



A novel non-linear orthotropic constitutive model with deformation induced fiber reorientation effect for composite laminate simulation

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ABSTRACT

Fiber reorientation plays an important role in the mechanical behavior of composites, since this phenomenon is independent from the applied constitutive laws and can lead to stiffer or softer nature. Previous general approaches have been relied on the small time step size to model such effect in explicit simulations. A novel non-linear in-plane elastic constitutive model is developed for continuous fiber reinforced laminates to account for this problem. The model consists of a non-linear elastic constitutive model with distinguished tensile and compressive behavior along with deformation induced fiber reorientation, which is not handled by the finite element formulations and extended for implicit approach as well. The proposed material model is also implemented as a user-defined material subroutine in the commercial finite element software LS-DYNA. The implementation includes both explicit and implicit formulation approaches for thin shell and solid elements, respectively. Moreover, the proposed material model is also validated via simulations of measurement results performed on Toray Cetex[®] TC1225 - Standard Modulus Carbon 145 gsm UD Tape 34% RC and glass fiber reinforced polyester materials.

1. Introduction

Application of continuous fiber reinforced composites plays an increasing role in different industries, such as in aerospace [1,2] or in automotive industry [3]. This trend arises due to the superior strength-to-weight ratio of fiber materials, e.g. modern carbon fibers, which have a modulus four times higher than steel, while their density is only 25% that of steel [4]. To assess the risk factors, that may occur during the development of composite structures, it is common, that the so-called building block approach is utilized [5]. In this step by step method the advancement to the next level can be made by carrying out large number of measurements to recognize the technological risks before proceeding to a more complex level. With proper numerical or analytical models of mechanical behavior of composites, the number of required measurements can be decreased. Therefore, the cost can be reduced greatly, while accelerating the process and maintaining the same design quality.

In this contribution the term *draping* refers to the deformation induced reorientation process of fibers. It is common, that fiber reorientation occurs in transverse isotropic materials due to mechanical fields [6]. This phenomenon also plays a big role in the nature of composites, as the reorientation of fibers in the direction of the load makes

the material appear stiffer, than expected. Sun and Zhu [7] showed this important stiffening behavior in their study, where draping was paired with an overstress viscoplasticity model. In their case, the consideration of fiber reorientation improved the model fit to the measurements results, which consist of specimens of symmetrical laminates subject to tensile loading. Fig. 1 gives two examples of the apparent stiffening nature as a consequence of draping. They also gave a procedure to account fiber reorientation in the case of rectangle specimens.

Eskandari et al. [8] investigated the effect of draping in the case of a damage model. They also proved that, the reorientation of fibers is a significant contributor to the evolution of damages in different directions. Moreover, their derived formula is based on time-integration of fiber rotation tensor.

Furthermore, Tabiei and Ivanov [9] demonstrated, that it is also possible, that the transverse direction aligns to the loading direction, which results in a less stiff behavior.

In addition, this reorientation phenomenon results in the loss of the initial orthogonality of the composite laminate. The previously showed examples, where draping is taken into account in the case of linear [9] and non-linear [7] material models, suggest, that it is worth to consider such effect. One possible pragmatic way to deal with this effect is to use

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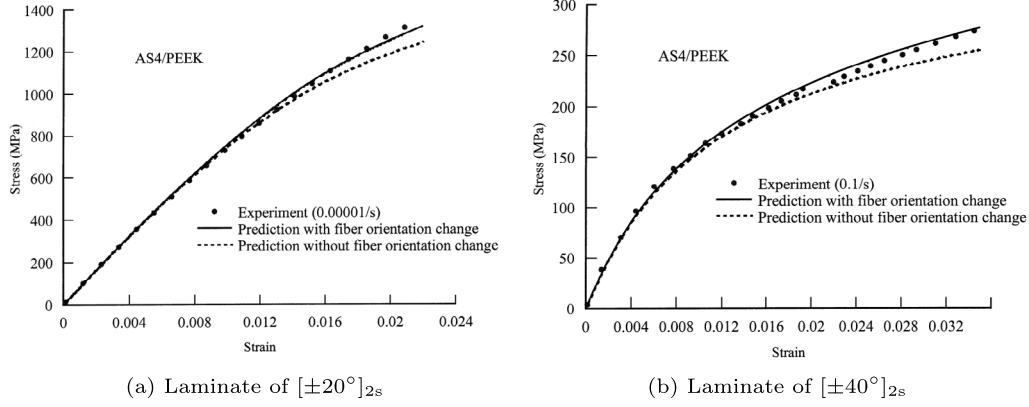


Fig. 1. The fitted model to measurement data with draping [7].

a non-linear elastic material model built-in a finite element software, e.g. MAT 58 in LS-DYNA [10] and define the shear stress - shear angle curve such that it captures the apparent effect of this phenomenon. Although, with this approach the material response to mechanical loads can be accurate, but tuning up the parameters of failure models can be challenging, since the underlying physical phenomenon is changed (in reality the fibers alleviate the load from the matrix material). Moreover, the draping effect is independent from the applied reference stress-strain curves, thus this hardening curve depends on the measurement type. Which means, that this phenomenological approach can lead to different input curves for different layups, consequently it is not a general nor representative method. Therefore it is more natural to model the phenomenon itself, rather than its effect. Another approach could be using embedded bar or membrane model [11–14] instead of an orthotropic model. In this case, the embedded bar model would be able to account for fiber reorientation, as all fiber bundles are modeled separately, but this method comes with a large computational cost. Moreover, the membrane model (as an alternative to the embedded bar) is not capable to represent the draping effect, since the fiber orientations are fixed to the solid elements (in which the membrane elements are embedded). Fig. 2 depicts the draping phenomenon due to deformation.

A novel non-linear in-plane elastic constitutive model is developed for continuous fiber reinforced laminates to account for this problem. The model consists of a non-linear elastic constitutive model with distinguished tensile and compressive behavior along with deformation induced fiber reorientation, which is not handled by the finite element formulations and extended for implicit approach as well. The proposed material model is also implemented as a user-defined material subroutine in the commercial finite element software LS-DYNA. The implementation includes both explicit and implicit formulation approaches for thin shell and solid elements, respectively. Moreover, the proposed material model is also validated via simulations of measurement results performed on two continuous fiber reinforced composite materials, namely Toray Cetex® TC1225 - Standard Modulus Carbon 145 gsm UD Tape 34% RC and glass fiber reinforced polyester manufactured with vacuum infusion.

The application of the new material model offers more accurate designing concepts in areas, where symmetric layup of $\pm\theta$ degree occurs, since these configurations are more sensitive to fiber reorientation, than the conventional quasi-isotropic stack-ups. Such applications for example are design of wing structures [15] and pressure vessels [16] or even ply-angle optimization methods utilizing FEM can benefit from the new implemented material model [17].

The structure of the paper is the following, some modeling aspects of composite laminates are presented in Section 2. After that, the details of the proposed material model can be read in Section 3, while the numerical implementation is discussed in Section 4. At last, Section 5 describes the validation process of the proposed material model following by a brief conclusion in Section 6.

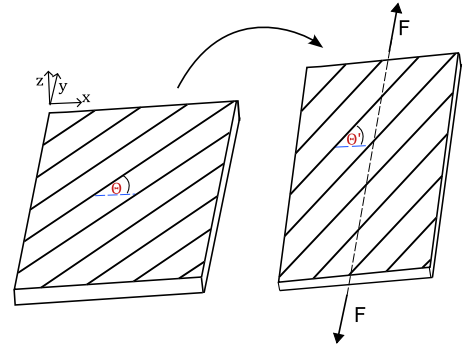


Fig. 2. Deformation induced fiber reorientation.

2. Modeling approaches of laminated composites

Many ply-wise (lamina level) constitutive models have been developed to capture the non-linear nature of composite laminates. Among these several constitutive models rely on failure mechanisms aided with damage laws, where the damage parameters decrease the stress bearing capability of the material. Continuum damage models (CDM) utilize internal (damage) variables to describe the propagation of the damage in a continuum constitutive model. One possible way to achieve such effect is by degrading the elastic moduli directly [18–23], as Eq. (1) characterizes the in-plane compliance matrix with Voigt notation,

$$\mathbf{H} = \begin{bmatrix} \frac{1}{(1-d_1)E_1} & H_{12} & 0 \\ H_{21} & \frac{1}{(1-d_2)E_2} & 0 \\ 0 & 0 & \frac{1}{(1-d_{12})G_{12}} \end{bmatrix}, \quad (1)$$

where $E_1, E_2, G_{12}, \nu_{12}, \nu_{21}$ are the in-plane elastic orthotropic properties (1 refers to the fiber direction and 2 to the transverse direction) and d_i are the associated damage variables respectively. One big difference of the models (apart from the formulation of the damage variables) is the treatment of the off-diagonal entries of H_{12} and H_{21} , although all models strive to hold the symmetric property of $\mathbf{H}$ (which is of course should be maintained, since it is a direct consequence of the Hooke's law), namely

$$H_{12} = H_{21}. \quad (2)$$

The trivial solution for that is to decouple the degradation of the principal material directions, by neglecting the Poisson's effect [18,20, 23], thus the off-diagonal elements read as,

$$H_{12} = -\frac{\nu_{21}}{E_2}, \quad (3)$$

$$H_{21} = -\frac{\nu_{12}}{E_1}. \quad (4)$$

Another approach to maintain the symmetric condition is to incorporate the degradation of the Poisson's terms into the model [19,21,22], with the following constraint,

$$\frac{1-d_{\nu_{12}}}{1-d_1} = \frac{1-d_{\nu_{21}}}{1-d_2}. \quad (5)$$

Moreover, other models decreasing the effective stresses (related to the undamaged state) as a function of the damage variables (Eq. (6)) to grasp the ongoing damage accumulation in the material, [24–26] by letting

$$\sigma_i = (1-d_i) \sigma_i^{\text{eff}}, \quad (6)$$

where, σ_i is an entry of σ in Voigt notation. It is worth to mention, that Eq. (6) is a result of a more general form of

$$\sigma^{\text{eff}} = \mathbf{M} : \sigma, \quad (7)$$

where σ is in a second-order tensor form and $\mathbf{M}$ is a fourth-order tensor called the damage effect tensor [27]. In both cases the damage variables are constrained as,

$$d_i \geq 0 \quad (8)$$

to guarantee the irreversibility of the damage evolution.

While other models apply linear or non-linear elasticity with viscous or plastic effects [28,29], for small-strain regime the non-linear formalism is based on orthotropic Hookean stiffness tensor. The augmentation to finite strain realm can be made with the help of hyperelasticity [30].

3. Proposed material model

As it was discussed in Section 2, there are multiple ways to model the non-linear behavior of fiber reinforced composite plates. The present model focuses on non-linear elastic behavior and implemented in LS-DYNA [31]. The choice of non-linear elasticity arises because damage-related material formulations cannot produce a non-linear response in the pre-loading state before the damage-initiation criterion is reached (except in the shear direction in specific cases [24]), as a consequence of the damage laws. However, distinguishing between the tensile and compressive directions is one appealing property of these damage models, which is also included in certain non-linear elastic materials (e.g. in LS-DYNA finite element software MAT 40 and 58 [10]). Another phenomenon, which serves as an improvement to the previously discussed models is the draping or fiber reorientation due to shear deformation, because there are implemented material models, which handle draping, but those are not applicable to laminated composite plates [10]. Overall, the three main features of the present orthotropic material model are the followings:

- elastic non-linear constitutive in-plane behavior,
- tensile and compressive directions are distinguished,
- draping phenomenon is taken into account.

It should be noted that the non-linearity of the model does not include any damage model.

3.1. The constitutive relations

The applied constitutive equations describe the ply-wise nature of composite plates. The inter-laminar behavior is not handled by this model, since in the case of thin plates, i.e. the thickness of the plate is negligible compared to the planar dimensions, this behavior does not play a significant role [32], thus plane-stress state can be assumed. Moreover, the same tangent modulus formulation is used as in the case of all material models, where moduli degradation happens (for example in the case of [33,34]). In other words, incremental approach is utilized

to describe the in-plane stresses $\sigma^T = [\sigma_1, \sigma_2, \tau_{12}]$ as a function of the in-plane strains $\epsilon^T = [\epsilon_1, \epsilon_2, \gamma_{12}]$, where ϵ_i are engineering strains, i.e.

$$\Delta \sigma = \mathbf{C}(\epsilon) \Delta \epsilon, \quad (9)$$

where Δ notation indicates the infinitesimal increments and $\mathbf{C}(\epsilon)$ is the stiffness matrix in accordance with the non-linear Hooke's law. Furthermore, in LS-DYNA the rigid body motion acts on the elements and the material subroutines are interpreted in the co-rotational or element-wise coordinate system of the element [35], which means that the applied constitutive equations do not need to deal with any (small or large) rigid body motions.

The literature contains many experiments [29,36,37], which on the one hand suggests, that in the fiber direction the behavior can be assumed as linear until failure occurs, therefore

$$H_{11} = (\mathbf{C}^{-1})_{11} = \frac{1}{E_{1(t \text{ or } c)}}, \quad (10)$$

where t stands for the tensile and c for the compressive directions. On the other hand, it is shown [36–39] that the order of strains remain in the small strain regime. In the transverse and shear direction the adopted version of the Ramberg–Osgood equation is applied [40,41] to describe the non-linear nature of the material, namely

$$\sigma = \frac{E\epsilon}{\left(\left(\frac{E\epsilon}{Y_0}\right)^p + 1\right)^{\frac{1}{p}}}, \quad (11)$$

where σ can stand for σ_2 or τ_{12} and ϵ for ϵ_2 or γ_{12} , while the Ramberg–Osgood parameters are the initial modulus E , asymptotic stress value Y_0 and the shape parameter p . Since the present material model distinguishes between the tensile and compressive behavior, the total number of six parameters correspond to the transverse directions, namely: $E_{2t}, E_{2c}, Y_{0t}, Y_{0c}, p_t$ and p_c . Three additional parameters are used for the shear direction: G, Y_0 and p . It is important, that in the present paper the values of the parameters must be greater than zero. Therefore, to obtain meaningful stress values for negative strains, Eq. (11) is modified to the following forms,

$$\sigma_2 = \begin{cases} E_{2t}\epsilon_2 \left(\left(\frac{E_{2t}\epsilon_2}{Y_{0t}} \right)^{p_t} + 1 \right)^{-1/p_t}, & \text{if } \epsilon_2 \geq 0 \\ E_{2c}\epsilon_2 \left(\left(\frac{E_{2c}|\epsilon_2|}{Y_{0c}} \right)^{p_c} + 1 \right)^{-1/p_c}, & \text{if } \epsilon_2 < 0 \end{cases} \quad (12)$$

$$\tau_{12} = G\gamma_{12} \left(\left(\frac{G|\gamma_{12}|}{Y_0} \right)^p + 1 \right)^{-1/p}. \quad (13)$$

By further simplifications Eq. (12) yields to

$$\sigma_2 = E_{2x}\epsilon_2 \left(\left(\frac{E_{2x}|\epsilon_2|}{Y_{0x}} \right)^{p_x} + 1 \right)^{-1/p_x}. \quad (14)$$

The corresponding secant moduli of the directions can be obtained from Eqs. (13) and (14), by taking the derivatives, which gives

$$E_2(\epsilon_2) = \frac{d\sigma_2}{d\epsilon_2} = E_{2x} \left(\left(\frac{E_{2x}|\epsilon_2|}{Y_{0x}} \right)^{p_x} + 1 \right)^{(-p_x+1)/p_x}, \quad (15)$$

$$G_{12}(\gamma_{12}) = \frac{d\tau_{12}}{d\gamma_{12}} = G \left(\left(\frac{G|\gamma_{12}|}{Y_0} \right)^p + 1 \right)^{(-p+1)/p}, \quad (16)$$

where x stands for t or c, which depends on the direction, i.e. the sign of ϵ_2 . The formulation of Eqs. (15) and (16) has a similar approach as the material models with damage, since equations can be transformed into the form of

$$E_2(\epsilon_2) = E_{2x}(1-d_2(\epsilon_2)), \quad (17)$$

$$G_{12}(\gamma_{12}) = G(1-d_{12}(\gamma_{12})), \quad (18)$$

but now d_i are not a "real" damage variables, since to achieve elastic behavior without hysteresis between the loading and relaxing phase d_i cannot be constrained as in Eq. (8).

3.2. The stiffness matrix

The problem with the above derived relation is the consideration of the Poisson-effect, since to maintain the symmetric property of the compliance matrix as in Eq. (2) the following equation must hold (which is the consequence of the secant moduli at undeformed state),

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}, \quad (19)$$

but during the deformation process the ratio $E_{1(t \text{ or } c)}$ and E_2 is not constant due to the changing secant moduli, while the Poisson's ratios remain the same. A possible solution is proposed by [42], which method uses the geometric mean to keep the symmetry condition. Therefore, the assembly of the compliance matrix yields (at a given ϵ)

$$\mathbf{H} = \mathbf{C}^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\sqrt{\frac{\nu_{21}\nu_{12}}{E_2 E_1}} & 0 \\ -\sqrt{\frac{\nu_{21}\nu_{12}}{E_2 E_1}} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}. \quad (20)$$

It should be mentioned here, that the calculation of ν_{12} from ν_{21} uses the initial moduli (which depend on the direction of the load) [42] while E_3 is assumed to be constant. It can be seen, that if Eq. (19) holds (and the moduli do not change), then $\mathbf{H}$ in Eq. (20) reduces back the form of the classical linear orthotropic compliance matrix. From Eq. (20) the stiffness matrix can be obtained as,

$$\mathbf{C}(\epsilon) = \begin{bmatrix} E_1/\nu & \sqrt{E_1 E_2 \nu_{12} \nu_{21}}/\nu & 0 \\ \sqrt{E_1 E_2 \nu_{12} \nu_{21}}/\nu & E_2/\nu & 0 \\ 0 & 0 & G_{12} \end{bmatrix}, \quad (21)$$

where $\nu = 1 - \nu_{12}\nu_{21}$.

It should be noted, that Eq. (20) is the consequence of the assumed plane-stress state. Consider the case, where all three dimensions are taken into account [42], then

$$\mathbf{H}^* = \begin{bmatrix} \frac{1}{E_1} & H_{12} & H_{13} & 0 & 0 & 0 \\ H_{21} & \frac{1}{E_2} & H_{23} & 0 & 0 & 0 \\ H_{31} & H_{32} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{31}} \end{bmatrix}, \quad (22)$$

where,

$$H_{12} = H_{21} = -\sqrt{\frac{\nu_{21}\nu_{12}}{E_2 E_1}}, \quad (23)$$

$$H_{13} = H_{31} = -\sqrt{\frac{\nu_{31}\nu_{13}}{E_3 E_1}}, \quad (24)$$

$$H_{23} = H_{32} = -\sqrt{\frac{\nu_{23}\nu_{32}}{E_2 E_3}}. \quad (25)$$

Which tensor gives the relations between the strain and stress increments, as

$$\begin{bmatrix} \Delta\epsilon_1 \\ \Delta\epsilon_2 \\ \Delta\epsilon_3 \\ \Delta\gamma_{12} \\ \Delta\gamma_{23} \\ \Delta\gamma_{31} \end{bmatrix} = \mathbf{H}^* \begin{bmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\sigma_3 \\ \Delta\tau_{12} \\ \Delta\tau_{23} \\ \Delta\tau_{31} \end{bmatrix}. \quad (26)$$

In the plane-stress state $\Delta\sigma_3 = 0$, $\Delta\tau_{23} = 0$ and $\Delta\tau_{31} = 0$, thus the corresponding rows and columns of $\mathbf{H}^*$ can be deleted (with one condition detailed below), therefore the obtained reduced form of $\mathbf{H}^*$ is $\mathbf{H}$ defined in Eq. (20). However, the constitutive relation in Eq. (9) should be extended with and extra equation to account for the out-of-plane strain as the result of the Poisson effect,

$$\Delta\epsilon_3 = -\sqrt{\frac{\nu_{31}\nu_{13}}{E_3 E_1}} \Delta\sigma_1 - \sqrt{\frac{\nu_{23}\nu_{32}}{E_2 E_3}} \Delta\sigma_2. \quad (27)$$

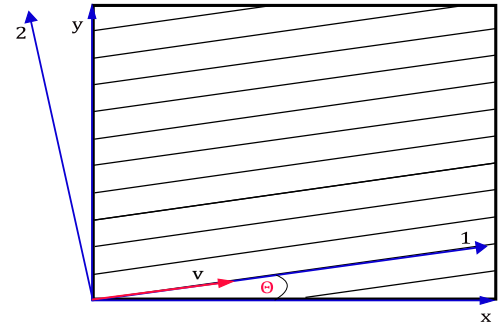


Fig. 3. The orientation of the fiber coordinate system.

4. Finite element implementation in LS-DYNA

As it was mentioned before for anisotropic materials strains and stresses are rotated into the element coordinate system, prior passing it to the material subroutine. Therefore, the updated stress should be also considered in this local coordinate system. As it was discussed in Section 3.1, incremental approach is applied, which means that, the stress vector σ (which comes from the Voigt notation of the Cauchy stress tensor) in step $n + 1$ is updated as,

$$\sigma^{n+1} = \sigma^n + \Delta\sigma^n, \quad (28)$$

where $\Delta\sigma^n$ is the stress increment in step n .

If draping is not considered, then the fiber coordinate system is identical with the element coordinate system, thus the stress increment can be calculated according to discretized form of Eq. (9), i.e.

$$\Delta\sigma^n = \mathbf{C}(\epsilon^{n-1}) \Delta\epsilon^n, \quad (29)$$

where $\Delta\epsilon^n$ is an input of the material subroutine [35] (from the solver) and ϵ^{n-1} is the total strain in step $n - 1$, which can also be tracked with an incremental approach. Finally, $\Delta\epsilon_3^n$ is overwritten, according to Eq. (27) (in case of shell implementation it would not be an issue if this equation were disregarded, as LS-DYNA does not, by default, change the thickness of thin shells [35]). In order to run implicit simulations, the material tangent modulus should also be provided, which in the case of non-linear elasticity equals to the stiffness matrix defined in Eq. (21).

Apart from implementing Eq. (11) (for the sake of simplified usage) into the material subroutine, arbitrary reference stress-strain curves can be fed to the model for the transverse (tensile and compressive) and shear direction, then the secant moduli are calculated with the help of built-in differentiating subroutines [35].

If draping is considered, then the original element coordinate system (x, y) and the fiber coordinate system ($1, 2$) may alter from each other during the simulation process. Since in the present model infinitesimal strain increments are used, the incremental gradient tensor can be approximated with the same manner as in [9]. The approximation in planar case (interlaminar strains are neglected) takes the following form,

$$\Delta\mathbf{F}^n \approx \begin{bmatrix} 1 + \Delta\epsilon_x^n & \Delta\gamma_{xy}^n/2 \\ \Delta\gamma_{xy}^n/2 & 1 + \Delta\epsilon_y^n \end{bmatrix}. \quad (30)$$

In this case, beyond the explicit formulation, the formulation can also be applied to implicit simulations by constraining the time steps in the material subroutine so that the incremental strains remain significantly less than unity.

With the help of Eq. (30) the orientation of the fiber coordinate system in time step $n + 1$ can be obtained, if the orientation in time step n is known, such as

$$\mathbf{v}^{n+1} = \Delta\mathbf{F}^n \mathbf{v}^n, \quad (31)$$

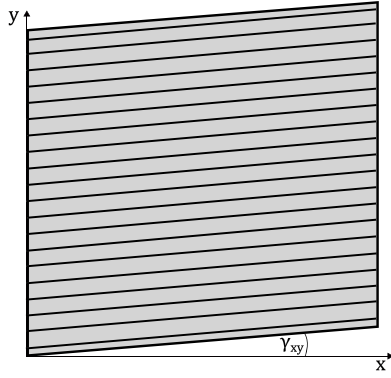


Fig. 4. Fiber reorientation induced by angle distortion of an element.

where $\mathbf{v}^n$ indicates the fiber direction in the element coordinate system, the initial value of this vector is $\mathbf{v}^0 = [1 \ 0]^T$, because there is no deformation at the initial step, thus the fiber coordinate system aligns with the element coordinate system. Fig. 3 shows the orientation of the fibers in the element coordinate system (in the undeformed configuration fibers aligned with the edges of the element). With the help of $\mathbf{v}$ the angle θ can be determined. Consequently, the x, y coordinate system can be rotated about the common z axis with the obtained angle to get the fiber coordinate system. The strain increments of $\Delta\epsilon_{x,y}^n$ should be transformed into the fiber coordinate system to calculate $\Delta\sigma_{1,2}^n$, since the non-linear stiffness matrix derived in Eq. (21) is interpreted in the fiber coordinate system. Finally, the stress increments need to be transformed back to the element coordinate system. The well-known transformation matrix of the strain increments can be constructed as,

$$\mathbf{T} = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix}, \quad (32)$$

where $c = \cos \theta$ and $s = \sin \theta$. Calculating the angle of vector $\mathbf{v}$ and then using the sin and cos functions in every time step would be computationally costly, but after calculating the unit vector $\mathbf{e} = \mathbf{v}/|\mathbf{v}|$, the values of c and s can be easily read from the entries of $\mathbf{e}$.

It is also known, that the transformation matrix, which rotates the stress increments back to the element coordinate system is $\mathbf{T}^T$. Overall, Algorithm 1 is done to calculate the stress response at the time step $n + 1$ in case of fiber reorientation.

The material model is chosen to be implemented for thin shells and solid elements. The latter is applicable as long the plane-stress state approximation holds.

It is worth to mention that in LS-DYNA certain shell formulations, like the Belytschko–Lin–Tsay shell element [43], are capable of handling fiber reorientation if the elements have angle distortion (via the definition of co-rotational system, which varies from type to type of elements), as Fig. 4 illustrates, which means that the material model augments the applications of these elements to account for draping of the type in Fig. 2.

5. Validation

The validation of the material model has been carried out in two steps, the first step consists of the validation of the five distinguished material directions by reproducing the measurement results of [44,45] with simulations, where the measured material is Toray Cetex® TC1225 - Standard Modulus Carbon 145 gsm UD Tape 34% RC. In the second step the fiber reorientation part is validated via glass fiber reinforced polyester material, since it has a less brittle nature, thus the draping phenomenon can be emphasized.

Algorithm 1: Stress response with draping

- 1: Calculating the new orientation of the fibers from $\Delta\mathbf{F}^n$ and $\mathbf{v}^n$.
- 2: Obtaining the strain increments in the fiber coordinate system, such as

$$\Delta\epsilon_{1,2}^n = \mathbf{T}^n \Delta\epsilon_{x,y}^n.$$

- 3: Computing the stress increment in the fiber coordinate system:

$$\Delta\sigma_{1,2}^n = \mathbf{C} \left(\epsilon_{1,2}^{n-1} \right) \Delta\epsilon_{1,2}^n.$$

- 4: Calculating $\Delta\epsilon_3^n$ as described in Eq. (27) (no need to rotate back this quantity, since axis z and 3 are identical).
- 5: Rotating back the stresses to the element coordinate system:

$$\Delta\sigma_{x,y}^n = \left(\mathbf{T}^T \right)^n \Delta\sigma_{1,2}^n.$$

- 6: Updating the stress values in the usual way:

$$\sigma_{x,y}^{n+1} = \sigma_{x,y}^n + \Delta\sigma_{x,y}^n.$$

For implicit implementation the material tangent modulus can be obtained as,

$$\mathbf{D}_{x,y}^n = \left(\mathbf{T}^T \right)^n \mathbf{C} \left(\epsilon_{1,2}^{n-1} \right) \mathbf{T}^n.$$

It should be emphasized, that the total strains are in the fiber coordinate system, which are tracked in an incremental approach as well:

$$\epsilon_{1,2}^{n-1} = \epsilon_{1,2}^{n-2} + \Delta\epsilon_{1,2}^{n-2}.$$

Table 1

The linear material properties used in the validation.

E_{1t}	126	[GPa]
E_{1c}	124	[GPa]
E_{2t}	8137	[MPa]
E_{2c}	2500	[MPa]
ν_{21}	0.0229	[-]

5.1. Validation of the distinguished material directions

The Toray Cetex® TC1225 - Standard Modulus Carbon 145 gsm UD Tape 34% RC material has been found to be linear in the fiber and transverse direction with different tensile and compressive behavior. Table 1 sums up the corresponding linear material properties of the measurements.

To model the non-linear in-plane shear behavior the data of [46] was used, in which the shear stress-shear angle data points are available to fit Eq. (16). The data fit was carried out with the least squares method, which can be seen in Fig. 5. To quantify the goodness of fit, a normalized version of the root mean square error was used:

$$\text{RMSE} = \sqrt{\frac{\sum_n (y_i - \hat{y}_i)^2}{n}}, \quad (33)$$

$$\text{NRMSE} = \frac{\text{RMSE}}{\max y_i - \min y_i}, \quad (34)$$

where y_i are the measured and $\hat{y}_i$ are the predicted data points. Besides the NRMSE measure the maximum relative deviation is also computed as,

$$\text{MRD} = \max \left(\frac{|y_i - \hat{y}_i|}{y_i} \right). \quad (35)$$

The obtained Ramberg–Osgod parameters are the followings (see Table 2),

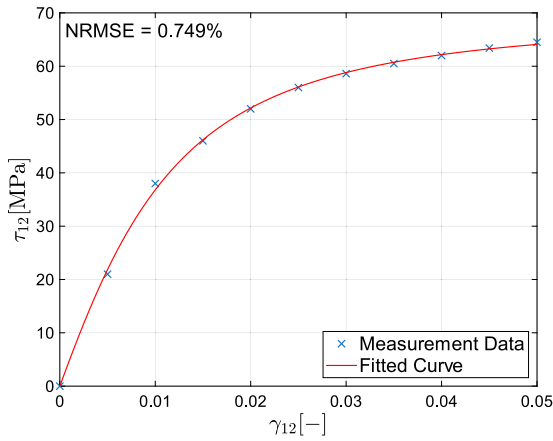


Fig. 5. The measurement data [46] and the fitted Ramberg–Osgood equation with 0.7485% error.

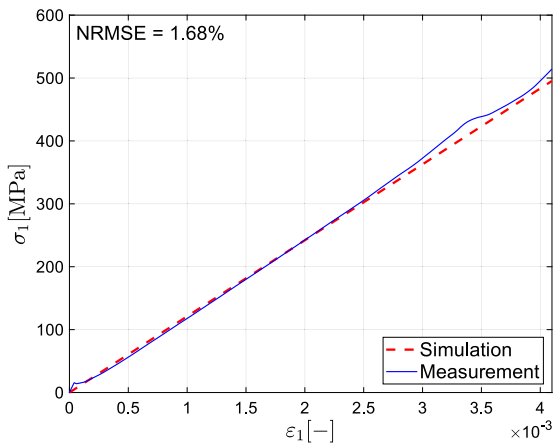


Fig. 6. Validation in tensile fiber direction.

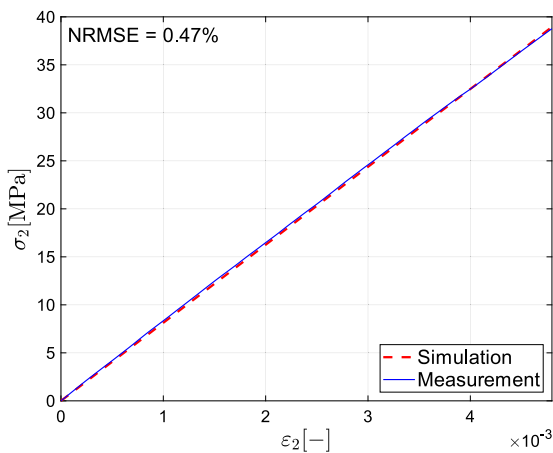


Fig. 7. Validation in tensile transverse direction.

Table 2
The Ramberg–Osgood parameters of the shear direction.

G	4854	[MPa]
Y_0	69.1	[MPa]
p	1.61	[–]

Table 3

The summarizing table of the simulations of validation.

Type	Ply-layup	Direction of Validation
Tensile	1.1 [mm], 0°	Fiber in tensile
Tensile	1.1 [mm], 90°	Transverse in tensile
3-Point bending	1.1 [mm], 0°	Fiber in tensile and compression
3-Point bending	1.1 [mm], 90°	Transverse in tensile and compression
4-Point bending	1.1 [mm], 0°	Fiber in tensile and compression
4-Point bending	1.1 [mm], 90°	Transverse in tensile and compression
Tensile	1.1 [mm], 30°	Fiber in tensile and compression, Transverse in tensile and compression and Shear
Tensile	1.1 [mm], 45°	Fiber in tensile and compression, Transverse in tensile and compression and Shear

Table 3 presents the standardized tests along with the validation purpose of the simulation. The relevant standard for the tensile type is ISO 527-5: 2021 [47], while for the bending type is ISO 14125: 1998 [48]. In case of tensile tests of 0° and 90°, direct strain measurement was possible, as regards to the others measurements the measured crosshead displacement was corrected by the stiffness of the testing frame to get the real specimen elongation. In order to visualize the measurement results, the so-called LOWESS method was applied [49], since the endpoint of the measurement curves are different due to the deviation of the material strengths. The validation results of the tensile fiber and transverse direction are displayed in Fig. 6 and Fig. 7.

The next step in the validation process is to validate the compressive directions (in order to increase gradually the directions), which can be carried out via the bending tests, due to the lack of compression tests with direct strain measurements. The finite element models of the bending simulations are shown in Fig. 11, while the results of the simulations with fitted parameters along with a comparison of the developed model with simple linear orthotropic model can be seen in Fig. 10. As it is expected, the curves corresponding to the developed model lie between the linear models, which cannot distinguish between tensile and compressive moduli. It should be noted that, in the case of bending simulations with a 0° orientation, the curves are within the margin of error at smaller displacements, due to the tensile and compressive moduli being close to each other ($E_{1t}/E_{1c} \approx 1.02$). However at larger displacements the reaction force curves corresponding to the developed model stay between the linear models. At last, the results of the off axis tensile measurements are presented in Fig. 8 and Fig. 9.

It is worth to mention, that no convergence problem occurred during the simulations. Moreover, with the help of the bending tests the material model is tested in contact problems with different tensile and compressive behavior. The applied convergence criteria are according to the LS-DYNA built-in algorithms [31], which checks the norms of the displacement, energy (computed from the residual force) and residual force. The adjusted tolerances were (10^{-4}) stricter, than the default values. Moreover, if convergence issues arise with the model, then built-in methods (e.g., stiffness matrix reformation or a reduced time step) can be used to achieve convergence.

The results of the validation can be seen in Table 4. It should be mentioned, that the “bump” in Fig. 6 is the consequence of the smoothing method applied to the original data points. As it can be concluded the errors are within the range of acceptable values, although the tensile test of 45° has a higher error, as this measurement was overestimated by the FEM. This due to the fact that, in the larger deformation range, only one measurement curve remained, which likely represented the lower bound of the stiffness response. Therefore, the measurement data in that region are no longer representative, particularly given that the predicted values are close to the measurements in the smaller deformation range.

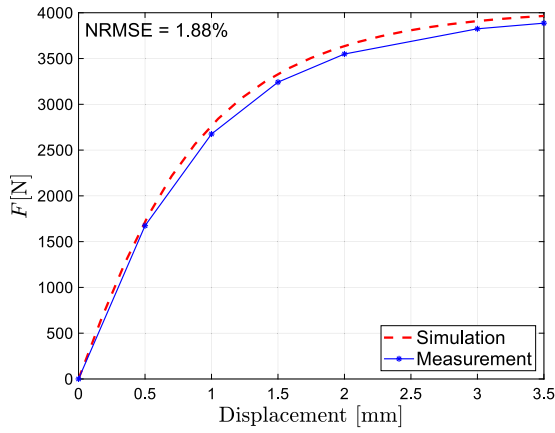


Fig. 8. Validation with off axis simulation of 30°.

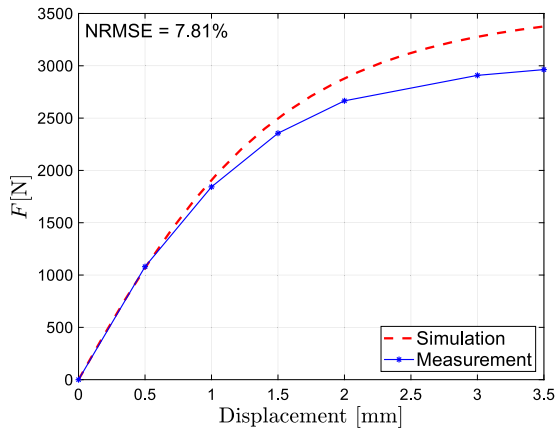


Fig. 9. Validation with off axis simulation of 45°.

5.2. On the remark of ISO14129

As previously discussed, the effect of fiber reorientation is worth to taken into account. The developed material model can demonstrate this effect on the standardized ISO14129 test [50], which test is valid until 5% shear angle according to the relevant standard. By using the previously validated material properties Fig. 12 illustrates the difference if the draping effect is considered. Moreover, the displacement corresponding to the recommended 5% shear angle is also denoted to observe, that until this point the relative error is less than 2% (thus the measurement results in Fig. 5 can be used, without considering draping, as it has been done), but after increasing the axial displacement of the specimen this relative error significantly rises. This indicates that the shear stress-shear angle curve cannot be determined directly from the measurement results, but corrections would be necessary to be applied in order to obtain the curve for larger shear angles (but still in the small strain regime). The model could serve as a possible solution, by applying numerical optimization techniques to the parameters of the model, these material properties can be determined, while the draping is considered during the post-processing of the measurement results. Fig. 13 shows the different stress fields (due to the fiber reorientation) in the global coordinate system. The higher stress values in the fiber reoriented case indicate that the strengths may be underestimated if the draping effect is not considered.

Table 4

Error of the model.

Test type	Ply-layup	Normalized RMSE	Max relative deviation
Tensile	1.1 [mm], 0°	1.68%	4.86%
Tensile	1.1 [mm], 90°	0.47%	3.01%
3-Point Bending	1.1 [mm], 0°	1.03%	2.36%
3-Point Bending	1.1 [mm], 90°	3.80%	7.28%
4-Point Bending	1.1 [mm], 0°	1.7%	4.50%
4-Point Bending	1.1 [mm], 90°	1.22%	10.60%
Tensile	1.1 [mm], 30°	1.88%	3.25%
Tensile	1.1 [mm], 45°	7.81%	13.92%

Table 5

The obtained linear and non-linear tensile parameters.

E_{1t}	28.14	[GPa]
E_{2t}	21.94	[GPa]
Y_{0t}	130	[MPa]
p_t	1.61	[-]
v_{21}	0.275	[-]
G	1472	[MPa]
Y_0	23.1	[MPa]
p	1.44	[-]

5.3. Validation of the algorithm of fiber reorientation

In the previous Section 5.1 the model is validated in all directions, although there were no measurement data to test, whether the proposed algorithm of the fiber reorientation is accurate. It was shown in Section 5.2 only, that the ISO14129 standard postulates a reasonable shear angle limit. To validate the draping effect a more ductile material is chosen, thus failure occurs only at larger displacements.

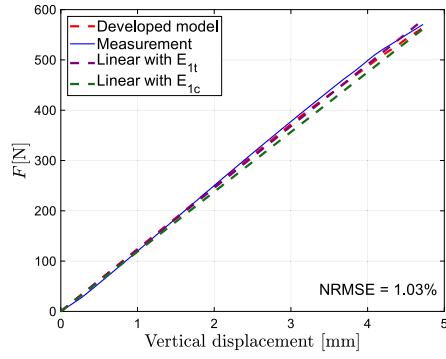
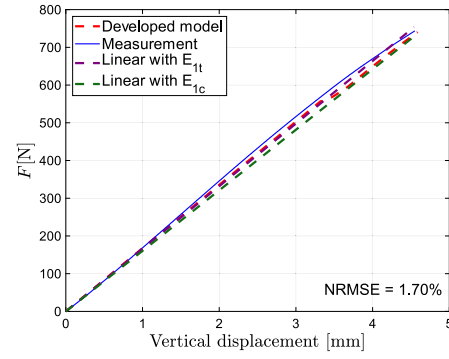
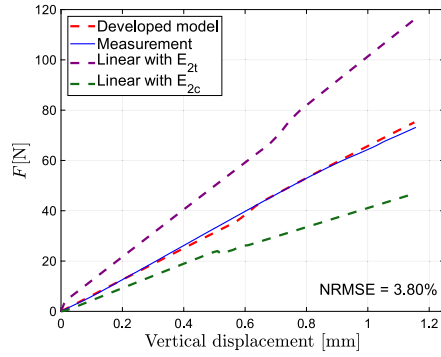
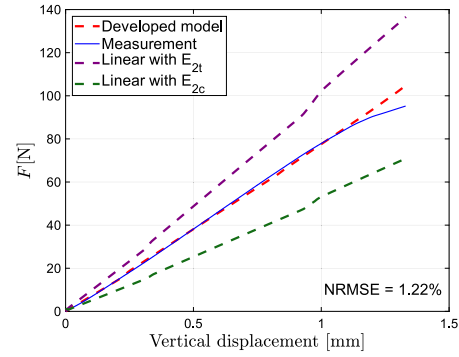
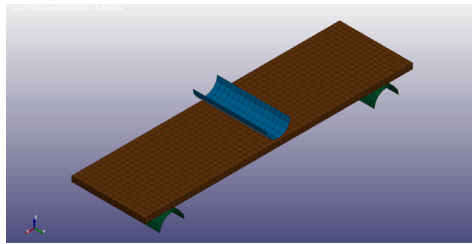
The validation is done via the ISO14129 measurement, but now data are available for the post 5% shear angle region. Consequently, fittings are carried out only to the tensile directions, since the compressive directions have no effect on the reaction force curve. The fittings are done with the same manner as in Section 5.1 (with the same standardized tests) and the material parameters are presented in Table 5. According to the ISO14129 standard the parameters of shear direction are fitted only to the 5% shear angle limit.

The goodness of fits are quantified by Fig. 14 and Fig. 15, while Fig. 16 shows, that after 5% shear angle (according to the standard: $\gamma_{12} = \epsilon_x - \epsilon_y$, if $\gamma_{12} \leq 5\%$) the model correctly predicted the measurement data, which is fully credited to the fiber reorientation effect. Fig. 16 also depicts the results of a simulation, where the obtained shear parameters are directly fed to a non-linear material model (MAT 58), which does not take draping into account. The obtained reaction force curve implies, that the algorithm of fiber reorientation is validated in an indirect way.

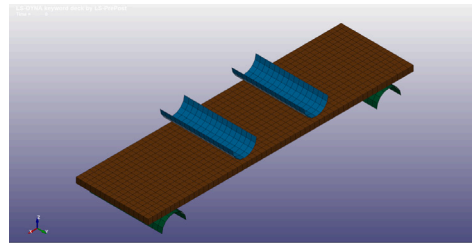
A more direct way is to compare the obtained simulated fiber orientation change with the measured one. This can be done with a simple analytic formula used in [7] with initial orientation of 45°, namely

$$\tan \theta = \frac{1 + \epsilon_x}{1 + \epsilon_y}, \quad (36)$$

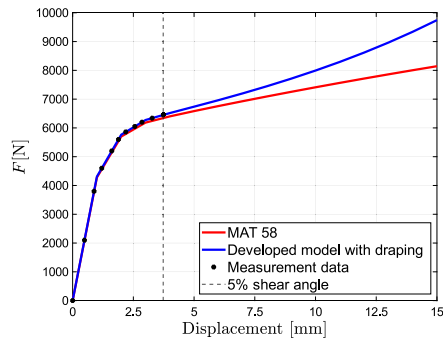
where θ is the orientation of the fibers at the strain state (ϵ_x, ϵ_y) , which strains are measured during the mechanical test. The comparison between the simulated and the calculated fiber orientation change can be seen in Fig. 17 with the RMSE of the model. It is worth to mention, that at the failure point the originally 45° fiber orientation almost altered to 38°, which is quite a significant change. In summary, the model successfully validated in a direct and indirect way with the measurement results of ISO14129 shear test.

(a) 3-point bending with orientation of 0° (b) 4-point bending with orientation of 0° (c) 3-point bending with orientation of 90° (d) 4-point bending with orientation of 90° **Fig. 10.** Results of the bending simulations.

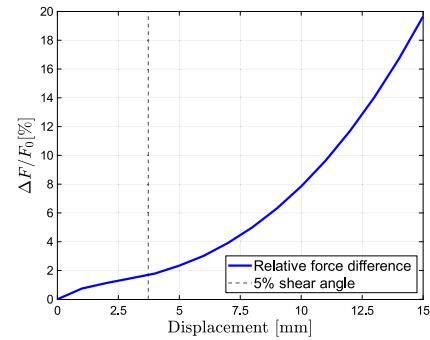
(a)

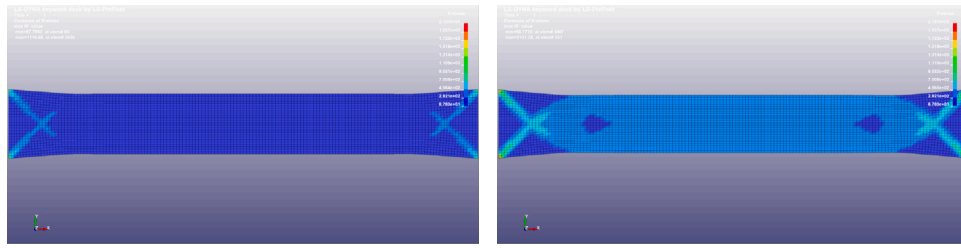


(b)

Fig. 11. The finite element models of the (a) 3- and (b) 4-point bending tests.

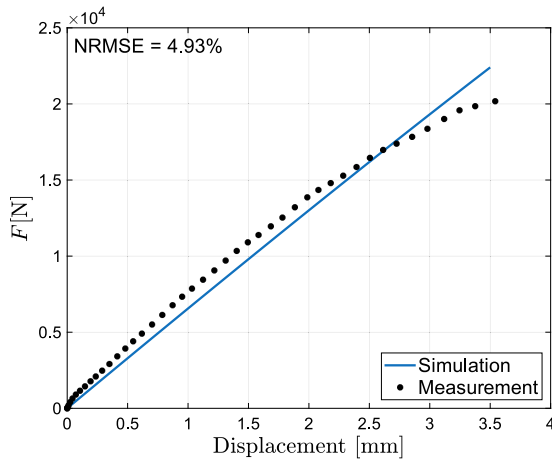
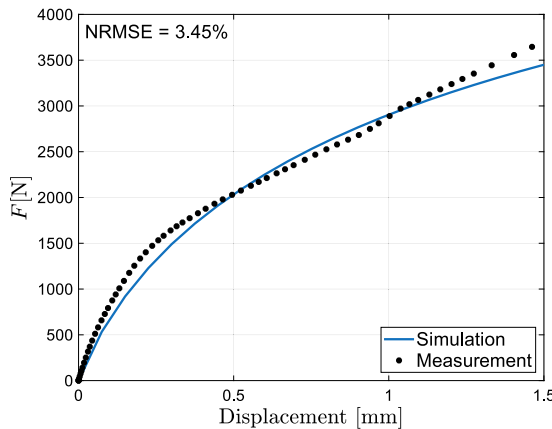
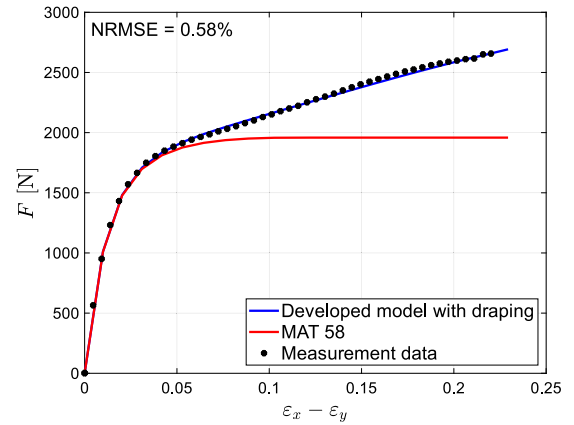
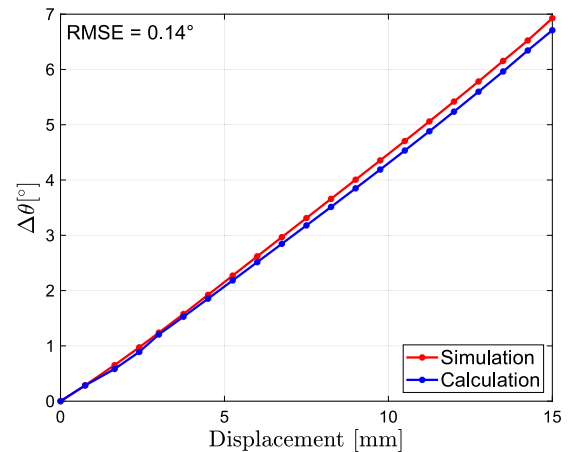
(a) The reaction force curves

(b) The relative error of the curves (F_0 is the case without draping)**Fig. 12.** The difference, that draping makes with ISO14129.



(a) Without considering draping

(b) With draping

Fig. 13. The stress fields (in MPa) of the model of ISO14129 in the global x direction.**Fig. 14.** Fitting in tensile direction.**Fig. 15.** Fitting in transverse direction.**Fig. 16.** Validation of draping with ISO14129 test.**Fig. 17.** The angle distortion during the ISO14129 measurement.

6. Conclusions

This work presents a non-linear elastic orthotropic material model with optional fiber reorientation for laminated CFRPs. Apart from presenting constitutive relations and numerical algorithms, the model has been implemented in the LS-DYNA finite element software for both explicit and implicit simulations. Furthermore, the material model is available for shell elements and for solid elements, as long the plane stress-state assumption holds, since the interlaminar behavior is neglected. This orthotropic model requires minor computational cost compared to the embedded bar method, as much less finite elements are needed and shell formulations can be applied, although more type of mechanical tests are necessary to obtain the material parameters compared to the embedded bar model. The developed material subroutine

offers the possibility to the user to utilize the built in Ramberg–Osgood equations or to define arbitrary reference curves. In addition, the reason behind the included draping phenomenon is well established and shown to have significant effect in certain scenarios. Finally, the model is demonstrated to be able to reproduce real-world measurements with appropriately fitted parameters, which can be obtained from simple tests.

CRediT authorship contribution statement

Roland Horváth: Writing – original draft, Visualization, Software, Methodology, Funding acquisition, Formal analysis, Conceptualization. **László Kovács:** Writing – review & editing, Validation, Supervision, Methodology, Conceptualization. **Szabolcs Berezvai:** Writing – review

& editing, Supervision, Methodology, Funding acquisition, Conceptualization.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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