	1604.44	1597.92	1762.42
γ_{m}	0.97							
$\gamma_{\text{Rd,num}}$	1.13							
$R_{d,2a}$	2282.32	1958.17	1825.58	1720.35	1600.07	1459.20	1453.27	1602.88

4.1.4. Results of the deterministic analysis

Before executing stochastic analysis, deterministic calculations are also executed. For the slenderness values presented in Section 3.1, the ultimate load is determined by calculating the nominal value of the material and geometric parameters and mean value of the geometric imperfections. The results are given in Table 9.

4.1.5. Results of 'method 1' – γ_{M^*}

As described in Section 3.4., the design value is determined according to 'Method 1'. The results are given in Table 10.

For the case of one slenderness, Fig. 10 presents the results and the relevant values of the resistance in a histogram.

The results of 'Method 1' are shown in Fig. 11 depending on the slenderness ratio. The vertical axis shows the load capacity (bending resistances); the horizontal axis represents the relative slenderness ratio of the flange plates applied. The lower 5 % and 0.1 % percentile values per slenderness from the stochastic analysis are presented in the diagram. Results of the deterministic analysis are also shown for direct comparison for all the 8 analyzed slenderness values. The results are also divided by their relevant partial safety factors γ_{FE} and γ_{M^*} ; the relevant values are also presented on the diagram.

As shown in Table 10, the value of γ_{FE} is independent of slenderness, but the partial factor γ_{M^*} was determined per slenderness according to Eq. (16). The values of the partial coefficient γ_{M^*} per slenderness are shown in Table 10. The results shown in Fig. 11 prove that the resistances achieved with the deterministic analysis are close to the characteristic resistances taken from the stochastic analysis. This is a good test and control to prove the applicability of the applied calculation results thus the equivalent geometric imperfections are determined to provide close values of the characteristic resistance. Results also show, that this evaluation method leads to safe side resistances since all resistance values by applying the partial factors are lower than the design values taken from the stochastic analysis.

4.2. Calculations according to 'method 2'

4.2.1. Calibration of γ_{m}

The material grade used in the present study is S355. The expected value and standard deviation of the material models are taken from Annex E of prEN 1993-1-1:2020 [19]. Following the procedure described in Section 2.2: $f_{y, \text{mean}} = 1.2 \cdot 355 \text{ MPa} = 426 \text{ MPa}$ and the coefficient of variation $V_X = 0.05$. According to Eq. (9) the material related partial factor can be determined by: $R_{d, \text{fy}} = 365.93 \text{ MPa}$, so $\gamma_{\text{m}} = 355/365.93 = 0.97$.

Table 12
Results of 'Method 2b'.

	BOX-1	BOX-2	BOX-3	BOX-4	BOX-5	BOX-6	BOX-7	BOX-8
$\bar{\lambda}$	0.541	0.676	0.773	0.902	1.082	1.352	1.515	1.731
R_{det}	2509.49	2153.07	2007.29	1891.58	1759.3	1604.44	1597.92	1762.42
γ_m	0.97							
$\gamma_{FE,d}$	1.10							
$\gamma_{Rd,method}$	1.08							
$R_{d,2}$	2187.22	1876.57	1749.51	1648.66	1533.39	1398.40	1392.71	1536.09

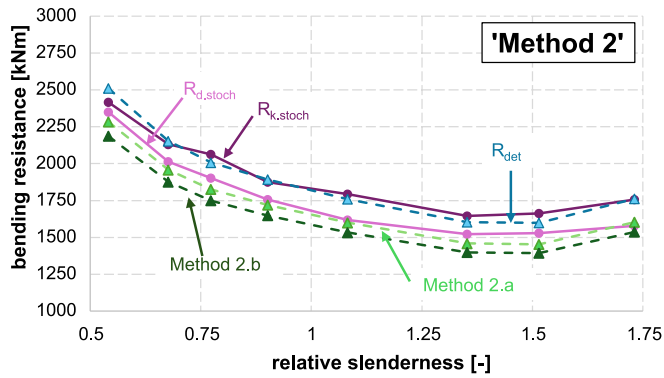


Fig. 12. Results of 'Method 2' depending on the local plate slenderness.

4.2.2. Results of 'method 2a'

The second task after the determination of γ_m is the calibration of γ_{Rd} . In the determination of the partial factor $\gamma_{Rd,num}$, the uncertainty of the resistance model is taken into account. The calculation of $\gamma_{Rd,num}$ is performed with Eq. (3), using $k_n = 3.04$, whereby the coefficient of variation and mean value calculated from $R_{exp} / R_{num,m}$ and $R_{k,stoich} / R_{det}$ ratios are evaluated together, as shown in Fig. 8. The result of the calculation leads to the value of $\gamma_{Rd,num} = 1.13$. The value of R_{det} should be divided by the product of γ_m and $\gamma_{Rd,num}$, which slenderness-independent product of the partial factors is equal to 1.10. The results are given in Table 11.

4.2.3. Results of 'method 2b'

In the 'b' version of 'Method 2' $\gamma_{FE,d}$ should be calibrated. The calculation is performed with Eq. (3) comparing R_{exp} and $R_{num,m}$, k_n is chosen to determine the design value according to EN1990 [1] Table D1. The result of the calculation is $\gamma_{FE,d} = 1.10$. The partial factor $\gamma_{Rd,method}$ should be also determined, in which the uncertainty of the calculation method (imperfection definition method) is taken into account, the calculation is performed with Eq. (3). The load capacity values obtained by the deterministic calculation are compared with the "reference" characteristic load capacity values determined by the stochastic calculation. Based on the ratio $R_{k,stoich} / R_{det}$ for 8 different slendernesses the mean value and the coefficient of variations are calculated, $k_n = 3.04$. The result of the calculation is $\gamma_{Rd,method} = 1.08$. The value of R_{det} should be divided by the product of γ_m , $\gamma_{FE,d}$ and $\gamma_{Rd,method}$, which slenderness-independent product of the partial factors is equal to 1.10. The results are given in Table 12.

4.2.4. Results of 'method 2'

The results of 'Method 2' are shown in Fig. 12. The vertical axis shows the load carrying capacity and the horizontal axis represents the relative slenderness ratios, similarly as presented for 'Method 1'. The resistances from the deterministic calculation, as well as the values divided by the γ_m , $\gamma_{Rd,num}$ and $\gamma_{Rd,method}$ partial factors, are shown on the diagram. Results show, that 'Method 2.b' leads to slightly smaller resistances than 'Method 2.a', but both values are smaller than the design values coming from the stochastic analysis, which means, this evaluation method leads also to safe side resistances for the analyzed failure

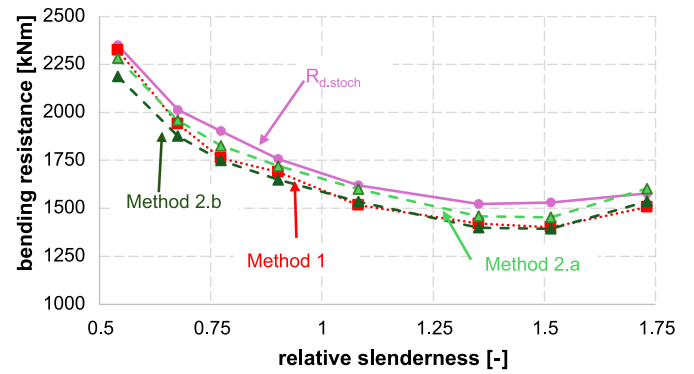


Fig. 13. Comparison of 'Method 1' and 'Method 2' results.

mode and resistance model.

4.3. Comparison of results obtained using the three methods

Fig. 13 summarises the results obtained from the two methods in relation to the design values determined from the stochastic calculation. It can be seen that the design values obtained from the two methods are close to each other and both of them envelop the lower 0.1 % lower percentile curve obtained from the stochastic analysis, i.e. they are on the side of safety.

The results also show that if the partial factor $\gamma_M = 1.10$ (slenderness-independent) is applied to the area of the bridge design and multiplied by the model factor ($\gamma_{FE} = 1.05$, as given in Section 4.1.1.), the results are on the side of safety.

5. Summary and conclusions

In the present paper the resistance-side model uncertainty is investigated for bending-type failure mode of welded box-section girders. A numerical parametric study is performed by deterministic and stochastic nonlinear analyses to determine the required partial safety factors. Based on the numerical study the following conclusions are drawn:

- Two methods are compared to properly consider the uncertainties on the resistance side. The methods are referred to as 'Method 1' and 'Method 2' (the latter has two variants). The methods differ in the way of the determination of partial factors taken into account on the resistance side. In 'Method 1', γ_M^* is determined without splitting material and resistance model uncertainties and a separate $\gamma_{FE,k}$ model factor is defined accounting for the uncertainties of the numerical model. In 'Method 2', γ_M^* is decomposed, and the uncertainty of the numerical model, the resistance model and material properties are determined separately.
- A deterministic analysis is also carried out to determine the characteristic resistances for different slenderness ratios, from which the design value of the resistance is determined using all three methods investigated.
- The design value calculated by the three methods are close to each other and all three are below the design value (reference) determined

by stochastic analysis. It means, all three methods are on the safe side and applicable for design purposes.

- Example for the calculating the model factor is given in the present paper, which value for the developed shell-type numerical model, FE-software and bending-type failure mode of steel welded box-section beam is 1.05. It is important to highlight that the conclusions of this paper are based on one set of experiments using a single software package. Research on the model factor needs to be extended to more experiments and to other FE software. The factor given in this point is not a general one, it is appropriate for the specific case presented in the current paper by using similar assumptions as used here. However, once the model has been validated, it can be relatively easily determined for other cases as described above.
- The aim of the study is not only to determine the model factor, but also to invent and present a model factor determination procedure and to raise questions about the appropriate description of the uncertainties on the resistance side.
- Three methods for the application of the model factor are also presented. It is proven, that the proposed method of the prEN 1993-1-14 leads to safe side resistances for the analyzed numerical model and failure mode.

CRediT authorship contribution statement

E. Bärnkopf: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation.
B. Kövesdi: Writing – review & editing, Supervision, Resources, Project administration, Investigation, Funding acquisition, Conceptualization.
A. Horváth: Supervision, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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