



Short Communication

Identification and variation of nonlinear elastic–plastic properties of poly(lactic acid) (PLA) foams produced by in-situ foaming additive manufacturing

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ABSTRACT

Extrusion-based additive manufacturing technologies have undergone significant development in recent years, enabling the cost-effective production of complex geometries, customized designs, and small production series. Recent studies have shown that multilayer foam systems composed of layers with different densities can provide improved impact absorption performance. 3D-printing with in-situ foaming filaments offers a promising approach to the production of such functionally graded foams with controlled density distributions. This relies on materials that expand during 3D-printing, while the foaming can be controlled through the printing temperature. The accurate numerical (finite element) prediction of the nonlinear mechanical behavior of these density-graded structures requires an appropriate constitutive model and a material characterization strategy for in-situ foaming polymer foams printed at different temperatures. This paper focuses on the experimental characterization and constitutive modeling of an in-situ foaming poly(lactic acid) (LW-PLA) filament printed over a temperature range of 190 – 250 °C. In compression (which is the characteristic loading of 3D-printed foams), the material exhibits finite strain deformation with yielding and nonlinear hardening properties. In this contribution, the isotropic Deshpande-Fleck crushable foam model with an associative flow rule is adopted for characterization of the measured material behaviour. Through carefully designed uniaxial mechanical tests and a dedicated post-processing method, the mechanical behavior of the foam is captured with high accuracy. Finally, the variation of the nonlinear foam properties as a function of the printing temperature is also identified.

1. Introduction

Extrusion-based additive manufacturing technologies have undergone significant development in recent years, enabling the cost-effective production of complex geometries, customized designs, and small production series [1]. In parallel with these advancements, there has been active research on enhancing the energy-absorption capability of polymers through the use of cellular architectures. In particular, multilayer foam systems composed of layers with different densities are known to provide improved impact absorption performance [2]. However, it is important to note that these multilayer structures are typically produced using conventional processing technologies (e.g., extrusion), where separately manufactured foamed sheets are subsequently joined or laminated to form the final structure [3,4]. As a result, the achievable

density distribution remains limited to discrete layers, and the overall process requires multiple manufacturing steps, increasing complexity and reducing design flexibility. These limitations highlight the need for alternative manufacturing methods that allow the direct fabrication of density-graded structures within a single process step. A promising strategy to address this challenge is the use of in-situ foaming filaments in extrusion-based 3D printing, where the foam structure forms directly during printing. In this case, the degree of expansion and the resulting cellular structure can be effectively controlled through processing parameters, most notably the printing temperature, which controls the activation of the blowing agent and the evolution of the foam morphology [5].

Poly(lactic acid) (PLA) is a widely used rigid material in 3D printing, characterized by a low melting temperature ($T_m \approx 150 - 180$ °C),

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which enables straightforward processing with fused deposition modeling/fused filament fabrication (FDM/FFF) technology due to its good processability, relatively low required printing temperatures, minimal tendency to warp, which allows printing on open-frame machines without an enclosed build chamber, and the adequate stiffness of the resulting printed parts [6–8]. The manufacturing and application of in-situ foamed PLA have been extensively studied, particularly for energy-absorbing components such as shock absorbers and honeycomb structures. Recent developments of LW-PLA filaments have also enabled their application in customized orthotic devices and vibration- or damping-related systems operating under low-pressure conditions [9–11]. Several studies have focused on the influence of processing parameters on the resulting microstructure [12–14] as well as on the linear elastic behavior and energy-absorption characteristics of in-situ foamed PLA filaments [15–17]. However, the existing literature lacks a comprehensive investigation of the relationship between foaming (primarily controlled by the printing temperature) and the finite-strain elastic–plastic behavior of such structures.

To predict the mechanical behavior of variable-density structures through numerical simulations (finite element simulations), it is necessary to accurately model the constitutive behavior of polymer foam materials foamed at different temperatures and describe the dependence of material parameters on printing temperature. Based on our previous research findings, the mechanical behavior of 3D-printed in-situ foaming materials (e.g., LW-PLA) involves large deformations and shape changes, as well as significant residual deformations due to their nonlinear hardening [18].

Several approaches in the literature describe the nonlinear elastic–plastic behavior of polymer foams. The Gibson–Ashby model describes the nonlinear stress–strain characteristics of foams through a micromechanical approach as a function of relative density (ρ_{rel}) and empirically determined geometric parameters [19]. These micromechanical models are suitable for estimating fundamental material structural characteristics (e.g., compressive strength, yield effect, densification strain). However, they cannot be applied in numerical simulations based on the formalism of finite deformations. In addition to micromechanical approaches, the so-called phenomenological description is widely used, which relies on fitting a constitutive model to the observed deformation components. In the model of Liu et al. [20], a phenomenological model is proposed for the entire strain domain, yet it still uses relative density as the primary parameter that determines the overall mechanical behavior and stress response [21]. Furthermore, this model provides a mathematical formula for the measured stress–strain curve without adopting either finite strain formalism or the theory of elastoplasticity.

For describing the elastic–plastic deformation of foams, the so-called crushable foam models can be applied [22]. Their theoretical foundation is the Deshpande–Fleck crushable foam model, initially developed based on experimental results for aluminum foams [23]. Although the material model was developed for metal foams, it is widely applied in the literature to model the elastic–plastic behavior of polymer foams as well [22].

Recent progress in foam constitutive modeling has focused on nonlinear elastic–plastic formulations [24–26], including crushable-foam and data-driven approaches [27]. In parallel, recent studies have shown that printing temperature strongly influences the mechanical response of 3D-printed PLA foams and their energy absorption performance [28–30]. However, the printing temperature-dependent characterization framework for in-situ foamed LW-PLA remains lacking, and the applicability of the Deshpande–Fleck model for such material has not yet been investigated. This contribution addresses this gap by providing a systematic modeling framework for characterizing the uniaxial elastic–plastic behavior of 3D-printed PLA foam using finite-strain formulation and the variation of nonlinear material properties with printing temperature. The proposed approach allows for accurate FE modeling and supports the design of density-graded foamed structures.

This paper is organized as follows. In Section 2, the investigated PLA material, specimen preparation and the uniaxial mechanical tests are presented. In Section 3, the nonlinear elastic–plastic constitutive model is introduced and summarized, and in Section 4, the parameter fitting strategy is presented, and the performance of the material model is evaluated. Finally, Section 5 concludes the paper.

2. Mechanical measurements

2.1. Material and specimen preparation

In this work, a commercially available 1.75 mm diameter in-situ foamable polylactic acid (PLA) filament (LW-PLA, ColorFabb B.V., Belfeld, The Netherlands) was used (melt flow index of the filament (210 °C/2.16 kg) is 6 g/10 min). The extent of foaming of this material strongly depends on the applied printing parameters. Thermal characterization was performed by differential scanning calorimetry (DSC, Q2000, TA Instruments) between room temperature and 300 °C and by thermogravimetric analysis (TGA, Q500, TA Instruments) between room temperature and 600 °C under nitrogen atmosphere, both using a heating rate of 10 °C/min (see Fig. 1). In the DSC curve, a weak endothermic shoulder was observed around 195 °C; however, no significant mass loss appeared in the TGA curve at this temperature, suggesting that the foaming mechanism is related to thermally expandable microspheres (TEMs) rather than a chemical blowing agent (CBA). Similar DSC characteristics were reported in [31] which also identified a maximum $\tan\delta$ value of 1.41 at approximately 210 °C, indicating the temperature range of enhanced molecular mobility relevant for foaming.

The foaming mechanism can be divided into four stages, as illustrated in Fig. 2. Upon approaching the heated nozzle, the filament melts, and above a specific temperature, the thermoplastic shell of the TEM softens and the pressure of the encapsulated gas increases (Step 1). After extrusion through the nozzle, the sudden pressure drop promotes the rapid expansion of the microspheres, and the formation of the cellular structure starts (Step 2). This is followed by cell growth (Step 3) and, finally, stabilization of the cellular structure during cooling (Step 4). The nozzle temperature influences all these steps; therefore, changing the temperature results in different degrees of foaming and distinct cell morphologies. Cell expansion is moderate at low temperatures, peaks at the optimal temperature (minimizing relative density), and declines at higher temperatures due to excessive softening of TEM shell, causing gas loss, cell rupture, coarsening, irregularity, and partial collapse.

The specimens used in this study were fabricated by material-extrusion 3D printing on a CraftBot+ printer (CraftUnique Ltd., Budapest, Hungary). Slicing was carried out using CraftWare software. The printing speed was 15 mm/s, the layer height and width were 0.40 mm, and a rectilinear infill with 0° raster orientation was applied. The build-plate temperature was set to 60 °C for all prints. To cover the temperature range relevant for in-situ foaming, the nozzle temperature was varied between 190 and 250 °C in 10 °C increments, i.e., $T = 190, 200, 210, 220, 230, 240$, and 250 °C.

To compensate for temperature-dependent expansion and obtain dimensionally accurate specimens, the extrusion multiplier was adjusted for each printing temperature according to the procedure described in [33]. The extrusion multiplier was reduced with increasing foaming ratio to maintain the nominal specimen geometry, layer thickness, and bead overlap, thereby minimizing geometrical variations between specimens printed at different temperatures. Since the nominal specimen geometry was kept constant, the required extrusion multiplier approximately followed the relative density variation of the foamed material (see Fig. 3/a).

All samples were printed with a 0.4 mm MK8 steel nozzle. It should be noted that the degree of foaming is influenced not only by temperature but also by nozzle diameter: smaller diameters increase residence time, shear rate, and pressure drop rate [18]. Nozzle geometry also affects thermal history and shear-induced crystallization behavior of the

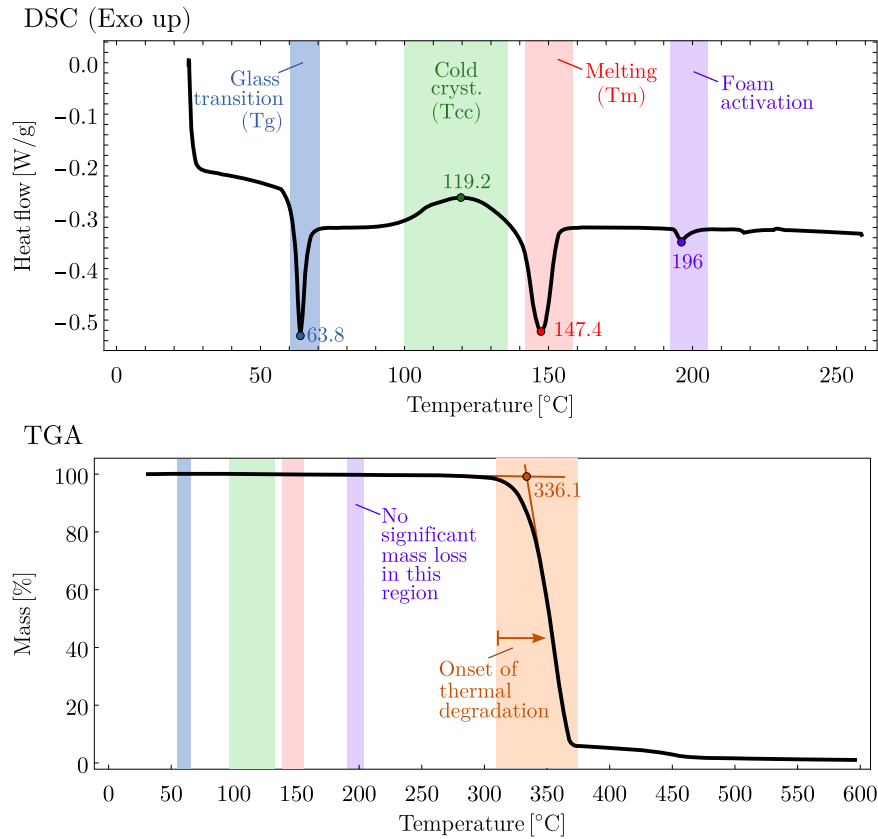


Fig. 1. DSC and TGA curves of the investigated LW-PLA indicating the most important thermal transitions.

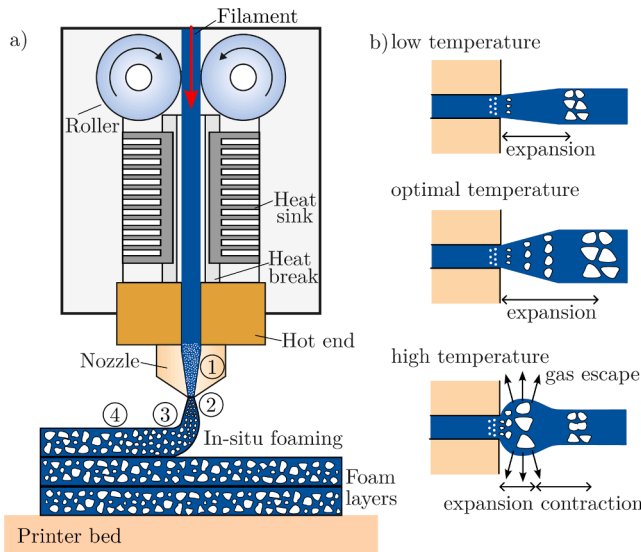


Fig. 2. The (a) schematics and steps of in-situ foaming 3D-printing: (1) shell softening, (2) microsphere expansion, (3) cell growth, and (4) stabilization of cell structure; and (b) the schematics of the cell evolution mechanism at low, optimal and high temperatures (redrawn from [32]).

polymer melt, affecting foam morphology and mechanical response [34]. A detailed investigation of these effects, however, is beyond the scope of the present work.

2.2. Material properties

Since the external dimensions of the printed specimens were main-

tained constant by adjusting the extrusion multiplier (EM), the reduction in material consumption directly reflected the degree of foaming. Therefore, the relative density (see Fig. 3/a) can also be approximated from the ratio of the extrusion multipliers as

$$\rho_{\text{rel}} = \frac{\rho_{\text{foam}}}{\rho_{\text{matrix}}} \approx \frac{EM_T}{EM_{\text{solid}}}, \quad (1)$$

where ρ_{foam} and ρ_{matrix} are the densities of the foam material and the matrix material, while EM_T and $EM_{\text{solid}} = 1$ denote the extrusion multipliers at the given printing temperature and the non-foamed reference state. After printing, the dimensional accuracy of the specimens was verified by measuring the outer dimensions of the cubes using a caliper. In all cases, the deviation between the measured and nominal dimensions remained below 2 %.

The cellular morphology of the specimens printed at the different nozzle temperatures was analyzed by scanning electron microscopy (JSM-6380LA, JEOL Ltd., Tokyo, Japan). Before imaging, the specimens were fractured under cryogenic conditions in liquid nitrogen to obtain representative fracture surfaces, followed by sputter-coating with a thin gold-palladium layer to provide adequate electrical conductivity.

Fig. 3/b-h presents the corresponding cellular morphologies (SEM images showing both the printed layered structure and the foam microstructure) at the different nozzle temperatures, while the corresponding cell size distribution and aspect ratio results are summarized in Fig. 4. In the 190–210 °C temperature range, no significant expansion of the TEM occurred, resulting in a relatively solid-like structure. At 220 and 230 °C, the average cell size remained relatively narrow; however, the noticeably higher cell density at 230 °C indicates that a larger fraction of the microspheres expanded due to the increased thermal exposure. The most homogeneous cellular morphology was obtained at 240 °C, where the cell size increased while the width of the cell size distribution remained approximately unchanged. At 250 °C broader cell

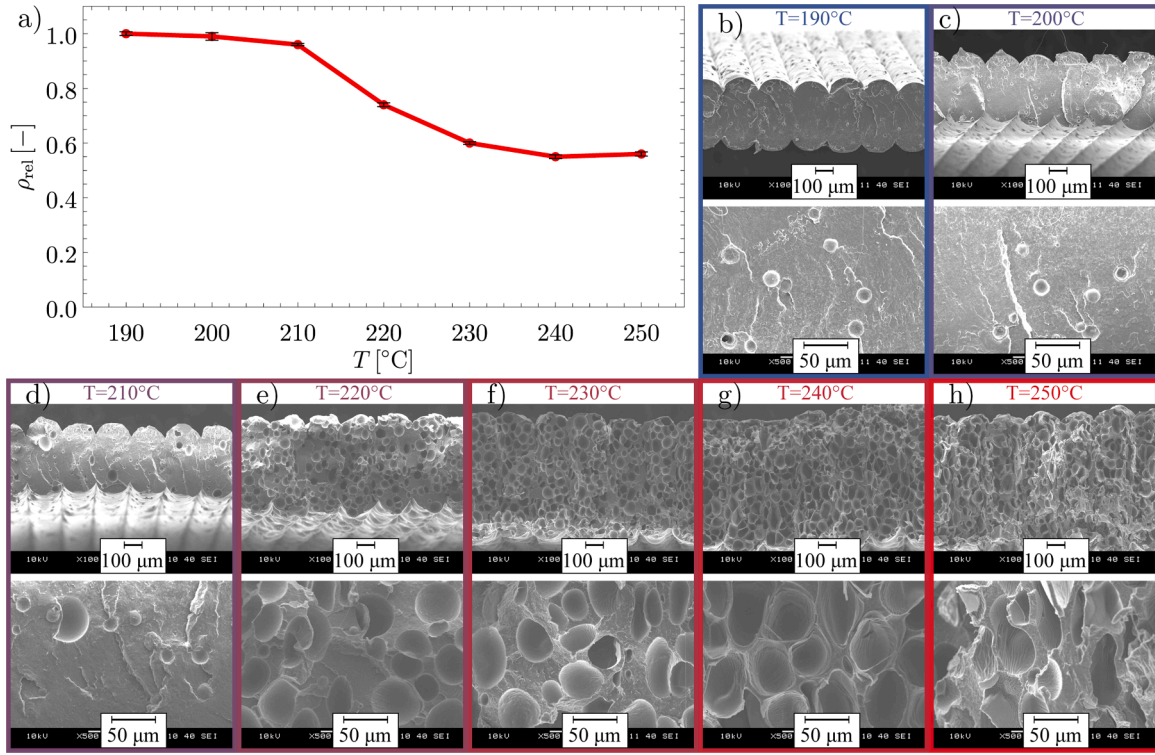


Fig. 3. The variation of (a) the ρ_{rel} relative density and (b)-(h) the cellular structure of the 3D printed LW-PLA filaments at different printing temperature levels.

size distribution and increased aspect ratio values indicate more irregular cell morphologies. This behavior is likely associated with excessive softening or rupture of the microsphere shells. Future studies may extend the cellular characterization by X-ray computed tomography (CT) to investigate process-induced defects, interlayer voids, and anisotropic structural features in greater detail [35,36].

The optimal printing temperature for maximum weight reduction is also affected by the nozzle's thermal conductivity. For the steel nozzle used in this study, the optimum temperature was shifted towards higher values compared with our previous experiments performed with a copper nozzle [33,37]. This effect must be taken into account when defining the processing window for in-situ foaming filaments.

2.3. Compression tests

For the constitutive modeling of the investigated 3D-printed foam specimens, uniaxial compression tests were performed using a Zwick Z050 dual-column material testing system equipped with an AST KAP-E load cell with a capacity of 50 kN. The dimension of the tested cubic specimens was $20 \times 20 \times 20$ mm. The schematics of the compression test are illustrated in Fig. 5/a. The initial separation of the compression platens was $H_0 = 25$ mm, while the applied crosshead speed was $v = 5$ mm/min to minimize the viscoelastic effects. At each printing temperature, 3 specimens were tested.

From the measured force-displacement ($F - u$) curves, the engineering (nominal) stress-compressive strain curve was determined using the relations

$$P = \frac{F}{A_0}, \text{ and } \varepsilon_c = \frac{u}{L_0}, \quad (2)$$

where F is the measured compressive force, $A_0 = 400 \text{ mm}^2$ is the initial cross-sectional area of the specimen, P is the compressive stress, u is the crosshead displacement, L_0 is the specimen height, while ε_c is the compressive strain. During processing of the measurement results, the so-called slack-correction method was adopted to compensate for the

initial convex-upward phase of the loading curve, caused by gradual contact between the compression platens and the entire cross-section of the specimens.

2.3.1. Transverse stretch characteristics

Furthermore, the compression tests were also video-recorded, and the transverse stretch characteristics were obtained using an image-processing method. As shown in Fig. 6/a, a slice of the video (in the middle part of the specimen) was cropped and binarized, while the transverse stretch at the i^{th} frame was obtained as

$$\lambda_T[i] = \frac{A_{wh}[i]}{A_{wh}[0]}, \quad (3)$$

where $A_{wh}[i]$ and $A_{wh}[0]$ are the area of the white pixels (the specimen) corresponding to the i^{th} and the first frame, respectively [38,39]. Then, the ground state Poisson's ratio (ν_0) was calculated for $\lambda \in [0.92, 1]$ domain using the relation

$$\nu_0 = \frac{\varepsilon_T^{\text{true}}}{\varepsilon^{\text{true}}} = -\frac{\ln(\lambda_T)}{\ln(\lambda)}. \quad (4)$$

Following the finite-strain generalization of the relation in Eq. (4) proposed by Blatz and Ko [40,41], the so-called generalized Poisson's ratio can also be obtained by fitting (ν^*) for the entire λ measurement domain (see Fig. 6/b).

2.3.2. True stress characteristics

Based on the transversal stretch characteristics, the current cross-sectional area can be expressed as

$$A = A_0 \cdot (1 - \varepsilon_c)^{-2\nu^*}, \quad (5)$$

while the true stress - true strain ($\sigma - \varepsilon_c^{\text{true}}$) characteristics can be obtained using

$$\sigma = \frac{F}{A} = P(1 - \varepsilon_c)^{2\nu^*}, \text{ and } \varepsilon_c^{\text{true}} = -\ln(1 - \varepsilon_c). \quad (6)$$

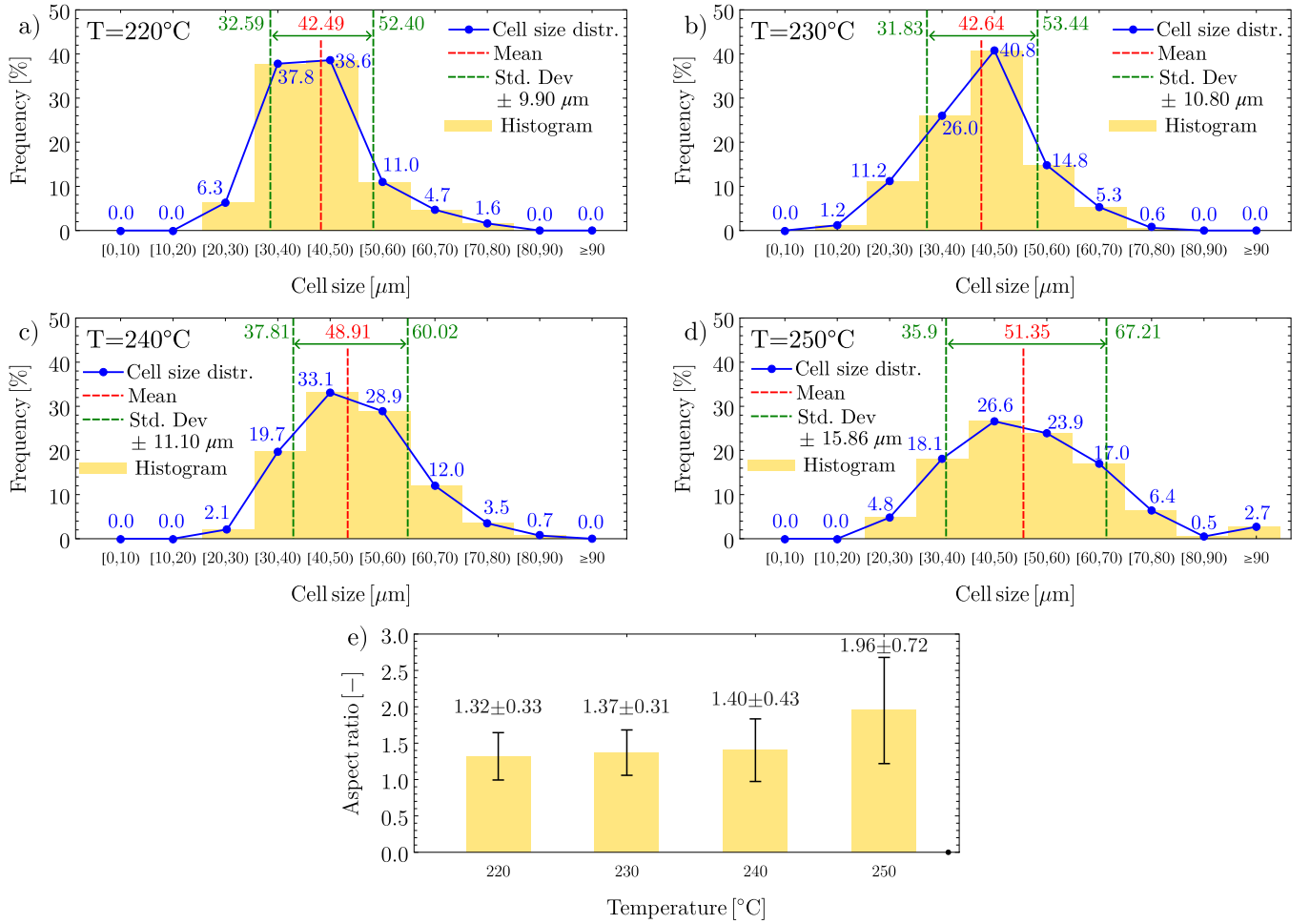


Fig. 4. The (a)-(d) cell size distribution and (e) the aspect ratio (ratio of major to minor cell axis) of the cells for the temperature range 220 – 250 $^{\circ}\text{C}$.

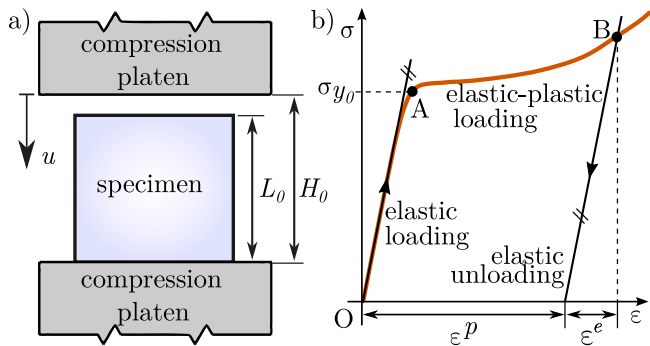


Fig. 5. The (a) compression test layout and (b) the post-processing of the stress-strain curve using additive strain increment decomposition.

The true stress–true compressive strain response measured at different printing temperatures is shown in Fig. 7/a, based on the measured curves of one representative specimen for each printing temperature. The results demonstrate that increasing the printing temperature initially leads to considerably lower stresses; however, above a printing temperature of 230 $^{\circ}\text{C}$, the stress values begin to rise again. The evolution of the mechanical response showed good agreement with the observed cellular morphology. Increasing microsphere expansion up to

240 $^{\circ}\text{C}$ reduced the relative density and stiffness, while at 250 $^{\circ}\text{C}$ the broader cell size distribution was accompanied by increasing elastic modulus and hardening response.

The initial elastic modulus (E) was calculated from the tangent of the true stress–true compressive strain curves for the domain $\epsilon_c^{\text{true}} \leq 0.02$. The numerical values are listed in Table 1, and the variation is illustrated in Fig. 6/c.

2.3.3. Hardening curve

Using the above, the hardening curve for the elastic-plastic material behavior can be constructed, which shows the relationship between the yield stress and the plastic compressive strain. It should be noted (as it is described in detail in Chapter 3) that we assume that the total strain increment is divided into elastic and plastic contributions as

$$d\epsilon = d\epsilon^e + d\epsilon^p, \quad (7)$$

where $d\epsilon^e$ and $d\epsilon^p$ stand for elastic and plastic strain increments (see Fig. 5/b), respectively [42]. Close to the origin (O-A regime), the material response is purely elastic, until the initial yield stress σ_{y0} is reached (point A). Above that point, the material yields, and the deformation exhibits elastic-plastic behavior, as shown by the nonlinear hardening curve. Note that for this simplified (monotonous 1D loading) case, the additive split also holds for the total strains, letting $\epsilon = \epsilon^e + \epsilon^p$.

The yield point was identified as the first measurement point with a

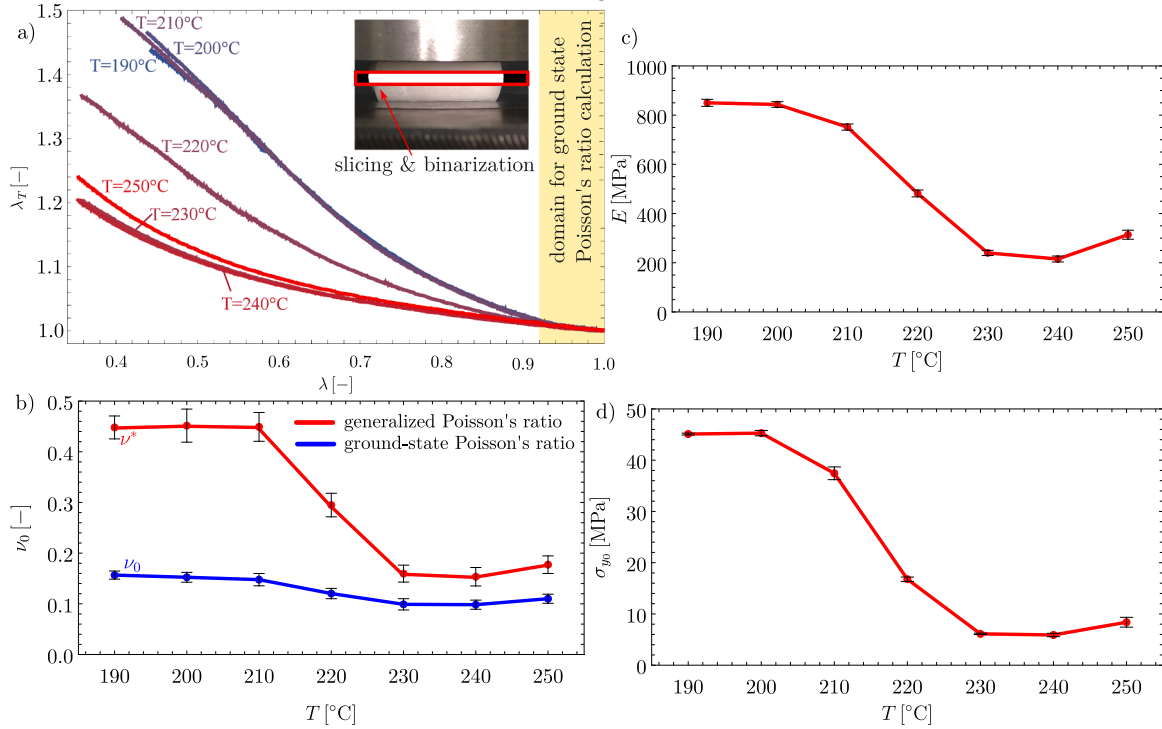


Fig. 6. The (a) image processing and the transverse stretch characteristics ($\lambda - \lambda_T$) (yellow region indicates ground-state Poisson's ratio domain); the variation of (b) ν and ν^* Poisson's ratios; (c) the E elastic modulus; (d) the initial yield stress σ_{y0} at different printing temperature levels.

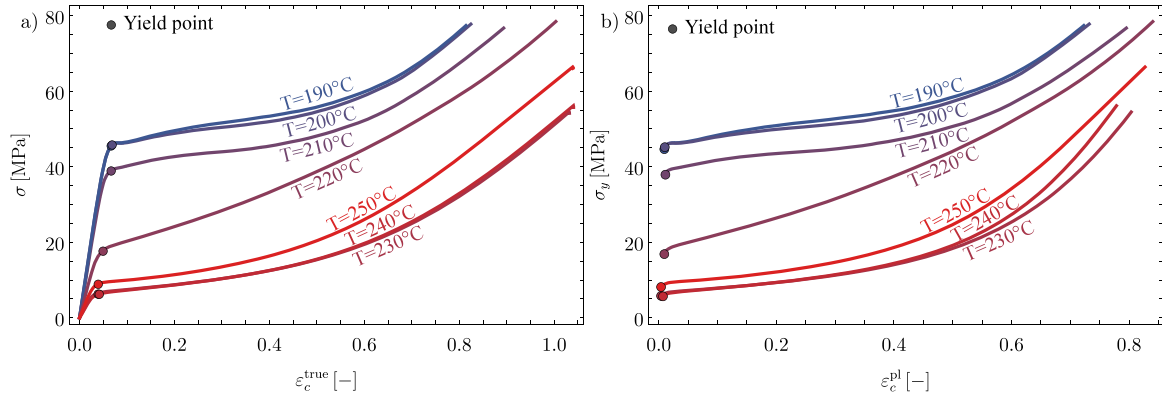


Fig. 7. The (a) measured true stress - true compressive strain curves and (b) the hardening curves at different printing temperature levels.

Table 1

Fitted numerical material parameter values at different printing temperature levels.

$T[^\circ\text{C}]$	$\rho_{\text{rel}}[-]$	$\nu_0[-]$	$\nu^*[-]$	$E[\text{MPa}]$	$\sigma_{y0}[\text{MPa}]$
190 °C	1.000 ± 0.007	0.1567 ± 0.0081	0.448 ± 0.0226	850.02 ± 14.31	45.101 ± 0.182
200 °C	0.997 ± 0.014	0.1524 ± 0.0097	0.452 ± 0.0326	843.33 ± 11.32	45.275 ± 0.525
210 °C	0.964 ± 0.004	0.1477 ± 0.0121	0.449 ± 0.0284	751.48 ± 12.52	37.442 ± 1.253
220 °C	0.738 ± 0.007	0.1203 ± 0.0102	0.295 ± 0.0232	481.74 ± 13.98	16.775 ± 0.428
230 °C	0.597 ± 0.005	0.0991 ± 0.0110	0.159 ± 0.0167	239.89 ± 10.63	6.116 ± 0.131
240 °C	0.546 ± 0.007	0.0982 ± 0.0092	0.153 ± 0.0182	215.21 ± 12.03	5.897 ± 0.331
250 °C	0.563 ± 0.008	0.1105 ± 0.0089	0.177 ± 0.0173	313.95 ± 18.33	8.383 ± 0.963

10% relative error from the linear elastic 1D Hooke's law ($\sigma = E \cdot \epsilon_c^{\text{true}}$) (see Fig. 6/d). The 10% threshold was chosen to avoid sensitivity to pre-yield softening leading to underestimation of the hardening response and also to minimize the overestimation of the yield point. The points preceding the onset of plastic flow were discarded, and for the subsequent measurement points, the values of the plastic compressive strain

ϵ_c^{pl} were calculated using

$$\epsilon_c^{\text{pl}} = \epsilon_c^{\text{true}} - \epsilon_c^e = \epsilon_c^{\text{true}} - \frac{\sigma}{E}, \quad (8)$$

where ϵ_c^e is the elastic compressive strain.

To determine the hardening curve, the obtained plastic compressive

strain ϵ_c^{pl} and the corresponding yield stress must be plotted, as illustrated in Fig. 7/b. Here, the previously observed behavior is again evident: once foaming begins, the degree of hardening decreases, but after reaching its minimum at the printing temperature of 230 °C, the hardening curves begin to increase again due to reduced expansion. The calculated material parameters from the compression tests are listed in Table 1.

3. Constitutive model

To describe the material behavior presented in the previous section, the Deshpande-Fleck crushable foam model was adopted using the finite-strain formalism, which combines linear elastic behavior with nonlinear hardening [23,43,44]. Initially, this material model was developed based on experimental results from aluminum foams. However, it has since been widely used for modeling the permanent deformations of polymer foams as well [22]. The Deshpande-Fleck foam model assumes symmetric behavior in tension and compression, and the equivalent plastic strain governs the evolution of the yield surface (ϵ^{pl}), which contains both volumetric and deviatoric plastic strains. The model is also available in commercial finite element solvers (Abaqus [45], LS-DYNA [46]), although with slightly different formulations. The applied Deshpande-Fleck model captures nonlinear foam hardening under quasi-static loading; however, it omits anisotropy or rate-dependent effects, which are beyond the scope of this study. The isotropic formulation is adopted to assess the applicability of the model for uniaxial characterization. As detailed in Section 2, identical rectangular infill pattern, 0° raster orientation and loading direction were used to minimize anisotropic effects, while anisotropic modeling is left for future work.

3.1. Yield function and flow rule

In our research, we use the formulation implemented in the LS-DYNA MAT154 *Crushable Foam* material model [46], where the yield function Φ is defined as

$$\Phi = \sigma_{eq} - \sigma_y \leq 0, \quad (9)$$

where σ_{eq} is the equivalent stress, while σ_y the yield stress.

It should be noted that for finite strain formulation, Hooke's law can be expressed in rate form using the so-called hypoelastic constitutive model as

$$\dot{\sigma}^J = \mathcal{D} : d, \quad (10)$$

where $\dot{\sigma}^J$ is the Jaumann-stress rate, $\mathcal{D}$ the Hooke operator, and d the rate of deformation tensor [45,47]. For monotonous uniaxial loading with constant strain rate, the constitutive relation simplifies to

$$\sigma = E\epsilon^{true}, \quad (11)$$

where σ is the true stress, while ϵ^{true} is the true strain.

According to the Deshpande-Fleck theory, the equivalent stress can be expressed as

$$\sigma_{eq} = \sqrt{\frac{(\sigma_{eq}^{Mises})^2 + \alpha^2 \sigma_m^2}{1 + (\alpha/3)^2}}, \quad (12)$$

where σ_{eq}^{Mises} is the Mises equivalent stress, while $\sigma_m = \text{tr}(\sigma)/3$ is the

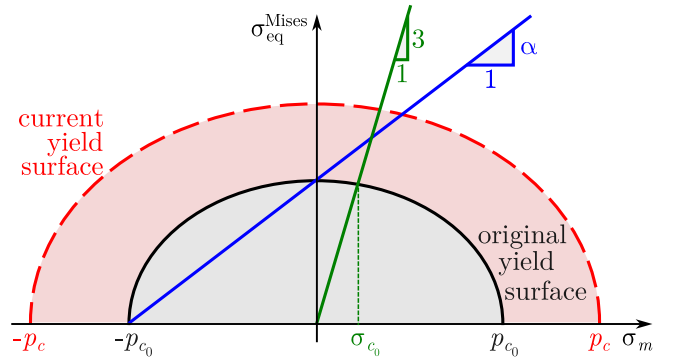


Fig. 8. The yield surface of the isotropic Deshpande-Fleck foam plasticity.

mean stress, which can be expressed as

$$\sigma_{eq}^{Mises} = \sqrt{\frac{3}{2}} s : s, \text{ and } s = \sigma - \sigma_m I. \quad (13)$$

This defines an elliptical yield surface (see Fig. 8), which is centered at $\sigma_m - \sigma_{eq}^{Mises}$ plane and evolves in a self-similar manner. The parameter α defines the shape of the yield ellipse, which can be calculated as

$$\alpha = \frac{3k}{\sqrt{9 - k^2}}, \text{ with } k = \frac{\sigma_{c0}}{p_{c0}}, \quad (14)$$

where σ_{c0} and p_{c0} stands for the initial yield stress in uniaxial and hydrostatic compressions, respectively. To have physically admissible deformations $0 \leq k \leq 3$ should hold.

Furthermore, an associative flow rule is assumed; thus, the plastic strain is perpendicular to the yield surface, namely

$$\dot{\epsilon}^{pl} = \dot{\lambda} \frac{\partial \Phi}{\partial \sigma}, \quad (15)$$

where $\dot{\lambda}$ is the plastic flow multiplier, for which $\dot{\lambda} \geq 0$ holds. This also gives that α is directly related to the plastic Poisson's ratio (ν_{pl}) as

$$\alpha = \sqrt{\frac{9(1 - 2\nu_{pl})}{2(1 + \nu_{pl})}}, \quad (16)$$

in which, the plastic Poisson's ratio is defined for large deformations with the true strain rates [48,49], as

$$\nu_{pl} = -\frac{\dot{\epsilon}_T^{true}}{\dot{\epsilon}^{true}} \quad (17)$$

Combining that for quasi-static tests ϵ_c varies linearly in time, with Eq. (6) and assuming that generalized Poisson's ratio relation in Eq. (4) holds, we get $\nu_{pl} = \nu^*$. Therefore, the measured ν^* can be adopted to approximate the plastic Poisson's ratio.

3.2. Nonlinear hardening model

In the proposed model, the evolution of the yield surface is characterized by a nonlinear hardening curve, which defines the relation of the yield stress and the equivalent plastic strain, namely $\sigma_y(\epsilon_c^{pl})$. The hardening curve should incorporate both the plateau region and the densification regime characteristic of foams. Since foams are typically subjected to compressive loading, the hardening law is expressed in

terms of the compressive plastic strain, ϵ_c^{pl} . Based on the formulation in [46], the hardening law can be written as

$$\sigma_y = \sigma_{y0} + \gamma \frac{\epsilon_c^{\text{pl}}}{\epsilon_D} + \alpha_2 \ln \left[\frac{1}{1 - (\epsilon_c^{\text{pl}} / \epsilon_D)^\beta} \right] \quad (18)$$

where σ_{y0} stands for the initial yield stress, while γ is the linear hardening modulus, α_2 the nonlinear hardening modulus, β the nonlinear hardening parameter, ϵ_D is the densification strain. The latter has different forms depending on the load case, but it can be directly derived from the relative density ρ_{rel} and the yield surface parameter α . For uniaxial loading,

$$\epsilon_D = -\frac{9 + \alpha^2}{3\alpha^2} \ln(\rho_{\text{rel}}). \quad (19)$$

4. Parameter fitting

In the proposed constitutive model, a total number of 8 material parameters should be determined for each printing temperature, namely: E , ν_0 , σ_{y0} , ν_p , ϵ_D , α_2 , β , and γ . The parameters were determined using the following method.

1. Parameters of linear elasticity (E, ν_0): Determined from the uniaxial compression test as the initial slope and the measured ground state Poisson's ratio in Eq. (4) (for the values see Table 1).
2. Initial yield stress (σ_{y0}): Calculated using the 10% relative error method in Section 2.3.3 (for the values see Table 1).
3. Plastic Poisson's ratio (ν_p): Due to the large compressive strains, plastic Poisson's ratio is approximated as the generalized Poisson's ratio determined from image processing for the entire compression test, letting $\nu_p \approx \nu^*$ (for the values see Table 1).
4. Densification strain (ϵ_D): Calculated using the relation in Eq. (19) that holds for associative and isotropic Deshpande-Fleck foam (for the numerical values see Table 2).
5. Parameters of the nonlinear hardening curve (α_2, β, γ): Determined using parameter optimization for the measured $\sigma_y(\epsilon_c^{\text{pl}})$ curves (see Table 2 for the numerical values).

4.1. Parameter fitting for the hardening curve

The hardening curve of the proposed material model was fitted to the $\sigma_y(\epsilon_c^{\text{pl}})$ hardening curves measured at different printing temperatures. The fitting was performed in *Wolfram Mathematica* using the *NMinimize* global optimization procedure, in which the error function of Q was defined as a sum of normalized squared differences (SNSD), as

$$Q = \sum_{i=1}^N \left(\frac{\sigma_y^{\text{meas}} - \sigma_y^{\text{fit}}(\epsilon_c^{\text{pl, meas}})}{\sigma_y^{\text{meas}}} \right)^2 \quad (20)$$

where N is the number of measurement points, and σ_y^{meas} and $\epsilon_c^{\text{pl, meas}}$ are the data points of the measured hardening curves. The fitted parameter

values for each printing temperature fit are listed in Table 2, and their variations with the printing temperature are depicted in Fig. 9/a-c.

The fitted model was evaluated also by performing an FE analysis on a unit cube ($1 \times 1 \times 1$ mm) in Abaqus [45], using a single eight-node brick element with reduced integration (C3D8R) for the mesh. The applied boundary conditions are shown in Fig. 10/a, while the comparison of the model prediction and the measurement data are illustrated in Fig. 10/b-h. The accuracy of the model prediction is quantified by the coefficient of determination R^2 defined as

$$R^2 = 1 - \frac{\sum_{i=1}^N (\sigma_y^{\text{meas}} - \sigma_y^{\text{fit}}(\epsilon_c^{\text{pl, meas}}))^2}{\sum_{i=1}^N (\sigma_y^{\text{meas}} - \bar{\sigma}_y^{\text{meas}})^2}, \quad (21)$$

where $\bar{\sigma}_y^{\text{meas}}$ denotes the average of σ_y^{meas} .

5. Results

The fitting results clearly demonstrate that the proposed constitutive model, based on the Deshpande-Fleck crushable foam formulation, can accurately capture the mechanical response of the investigated in-situ foaming LW-PLA material over the studied range of printing temperatures. The variation of the parameters shows good agreement with the microstructural characteristics. In particular, the printing-temperature dependence of the initial elastic modulus E , the initial yield stress σ_{y0} , and the densification parameter ϵ_D follow the relative density characteristics presented in Fig. 3/a. Moreover, the linear contribution of the hardening law, represented by the parameter γ , becomes increasingly dominant at higher temperatures and exhibits an approximately linear trend within the investigated temperature range. In contrast, the nonlinear hardening parameter β shows a decreasing tendency, while the nonlinear hardening modulus α_2 decreases sharply at elevated temperatures and displays local maxima in the transition region. Overall, the results indicate that the proposed printing-temperature-dependent material model can be readily and efficiently implemented in finite element solvers for the numerical analysis and optimization of variable-density sandwich structures. The FE validation is limited to uniaxial compression used for parameter identification; multiaxial and structural applications are left for future work.

6. Conclusion

In this study, the nonlinear elastic-plastic behavior of in-situ foamed LW-PLA produced by additive manufacturing was investigated over a range of printing temperatures. An optimal foaming regime was identified around 230 – 240 °C, with a low relative density (i.e., high foaming rate), low elastic moduli and initial yield stress, and the most homogeneous cellular structure, while even higher temperatures resulted in structurally less stable foam.

For the mechanical characterization the Deshpande-Fleck crushable foam model with an associative flow rule and nonlinear hardening was adopted. The proposed fitting strategy enables reliable determination of elastic, yielding, and hardening parameters directly from uniaxial compression tests. In this framework, the generalized Poisson's ratio gave a reliable approximation of the plastic Poisson's ratio

Table 2
Fitted model parameters for LW-PLA at different printing temperature levels.

T [°C]	ϵ_D [–]	α_2 [MPa]	β [–]	γ [MPa]	R^2 [%]
190 °C	0.9073 ± 0.106	66.48 ± 6.84	6.966 ± 0.249	23.005 ± 1.66	99.91
200 °C	1.0883 ± 0.175	70.25 ± 8.81	4.512 ± 0.271	23.867 ± 1.319	98.86
210 °C	1.3682 ± 0.162	71.58 ± 5.36	3.707 ± 0.071	37.904 ± 0.641	92.54
220 °C	1.0502 ± 0.061	82.23 ± 5.78	3.519 ± 0.119	37.124 ± 1.171	99.21
230 °C	0.9891 ± 0.024	95.43 ± 7.37	3.632 ± 0.209	14.691 ± 0.592	99.53
240 °C	1.1205 ± 0.028	101.45 ± 3.95	3.090 ± 0.258	12.976 ± 0.479	98.87
250 °C	1.1697 ± 0.029	106.87 ± 5.88	2.851 ± 0.009	16.566 ± 2.069	99.01

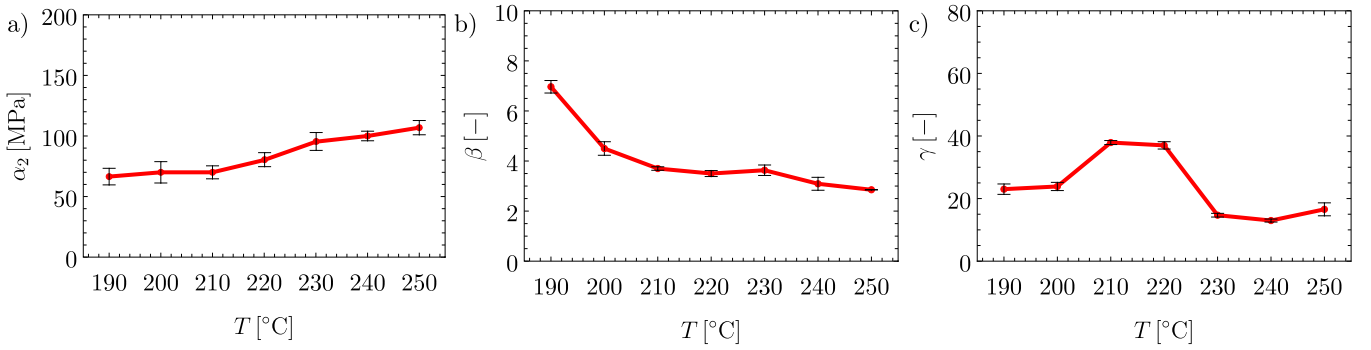


Fig. 9. The (a)-(c) variation of the hardening parameters α_2 , β and γ in the function of the printing temperature.

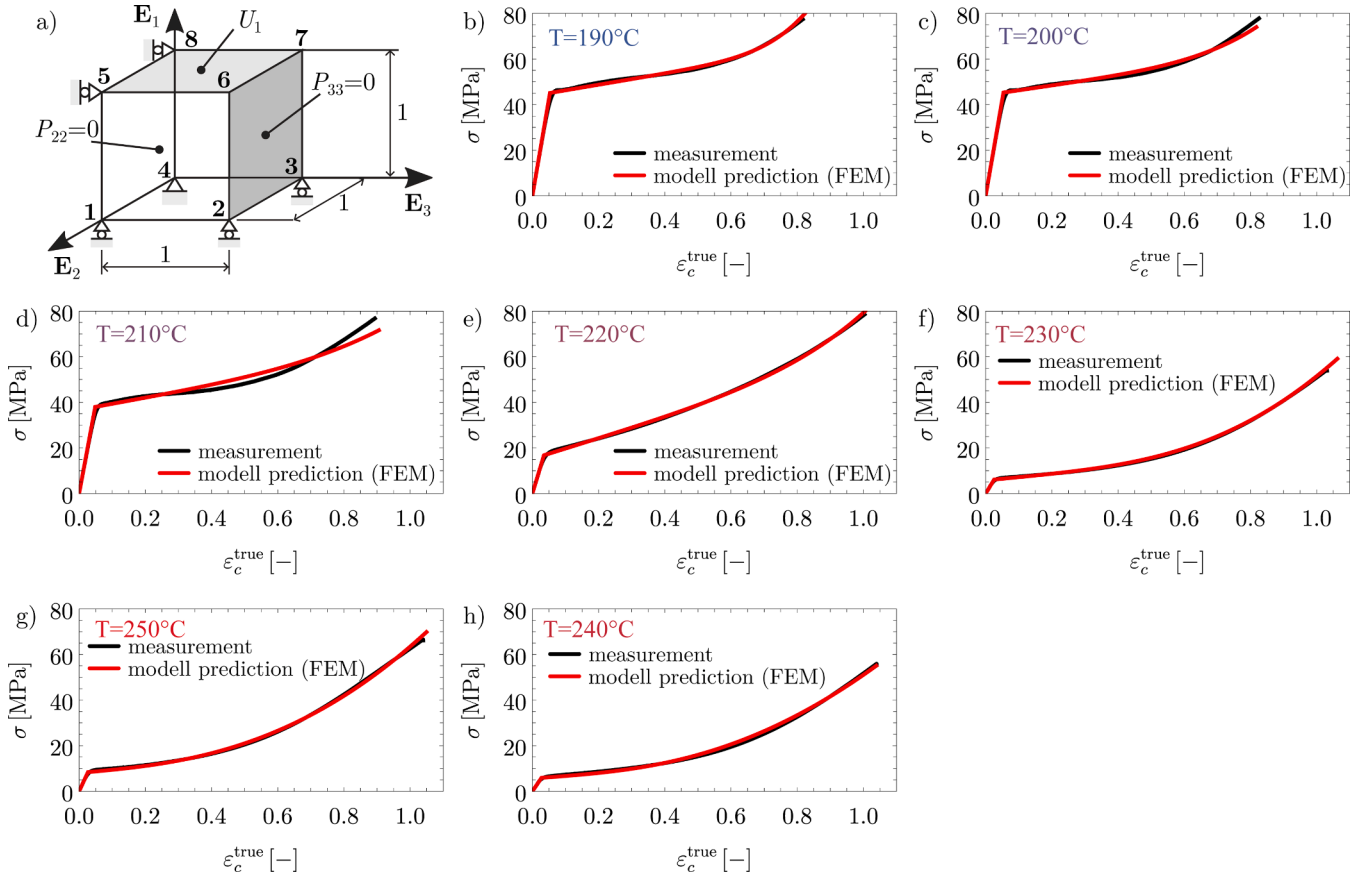


Fig. 10. The (a) applied FE-model for validation and (b)-(h) the comparison of the model prediction (FEA results) and the measured true stress - true compressive strain results for printing temperature levels.

characterizing both the yield surface and densification behavior. The model prediction shows excellent agreement with the measured stress-strain responses over the entire temperature range, as confirmed by finite element validation.

The main findings are the following: i) the elastic modulus, the ground-state Poisson's ratio and the initial yield stress show similar tendency as the relative density; ii) the linear contribution in the nonlinear hardening curve becomes increasingly dominant; iii) the order of nonlinearity in the hardening curves decreases monotonously and iv) the nonlinear contribution has a peak significance at activation

temperatures where foaming starts.

Overall, the results demonstrate that the Deshpande–Fleck crushable foam model, in combination with printing-temperature-dependent material parameters, provides a reliable and efficient tool for predicting the mechanical response of 3D-printed in-situ foamed PLA polymers. The proposed modelling strategy is well-suited for implementation in commercial finite element solvers and offers a solid basis for the numerical design and optimization of functionally graded and sandwich-type foam structures manufactured by additive manufacturing.

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CRedit authorship contribution statement

Szabolcs Berezvai: Writing – original draft, Visualization, Software, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Márton Tomin:** Writing – review & editing, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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