



Snap-through of soft elliptical shells under volume- and pressure-control

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ABSTRACT

Soft materials are widely used in engineering applications due to their flexibility, energy absorption capability, and low prototyping cost. These characteristics make such materials an ideal basis for designing complex, compliant mechanical systems, addressed in soft robotics. Additionally, they often exhibit significant material and structural nonlinearities, which can be exploited through snap-through instability to achieve rapid and enhanced mechanical responses. While snap-through behavior has been extensively studied in the literature for soft spherical shells, this contribution extends shell geometries considering an axisymmetric elliptical generalization using fluidic actuation. A comprehensive framework is proposed for capturing the snap-through behavior of soft axisymmetric elliptical shells through numerical and experimental methods, utilizing volume- and pressure-controlled methods. A general geometric formulation is introduced to describe elliptical shells, and finite element models are developed to analyze their nonlinear snapping behavior. The numerical results are validated using a dedicated experimental setup capable of reproducing both control methods. The results show that axisymmetric simulations provide an efficient and reliable tool for parameter studies, while dynamic 3D simulations closely follow the stable branches of the quasi-static equilibrium paths. Equivalent energy-release geometries are identified, revealing a linear relation between key geometric parameters that enables the structural response to be tuned without changing the overall actuator size. The experiments confirm that volume control provides a more robust characterization of snap-through, whereas pressure control is more sensitive to control-loop limitations. Overall, the results demonstrate that elliptical shells offer additional geometric flexibility while preserving the performance of spherical snap-through actuators.

1. Introduction

Elastic structures can exhibit multiple equilibrium configurations when their geometry, boundary conditions, or material properties allow more than one local minimum in the total potential energy landscape. Such systems are commonly referred to as multistable structures, and they are capable of maintaining distinct configurations without continuous external energy input. Among these, bistable structures, which possess two stable equilibrium states separated by an unstable configuration, represent one of the simplest and most widely studied classes of multistable systems [1]. Transitions between equilibrium states typically occur through elastic instabilities, during which the structure moves rapidly from one configuration to another. These transitions are driven by the accumulation of elastic strain energy under external loading. As the load increases, the system approaches a critical point where the current equilibrium path loses stability, triggering a sudden release of stored energy and a rapid transition to another stable configuration [1–5]. This phenomenon, often referred to as snap-through buckling, is characterized by highly nonlinear structural behavior and

dynamic motion. A simple example of snap-through buckling can be seen in Fig. 1(a).

The phenomenon is usually characterized by the force–displacement relationship, which is highly nonlinear even in the case of linear material behavior. Fig. 1(b–c) represents the characteristic curve of the presented spherical arc, which is the set of all the points that satisfy the static equilibrium equations [2,3]. Some of the equilibrium points are unstable, making them impossible to reach in a real situation (see dashed lines in Fig. 1(b–c)). In measurements, one can either use force or displacement controlled methods, which capture the snap-through and snap-back phenomena of the system, respectively. When the force is controlled, upon reaching the critical loading point (F_{cr}), the system jumps to its other stable state. The same happens during the displacement control method when the critical displacement point (D_{cr}) is reached.

The snap-through instability in shells and plates is a significant area of study in structural mechanics [7–14]. Traditionally, buckling and snap-through are considered undesirable failure modes in structural

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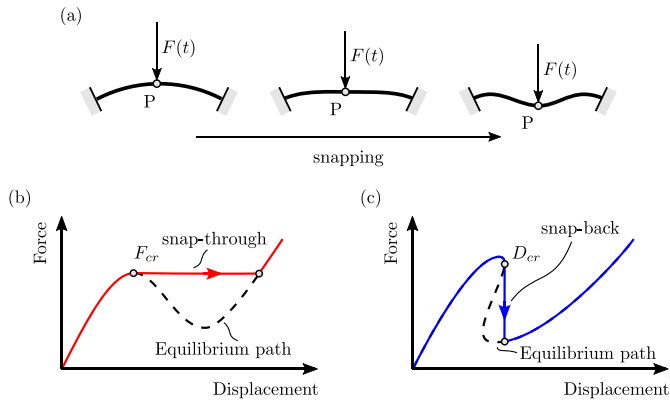


Fig. 1. The snap-through buckling of a spherical arc. (a) The snap-through buckling of the spherical arc is induced by the increasing force $F(t)$ at point P. (b) The force-displacement curve (red curve) of the spherical arc, regarding the vertical displacement of point P, using force-controlled method, showing the snap-through when the critical force F_{cr} is reached. (c) The force-displacement curve (blue curve) acquired using displacement-controlled method, showing the snap-back when the critical displacement D_{cr} is reached [6]. The dashed lines in (b–c) represent the unstable equilibrium path [3].

mechanics [15]. It is also remarkable that a systematic discrepancy was found between theoretically obtained and experimentally observed buckling pressures [8,16], which posed additional challenges for the accurate design of thin-walled structures. Consequently, extensive research efforts were undertaken to improve the theoretical models and enable more efficient and reliable design of such structures [17].

In contrast, recent developments demonstrate that these instabilities can be deliberately utilized as functional features [15]. Several examples can be mentioned in soft robotics: snapping can be utilized to achieve rapid actuation and enhanced performance. Rapid actuation can be reached, even for slow actuation speeds [2,3,5,18–22]. Recent research focused on the response of artificial muscles [23,24], bioinspired robots [25,26], elastic jumpers [27,28], grippers [29], energy harvesters [30] and energy dissipation mechanisms [31–34]. Soft robotic systems are constructed from a variety of components, leading to considerable variability in geometry [18]. Commonly, these geometries include spherical shells used in jumpers [27,35,36] and soft valves [37,38], curved plates, beams and panels [5,39–42], membranes [5,43] and rings [44]. Snapping has been also actively investigated in the field of mechanical metamaterials [45–54]. Further applications include morphing composite laminates used for aerospace structures [55–61] and vibration-based energy harvesting [62–65]. Snapping is also used in MEMS structures [66–69] and in Optical MEMS design [70–73].

Most recent studies further demonstrate that snap-through and buckling phenomena remain active research topics across shell structures, soft systems, and instability-based metamaterials. Patterned soft spherical shells have been shown to exhibit controllable buckling morphologies and multiple snap-through events under indentation [74], while contact-induced eversion of axisymmetric shells has been analyzed to clarify the transition between stable deformation and snapping [75]. Further developments include bistable dome shells with tunable energy barriers [76], rate-dependent snap-through in viscoelastic metamaterials [77], and spontaneous crumpling of active spherical shells [78]. Recent studies have also investigated fluid-induced snap-through of spherical shells [79], snap-through buckling of elliptic paraboloid shallow shells [80], imperfection sensitivity in multi-defect hemispherical shells [81]. These developments confirm the continued relevance of shell buckling and instability-driven design.

The present work builds upon these foundational studies by introducing a comprehensive framework for the numerical and experimental

analysis of snap-through instability in hyperelastic shells characterized by axisymmetric elliptical geometries. The elliptical formulation of the shells enables a more generalized geometric description of the problem, which opens the route for robust parameter studies. While snap-through behavior has been extensively investigated in spherical shells [27,36,82,83] and plates [84–87], alternative geometries can offer additional design flexibility. In particular, elliptical shell geometries introduce additional geometric parameters, which expand the accessible design space compared to spherical shells defined by a single curvature. This increased geometric freedom enables a more detailed tuning of the structural response, allowing the critical load, deformation path, and energy release associated with snap-through to be adjusted through geometry. From a design perspective, this flexibility may also enable more efficient structural solutions, where similar mechanical performance can be achieved with reduced material usage or more compact geometries compared to equivalent spherical shells. Such improvements are particularly relevant for applications where mass and volume constraints are critical. Another practical motivation arises from manufacturing considerations. In soft shell systems fabricated using molds, geometric deviations may occur due to mold wear, deformation during the curing process, or other fabrication imperfections. To account for these imperfections, researchers often used intentional surface defects and indentations in the mathematical models [74,88–90]. On the other hand, the elliptical formulation can be potentially used to account for, compensate, or even deliberately introduce controlled imperfections in future designs. Understanding how these parameters influence the mechanical response is therefore an important step toward robust and tunable multistable structures.

Despite the extensive literature on snap-through instabilities in spherical shells and related soft structures, the role of axisymmetric elliptical shell geometries has not been systematically addressed. This represents an important lack in the literature, since spherical shells provide only a limited geometric design space, whereas axisymmetric elliptical geometries introduce additional parameters that can be utilized to substantially affect the critical pressure, deformation path, and energy release during instability. The present work addresses this gap by formulating an axisymmetric elliptical generalization of soft shell snap-through and by combining parametric finite element analyses with experimental validation. This contribution aims to quantify the influence of ellipticity and provides relations for tuning the snap-through response in a wider geometric design space compared to purely spherical shell configurations.

To analyze the snap-through phenomena, axisymmetric simulation models are built in ABAQUS, which follow the quasi-static solution approach. These models are used to analyze the effect of the geometric parameters, thus they function as a basis of geometry selection for more advanced, 3D finite element simulations. The results of the numerical simulations are validated using measurements performed on a self-developed setup. The methodology of the work can be seen in Fig. 2. In this work, we are limiting the investigation to axisymmetric responses, because preliminary tests and observations indicate that, for the shell geometries considered, the snap-through behavior is predominantly axisymmetric. This simplification allows us to focus on the main deformation mechanisms while still providing a good approximation of the actual response.

The work is separated into six main sections, including the present Introduction. Section 2 presents the geometric formulation of the problem, while Section 3 focuses on the modeling of the material behavior. Section 4 details the implementation of the numerical simulations, and Section 5 presents the experimental methodology, which is employed to validate the numerical results both quantitatively and qualitatively. Finally, Section 6 summarizes the results, and Section 7 concludes the paper by highlighting the main findings.

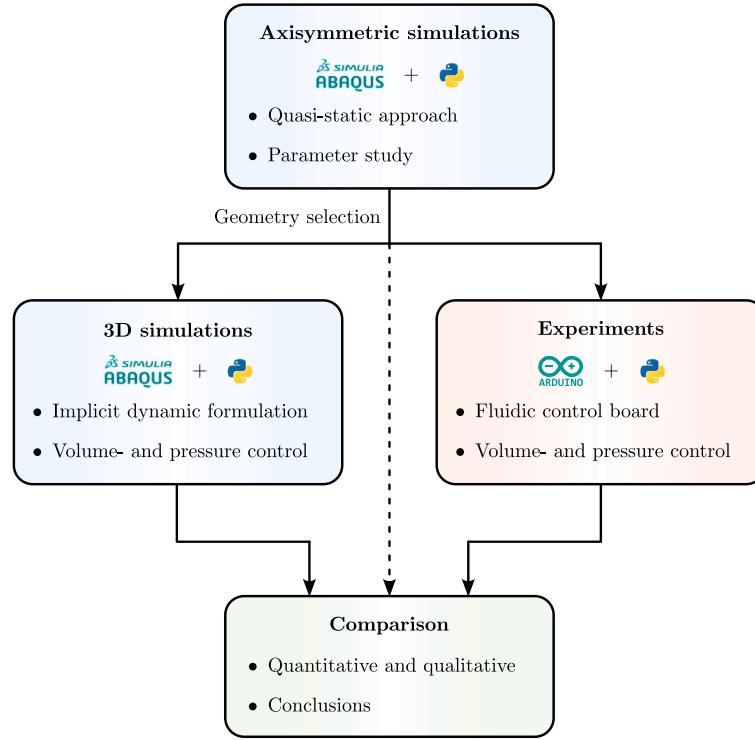


Fig. 2. The methodology of the present work. The parametric, quasi-static axisymmetric simulations provide the basis of the geometry selection, while implicit dynamic 3D simulations and experiments using a fluidic control board provide the main results for the quantitative and qualitative comparison, for both volume- and pressure-controlled scenarios.

2. Geometric formulation

This section introduces the geometric description of the investigated axisymmetric elliptical shell and the surrounding container. The formulation defines the main design parameters used later in the parameter studies and numerical simulations.

The elliptical characterization of the shell geometry provides a general framework to perform parameter- and sensitivity studies. This formulation provides the possibility to analyze the precise relation of the geometry and the characteristics of its snap-through. The elliptic form factor enables us to tune the geometry depending on the specific soft robotic application. In the proposed method the elliptical, silicone rubber shell is placed inside an ABS (Acrylonitrile Butadiene Styrene) plastic container.

The snap-through instability will be induced by the internal pressurization $P(V)$ of the cavity formed by the shell and the container. The proposed geometry (seen in Fig. 3(a)) can be completely defined by the minor- and major semi-axes of the elliptical shell (A , B), the shell's thickness (t), base radius (R), base thickness (t_b), its opening angle (θ), and the geometric dimensions of the container, namely the height (H) and the thickness (v).

To formulate this geometry in a reproducible parametric form, the coordinates of the visualized points are derived by analyzing the intersection points of primitive geometric lines. Although the main dimensions are illustrated in Fig. 3(a), the following analytical relations provide the basis for the automated model generation used in the FE (finite element) simulations. The coordinates of the points in the denoted (x, y) coordinate system are given by Eqs. (1) to (11).

$$\mathbf{P}_1 = \left(0, B - \frac{t}{2} \right), \quad (1)$$

$$\mathbf{P}_2 = \left(0, B + \frac{t}{2} \right), \quad (2)$$

$$\mathbf{P}_3 = \left(0, H \right), \quad (3)$$

$$\mathbf{P}_4 = \left(0, H + v \right), \quad (4)$$

$$\mathbf{P}_5 = \left(\sqrt{\left(1 - \frac{r_e^2 \cos^2 \theta}{\left(B + \frac{t}{2} \right)^2} \right) \cdot A_p^2 + v}, H + v \right), \quad (5)$$

$$\mathbf{P}_6 = \left(\sqrt{\left(1 - \frac{r_e^2 \cos^2 \theta}{\left(B + \frac{t}{2} \right)^2} \right) \cdot A_p^2}, H \right), \quad (6)$$

$$\mathbf{P}_7 = \left(\sqrt{\left(1 - \frac{r_e^2 \cos^2 \theta}{\left(B + \frac{t}{2} \right)^2} \right) \cdot A_p^2 + v}, r_e \cos \theta \right), \quad (7)$$

$$\mathbf{P}_8 = \left(\sqrt{\left(1 - \frac{r_e^2 \cos^2 \theta}{\left(B + \frac{t}{2} \right)^2} \right) \cdot A_p^2}, r_e \cos \theta \right), \quad (8)$$

$$\mathbf{P}_9 = \left(R, r_e \cos \theta \right), \quad (9)$$

$$\mathbf{P}_{10} = \left(R, r_e \cos \theta - t_b \right), \quad (10)$$

$$\mathbf{P}_{11} = \left(\sqrt{\left(1 - \frac{(r_e \cos \theta - t)^2}{\left(B - \frac{t}{2} \right)^2} \right) \cdot A_m^2}, r_e \cos \theta - t_b \right), \quad (11)$$

where A_p and A_m are

$$A_p = A + \frac{t}{2}, \quad A_m = A - \frac{t}{2}, \quad (12)$$

and r_e is the distance of an arbitrary point on the ellipse from the origin, defined by the opening angle θ . The relation of r_e and θ can be obtained

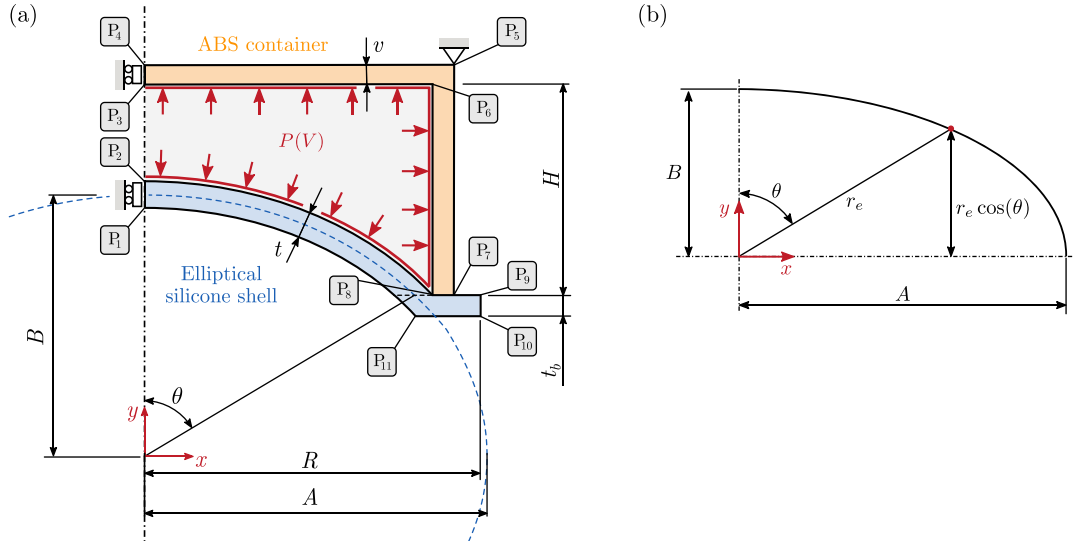


Fig. 3. Geometric description. (a) The proposed geometry of the elliptical shell and its environment, where the snap-through instability will be induced. The geometry is defined by the $P_1 - P_{11}$ points, whose coordinates are calculated by the geometric parameters as the minor- and major semi-axes (A, B), the thickness (t), base radius (R), base thickness (t_b), the opening angle (θ) of the shell, and the container height (H) and thickness (v). Note, that this is a generalization of the geometry proposed in the literature [27]. The model consist of two different materials: the container is made out of ABS plastic, while the shell is fabricated from silicone rubber. (b) The interpretation of the length of the vector pointing to an arbitrary point on the ellipse (r_e), defined by the opening angle (θ).

from the polar-form of the ellipse, given as

$$r_e = \frac{AB}{\sqrt{A^2 \sin^2 \left(\frac{\pi}{2} - \theta \right) + B^2 \cos^2 \left(\frac{\pi}{2} - \theta \right)}}, \quad (13)$$

which can be interpreted using Fig. 3/(b).

3. Material characterization

As previously mentioned, the analyzed model is composed of ABS and silicone rubber materials. This section presents the material characterization for the silicone shells and the ABS container. Since the snap-through response involves finite-strain elastic deformations, particular attention is given to the experimental characterization and hyperelastic modeling of the silicone materials.

3.1. Material models for the silicone rubber shells

In this study, a finite strain pure elastic behavior of the soft silicone material is assumed, which is widely applied in snap-through problems [3,5,27,37,38,91]. Constitutive material models for hyperelastic materials are derived from the so-called strain energy density function

$$W = W(\mathbf{C}), \quad (14)$$

where $\mathbf{C} = \mathbf{F}^T \mathbf{F}$ is the right Cauchy–Green deformation tensor and $\mathbf{F}$ is the deformation gradient. For isotropic materials, it can be shown that

$$W = W(\mathbf{C}) = \bar{W}(I_1, I_2, I_3) = \bar{W}(\lambda_1, \lambda_2, \lambda_3), \quad (15)$$

where I_1, I_2, I_3 are the principal invariants of $\mathbf{C}$, while λ_1, λ_2 and λ_3 are the principal stretches. The Cauchy stress tensor is given by

$$\boldsymbol{\sigma} = \frac{2}{J} \mathbf{F} \frac{\partial W(\mathbf{C})}{\partial \mathbf{C}} \mathbf{F}^T, \quad (16)$$

in which

$$J = \det(\mathbf{F}) = \lambda_1 \lambda_2 \lambda_3 \quad (17)$$

is the volume ratio, which takes the value of $J \equiv 1$ for incompressible materials. The principal stresses can be obtained as

$$\sigma_k = \lambda_k \frac{\partial W}{\partial \lambda_k}, \quad \text{and} \quad P_k = \frac{\partial W}{\partial \lambda_k}, \quad (18)$$

for $k = 1, 2, 3$, where σ and P denote the Cauchy and the nominal stress, respectively. In this study, the shells were fabricated from two kinds of materials: the *ELASTOSIL M 4601 A/B* silicone (which will be referred to as *ELASTOSIL* for short), and the *RTV2* silicone. Their properties are summarized in Table 1. The stress–strain characteristics of the materials were determined through uniaxial tensile and compression tests using an *Instron 3345 Single Column Universal Testing System*, equipped with an *Instron 2519-107* 5 kN load cell. The tensile tests were conducted on multiple dogbone shaped specimens, according to the ISO 37 standard, compression tests were performed on cylindrical specimens, in accordance with the ISO 7743 standard (see Fig. 4/(a)). The displacement–force characteristics were recorded by the testing system, with great reproducibility, as shown in Fig. 4/(b). The transverse behavior of the material is captured via high resolution video recording and binarization post-processing techniques, illustrated in Fig. 4/(c). The stretch in the longitudinal direction is denoted with $\lambda_1 = \lambda$, while the transverse stretches are denoted with $\lambda_2 = \lambda_3 = \lambda_T$. As stated previously, if the material is incompressible, then

$$J = \det(\mathbf{F}) = \lambda_1 \lambda_2 \lambda_3 \equiv 1 \quad (19)$$

holds, from where the transverse stretches can be calculated as:

$$\lambda_2 = \lambda_3 = \lambda_T = \frac{1}{\sqrt{\lambda_1}} = \frac{1}{\sqrt{\lambda}}. \quad (20)$$

A general expression for the transverse stretch is given by

$$\lambda_T = \lambda^{-\nu}, \quad (21)$$

where ν is the Poisson ratio, which takes the value of $\nu \equiv 0.5$ for incompressible materials. By fitting models to the experimental results in the form of Eq. (21), the Poisson ratio can be obtained, which are $\nu_E = 0.498$ and $\nu_R = 0.491$ for the *ELASTOSIL* and *RTV2* material, respectively. A graphical comparison of these experimental results to this theoretical relation is presented in Fig. 4/(d). As the materials are nearly incompressible, their behavior was captured with incompressible hyperelastic material models. This also means that the uniaxial tension is equivalent to equibiaxial compression, while the uniaxial compression is equivalent to equibiaxial tension, through the superposition of the hydrostatic stress. Viscoelastic effects are neglected in the present model, since the dynamic characteristics of the snap-through process are mainly governed by inertial effects, which are explicitly captured in

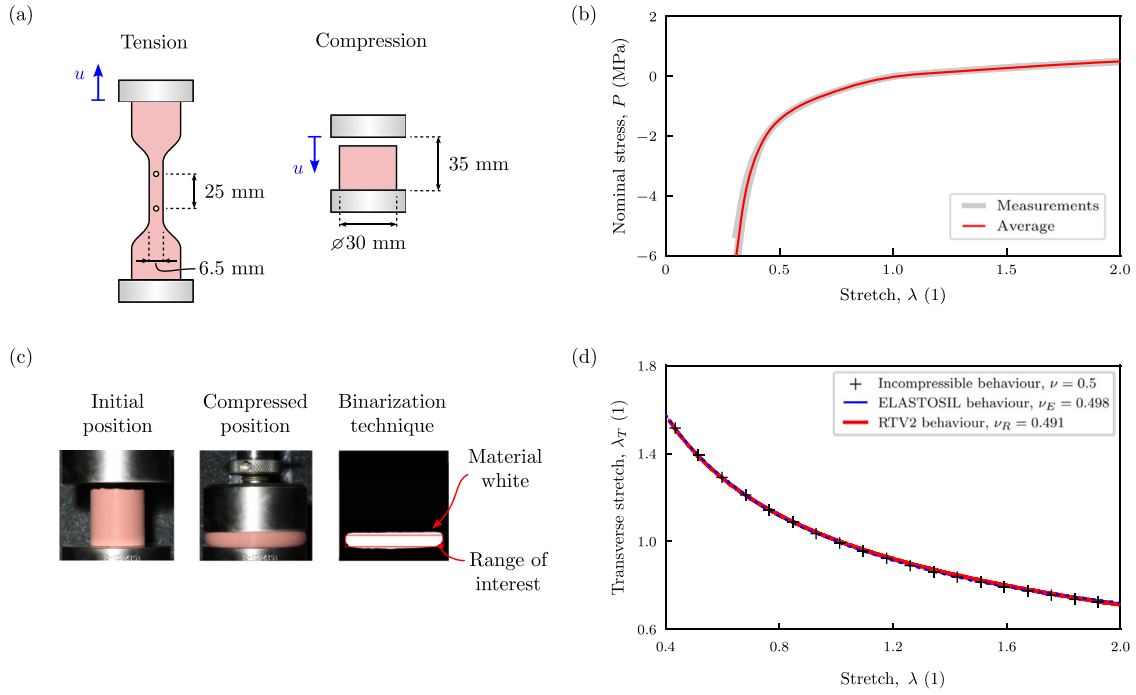


Fig. 4. Material testing by uniaxial tests. (a) Tension (dogbone) and compression (cylindrical) specimens and the relevant specimen dimensions used for uniaxial material tests. (b) Measured nominal stress–stretch ($P - \lambda$) curves (gray lines) for the RTV2 material with their average (red line). The measurements showed great reproducibility. (c) Binarization image processing technique applied on a cylindrical specimen. The number of the material white pixels in the region of interest is used to calculate the transverse stretch. (d) The acquired relation of the longitudinal and transversal stretches ($\lambda - \lambda_T$) for both tested materials (red and blue lines) and the theoretical curve (black markers) for incompressible behavior ($\nu = 0.5$). The fitted Poisson's ratios (ν_E, ν_R) are denoted in the legend, which indicated that the incompressibility assumption can be applied.

the dynamic finite element simulations. Including viscoelasticity would also lead to delayed snap-through phenomenon, which was already demonstrated for beam metamaterials [77] and which are beyond the scope of this study.

Multiple hyperelastic material models were fitted to the experimental data, including the neo-Hookean (NH), the Mooney–Rivlin (MR), and the Ogden (O) models, where the corresponding strain energy functions define as:

$$W^{\text{NH}}(I_1) = C_{10}(I_1 - 3), \quad (22)$$

$$W^{\text{MR}}(I_1, I_2) = C_{10}(I_1 - 3) + C_{01}(I_2 - 3), \quad (23)$$

$$W^{\text{O}}(\lambda_1, \lambda_2, \lambda_3) = \sum_{i=1}^N \frac{2\mu_i}{\alpha_i^2} (\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3). \quad (24)$$

First, second and third order Ogden models were fitted, from which the second-order model provided the best fit, while maintaining a relatively low number of parameters and capturing the stress stiffening properties in the tensile region. The fits were performed using a quality function with the help of the *minimize* function in SciPy, Python, using the *SLSQP* method. The quality function is defined as the root mean square error between the model predictions and the experimental results as

$$Q = \sqrt{\frac{1}{M} \sum_{i=1}^M \left(\frac{P_i^{\text{mod}} - P_i^{\text{exp}}}{P_i^{\text{exp}}} \right)^2}, \quad (25)$$

where M is the number of data points, P_i^{mod} is the nominal stress given by the material model, and P_i^{exp} denotes the nominal stress acquired by the experimental results. Between these models, the second-order Ogden model provided the best fit for both materials, which was validated by calculating the coefficient of determination R^2 as

$$R^2 = 1 - \frac{\sum_{i=0}^M (P_i^{\text{exp}} - P_i^{\text{mod}})^2}{\sum_{i=0}^M (P_i^{\text{exp}} - \bar{P})^2}, \quad (26)$$

Table 1

Important properties of the ELASTOSIL and RTV2 materials.

Silicone	Density at 23°	Mixing ratio (A:B)	Shore hardness	Color
ELASTOSIL	1.13 g/cm ³	9 : 1	28 Shore A	Reddish-brown
RTV2	1.3 g/cm ³	25 : 1	40–45 Shore A	White

where $\bar{P}$ is the mean of the experimental dataset. The fitted material parameters are summarized in Table 2, while the obtained fits are visualized in Fig. 5. The stability of every material model was evaluated with the Drucker-stability criterion. It is also known, that residual stresses could develop during the molding and curing process, which could influence the mechanical behavior of the specimens significantly [92–95]. In this study, these residual stresses are not considered, however a systematic study was performed to determine the sensitivity of the snap-through based on the specific material parameters (see Appendix A.1). The study is performed by analyzing the deviation in the output variable (pressure–volume curve) due to a disturbance in one of the parameters in the fitted second-order Ogden models. The analysis reveals that errors in the μ parameters (which are related to the material stiffness) lead to more pronounced deviations in the resultant critical pressures. This result highlights that the snap-through can be sensitive to molding imperfections, such as inhomogeneous regions or inaccurate geometry.

3.2. Material model for the ABS container

The plastic container, which is used to create an enclosed volume with the shell inside it (highlighted with orange in Fig. 3(a)) was a solid 3D print from Zortrax Z-ABS. This material is modeled using an isotropic, linear, elastic material model with a density of $\rho = 1195 \text{ kg/m}^3$, elastic modulus of $E = 1.08 \text{ GPa}$ and a Poisson's ratio of $\nu = 0.3$.

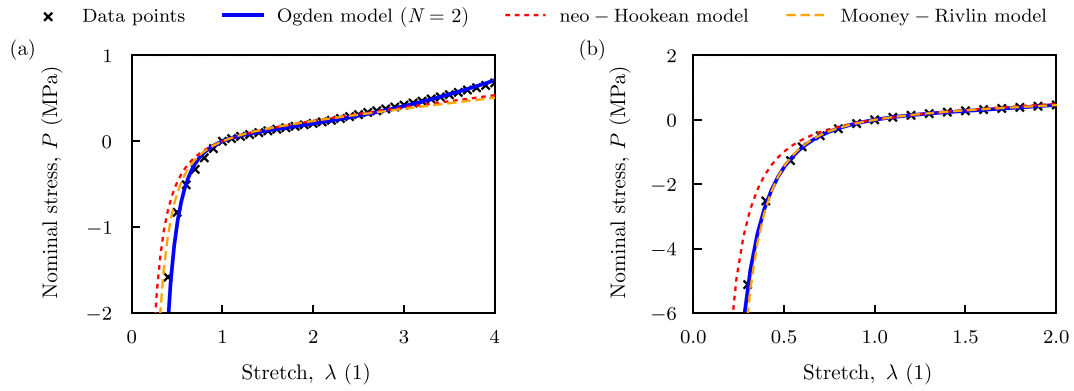


Fig. 5. Material models fitted to the experimental data. Fits produced for the measured nominal stress–stretch ($P-\lambda$) curve (black markers) for (a) *ELASTOSIL* and (b) *RTV2* materials using neo-Hookean (red dashed curve), Mooney–Rivlin (yellow dashed curve) and Ogden (blue curve) models. The acquired parameters are summarized in Table 2, where the coefficient of determination R^2 (calculated using (26)) showed that the second-order Ogden model provides the best fit for both of the investigated materials. Also important to note, that the Ogden model is capable to capture the stiffening effect in the tension range for the (a) *ELASTOSIL* material.

Table 2

Parameters of the fitted incompressible Ogden hyperelastic material model for the used silicone rubbers.

Silicone	μ_1 (MPa)	α_1 (1)	μ_2 (MPa)	α_2 (1)	R^2 (%)	Q (%)
ELASTOSIL	$1.05 \cdot 10^{-1}$	−2.325	$3.90 \cdot 10^{-2}$	3.339	95.86	11.04
RTV2	$2.77 \cdot 10^{-1}$	−0.671	$3.57 \cdot 10^{-2}$	4.769	99.97	2.44

4. Numerical simulations

This section describes the finite element framework developed to analyze the snap-through response of the shells. In this study, we will limit the investigations to axisymmetric snap-through, as initial experimental results showed that the shells tend to experience the instability with relatively good symmetry. Two distinct numerical simulation techniques are provided to characterize the snap-through behavior of the hyperelastic shell. The first, 2D method is based on a quasi-static approach, where the RIKS (arc-length) numerical method [14] is utilized to capture the equilibrium curve of the shell. To investigate the dynamics of the problem, more complex, 3D simulation models were also developed. A complete simulation environment was developed in ABAQUS 2020/STANDARD, through PYTHON scripting. The environment is capable of automatically building, calculating, and evaluating the problems based on the input parameters (geometry, materials, solver settings). The geometry is automatically created by using the derived analytical geometric equations for the significant points listed through Eqs. (1) to (11). In order to achieve numerically stable results, a mesh convergence study was performed for both approach, whose details can be seen in Appendix A.2. The study showed, that convergent solutions can be obtained if the thickness of the shell is discretized with at least 5 elements. In all developed simulations, the nonlinear geometric option was used with general solver tolerance of 10^{-3} . Aside from the boundary conditions and loading methods detailed in the subsequent sections, no additional contact interactions or constraint definitions were applied within the models.

4.1. Linear buckling analysis

To investigate the fundamental stability characteristics of the shell and select the appropriate modeling approach, a linear buckling analysis was performed. The numerical model was constructed using the three-dimensional geometry, ensuring that asymmetric mode shapes can appear. The 3D models were built by revolving the 2D sketches around the axis of symmetry. The parametric models were discretized with 8-node bilinear brick elements, using reduced integration and

hybrid formulation (C3D8RH). The applied boundary conditions reflect the conditions shown in Fig. 3/(a). During this analysis, a nominal distributed pressure of 1 Pa was applied directly on the surface of the shell. By solving the associated eigenvalue problem, the fundamental mode shapes and their corresponding load proportionality factors (λ_{LP}) were extracted. The obtained results indicate that the ratio of the semi-axes ($\hat{B} = A/B$) has an emphasized dependence on the initial buckling modes. Specifically, for low $\hat{B}$ values, the axisymmetric mode shape occurs at a lower load proportionality factor compared to the asymmetric modes. However, as the ellipticity increases, this behavior shifts significantly, and the asymmetric mode shapes become considerably more dominant. For these higher ellipticity geometries, the asymmetric modes occur at lower load levels and introduce complex spatial deformations. Because these robust asymmetric phenomena could deeply affect the highly nonlinear snap-through behavior of the shells, relying solely on two-dimensional axisymmetric simulations is insufficient. Therefore, the development of full 3D simulations is necessary to accurately capture the complete spatial and dynamic response of the system. It should be emphasized that while this linear analysis provides valuable preliminary insights, it inherently cannot capture the full complexity of the finite-strain snap-through phenomena. Consequently, these results serve primarily as qualitative guides.

4.2. Axisymmetric simulations

In order to explore the effect of the design parameters (A, B, t, θ), an axisymmetric model was developed, due to the lower computational cost compared to more refined models. The discretization was performed with the built-in medial axis meshing algorithm, using axisymmetric 4-node bilinear elements, with the hybrid formulation (CAX4H). The mesh size automatically adapts to the geometric parameters, ensuring that the thickness of the shell is discretized with at least five elements. The mesh quality was verified by monitoring the element aspect ratio, which was kept below 10 to avoid excessive distortion and ensure reliable numerical accuracy. The partitioned geometry, and the created mesh is illustrated at Fig. 7.

To simulate the enclosed fluid, the fluid–cavity interaction was used with a hydraulic fluid (water), which has a density of $\rho = 1000 \text{ kg/m}^3$ and a bulk modulus of $K = 2.1 \text{ GPa}$. The cavity surface was selected from the model accordingly, while a reference point was created, which is responsible to capture the pressure data. A symmetric and a fixed boundary condition was imposed on the model, as it can be seen in Fig. 8. The pressure of the cavity was set to 50 kPa, which was found high enough to induce the snap-through of the soft shells. The equilibrium solutions were determined using the arc-length method, which is accessed using a quasi-static RIKS step, with solver parameters

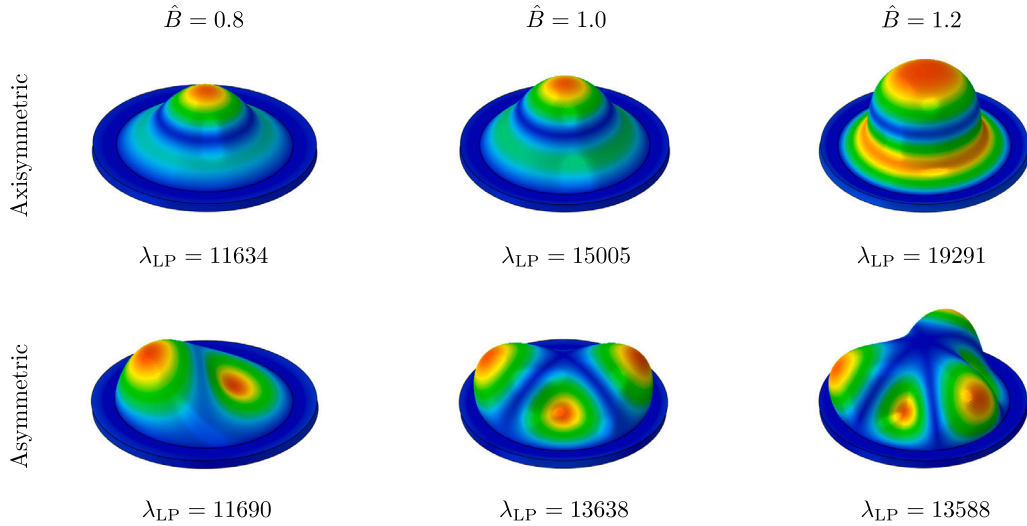


Fig. 6. Mode shapes from the linear buckling analysis. The resultant axisymmetric and asymmetric mode shapes and their load proportionality factor (λ_{LP}) from the linear buckling analysis. As the ratio of the semi-axes $\hat{B} = B/A$ increases (larger $\hat{B}$ corresponds to more 'elliptic' geometry), asymmetric modes occur at significantly lower load levels which could introduce complex deformation during the instability. The fringe color represents the total deformation, which is scaled by a factor of $5 \cdot 10^{-3}$.

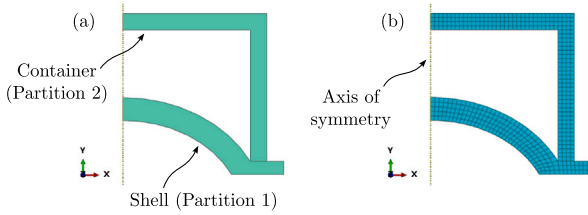


Fig. 7. The axisymmetric FE model. (a) The partitioned geometry of the shell (Partition 1) and the container (Partition 2) to achieve structured mesh. The partition is created by defining a section line at the intersection of the shell and the container. (b) The meshed geometry with CAX4H elements, with the medial axis algorithm.

as the minimum arc length increment size of $1 \cdot 10^{-32}$, maximum arc length increment size of $5 \cdot 10^{-3}$, initial arc length increment size of $1 \cdot 10^{-10}$ and maximum number of increments of 10000. With these settings, the numerical solution will evolve until the number of increments are reached. It should be noted, that a thicker shell require more increments, than shallow designs. After the shell snaps to its other stable position, the meaningful information from the simulation are acquired, the remaining increments will be used to inflate the geometry, which does not hold any important information regarding the snap-through phenomena. To stop the simulation after the snap-through, a stop condition was introduced for the simulations defined by the maximum vertical displacement of the pole point (v_{pole}). The stop criterion is defined as

$$\text{STOP, if } v_{pole} \geq 2H_{pole}. \quad (27)$$

where H_{pole} is the original pole height of the shell, measured from the base of the shell. Fig. 9 depicts the pole point and the stopping conditions imposed by the geometric relations. With this, the simulation will automatically stop, if the snap-through is reached, which saves computation time while performing parameter studies.

The nonlinear response of a shell can be characterized by the resultant equilibrium curve, by calculating the released energy during the snap-through. In this paper, the equilibrium curve of the shell is interpreted as the pressure–volume curve, which is the output of the fluid–cavity interaction. The evaluation is performed with

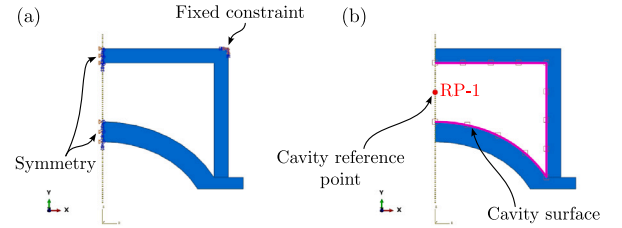


Fig. 8. Boundary conditions of the axisymmetric FE model. (a) The mechanical boundary conditions (symmetry and fixed constraints) (b) The defined fluid–cavity interaction with the cavity surface (purple lines) and its reference point (RP-1). For axisymmetric models, the axis of symmetry does not need to be selected, ABAQUS handles the formulation of the cavity according to the axisymmetric representation.

PYTHON, by detecting the limits points during the snap-through, then performing numerical integration using scientific packages. The visual interpretation of the ΔE energy is illustrated by Fig. 10.

4.3. Dynamic simulations

To capture the time-dependent behavior of the snap-through behavior, 3D dynamic simulations were developed. The same geometric modeling and meshing techniques were utilized as in the linear buckling analysis. The resultant geometry and the generated mesh is shown in Fig. 11. Structured mesh is achieved by partitioning the geometry, to drive the medial axis meshing algorithm. The global mesh size was calculated from the geometric parameters, using the same approach as in the 2D models (the thickness is discretized with at least 5 elements).

To model the fluid–structure interaction of the model, the same fluid–cavity interaction was defined using parameters from the 2D simulations. The required boundary conditions are illustrated in Fig. 12. The numerical solution is obtained by using an implicit dynamic step, with a time period of 1 s, with minimum, maximum and initial size of the time increment as $1 \cdot 10^{-10}$ s, $5 \cdot 10^{-3}$ s and $5 \cdot 10^{-3}$ s, respectively. The maximum number of increments was set to 10000. Note, that there is no extra condition to stop the simulation, because the dynamic simulation environment enables us to drive the simulation

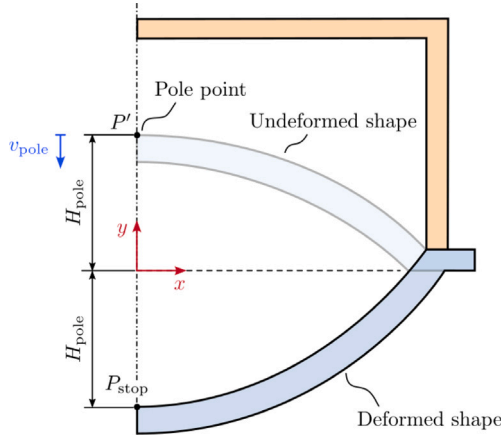


Fig. 9. Geometric interpretation of the pole point and the stop criterion. The pole point (P') is the inner point of the shell, located at the axis of symmetry. The vertical displacement (in the direction of the blue arrow) of the pole point is denoted as v_{pole} . The snap-through is identified and the simulation is stopped, when the pole point P' reaches the stop point P_{stop} . This is done by monitoring Eq. (27) in every timestep.

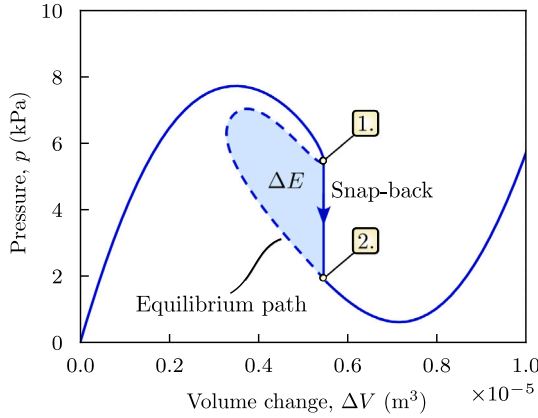


Fig. 10. The visual interpretation of the released energy during the snap-back. The pressure inside the cavity (p) is changing with respect to the volume change (ΔV), which gives the pressure–volume change curve, obtained by quasi-static simulations. The dashed line represent the unstable equilibrium path, which is the result of the quasi-static calculation. The snap-back instability initiates at Point [1], which corresponds to the primary limit point, where the quasi-static equilibrium path reverses direction in ΔV . Point [2] denotes the corresponding state on the lower stable branch, which is found by the intersection of the vertical line and the equilibrium path. The shaded area in light blue denotes the released energy ΔE during the snap-back, and can be calculated by integrating the curve between the designated points.

by controlling the pressure of the enclosed cavity. With this, important characteristic information can be achieved about the shells, as it was showcased in Fig. 1(b).

4.3.1. Pressure-controlled method

The pressure-controlled simulation is based on the explicit definition of the pressure evolution in the cavity, using a pressure boundary condition on the cavity reference point, with an amplitude function. A linear function was used to ramp the pressure, as this is the idealized scenario followed by the measurement approach.

4.4. Volume-controlled method

To perform volume-controlled simulation, the evolution of the cavity volume should be prescribed. The simulation framework is not

capable to do this in an explicit way, thus a thermal analogy is introduced. Specifically, the volumetric expansion of the cavity is driven by applying a prescribed temperature field to the enclosed fluid, utilizing its thermal expansion properties to control the internal volume. The thermal expansion of a fluid in ABAQUS is defined as

$$V(T) = V_0 [1 + 3\alpha (T - T_0) - 3\alpha (T_I - T_0)], \quad (28)$$

where $V(T)$ is the current volume at temperature T , while V_0 is the initial volume, α is the thermal expansion coefficient, T_0 is the reference and T_I is the initial temperature. If the thermal expansion coefficient is constant, the reference temperature can be emitted, which results to

$$V(T) = V_0 [1 + 3\alpha (T - T_I)]. \quad (29)$$

Assuming an artificial thermal expansion coefficient of $\alpha = 1/^\circ\text{C}$ and an initial temperature of $T_I = 0^\circ\text{C}$, the relative volume change is calculated as

$$\frac{V(T) - V_0}{V_0} = \frac{\Delta V(T)}{V_0} = 3T, \quad (30)$$

where $\Delta V(T)$ is the change of the volume due to temperature T . For example, if the cavity volume doubles, the left-hand side of Eq. (30) becomes 1, thus the required temperature to achieve this can be expressed as $T_D = 1/3^\circ\text{C}$. Using this methodology, the simulation can be controlled with a temperature boundary conditions. An initial $T_I = 0^\circ\text{C}$ temperature field is applied to fluid–cavity reference point, then this field is ramped to a value of $T = T_D$, using a linear amplitude function.

5. Experimental method

This section presents the experimental procedure used to validate the numerical predictions. The fabrication of silicone shell specimens and the fluidic measurement setup are described, with emphasis on the application of both volume- and pressure-controlled actuation.

5.1. Specimen fabrication

The shells were fabricated from the materials described in Section 3. The two components of the silicones were mixed in the ratio given by the datasheets, then degassed in a vacuum chamber to remove most of the air bubbles from the mixture. The silicones were poured into 3D printed Zortrax Z-ABS molds, according to the selected geometries. The ELASTOSIL material was cured in a drying chamber at 65°C for 30 min, while the RTV2 material was cured at room temperature for 24 h. After curing, the shells were removed from the molds, and any excess material was trimmed off. Some of the fabricated shells can be seen in Fig. 13.

5.2. Measurement setup

The experimental setup is based on the pressurization of the enclosed cavity, which is formed by the shell and the plastic container. The shell is placed inside a measurement container, which is sealed. The concept is based on the fluidic control board from Soft-Robotics Toolkit [96]. To apply the required distributed pressure on the shell, the cavity is inflated with water, using the Kamoer KLP01 diaphragm pump. The fluid flow in the system is controlled by a set of M&M solenoid valves, which are operated by an Arduino UNO R3 microcontroller using PWM signals. The pressure of the cavity is monitored by a pressure sensor at every time instance. The data acquisition and the control is performed by the microcontroller at a data rate of 400 Hz. The main sketch of the experimental setup is illustrated in Fig. 14, while the physical board can be seen in Fig. 15. The whole setup is operated by a laboratory power supply, which provides the DC voltage for the pump and the solenoid valves. Note, that the absolute volume of the cavity is not the same during the measurements and the simulations due to 3D

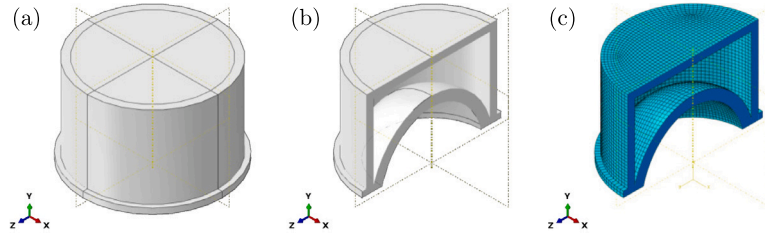


Fig. 11. The 3D (implicit dynamic) FE model. (a) The partitioned geometry to achieve structured mesh. Extra partitions were introduced to drive the meshing algorithm, so the model is basically formed by the identical 4 quadrants. (b) The partitioned geometry, sliced in half for visualization purposes. (c) The meshed geometry with C3D8RH elements, with the medial axis algorithm.

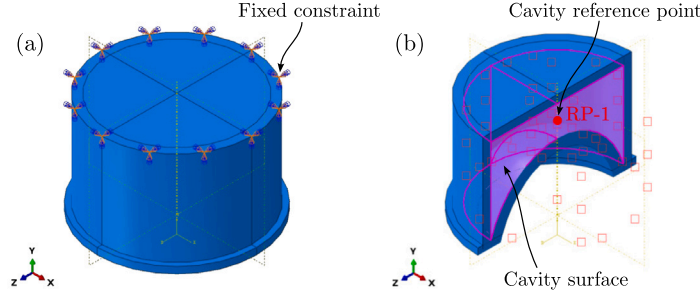


Fig. 12. Boundary conditions of the 3D (implicit dynamic) FE model. (a) The mechanical boundary condition (fixed constraint in the upper edge). (b) The defined fluid-cavity interaction with the cavity surface (highlighted in purple) and its reference point (RP-1). These definitions yield to a completely closed cavity.

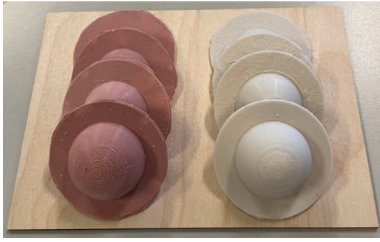


Fig. 13. Fabricated shells. Samples of the fabricated silicone shells, from the ELASTOSIL (reddish-brown) and RTV2 (white) materials, with geometric parameters: $A = 20$ mm, $t = 3.5$ mm and $\theta = 65^\circ$, while B was varied among 16 mm, 20 mm and 24 mm. Note that two specimens are present for the $B = 24$ mm case, resulting in the eight specimens depicted. These geometries yield to $\hat{B} = B/A$ values of 0.8, 1.0 and 1.2, respectively.

printed design. Therefore, to characterize the snap-through behavior, the relative volume change of the cavity $\Delta\hat{V}$ is used as

$$\Delta\hat{V} = \frac{V - V_0}{V_0}, \quad (31)$$

where V is the current volume of the cavity, while V_0 is the initial volume. This volume change during the simulations is calculated from the flow rate of the pump, which was determined experimentally, over a range of operating voltages, indicated in Appendix A.3. The experiments were performed using a flow rate of $\dot{Q} = 200$ ml/min. The linear characteristic observed for the pump volume flow-rate (see Fig. 25) provides an efficient case for the volume-controlled method, however, this benefit cannot be utilized during pressure-controlled measurements. In this case, the primary manipulated variable is still the volume, with pressure acting only as a secondary resulting metric.

The design of the measurement board provides the possibility to perform volume- and pressure-controlled experiments, similarly to the 3D simulations. The volume-controlled method regulates the system by controlling the flow rate of the pump. The pressure-controlled method is based on a feedback loop, where the recorded pressure data is

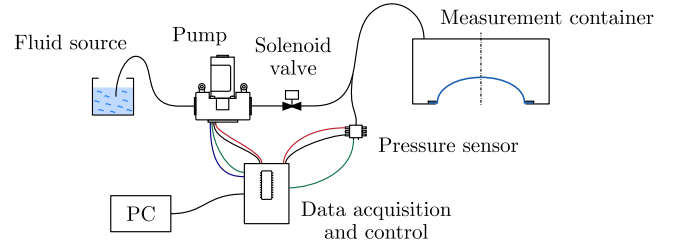


Fig. 14. Sketch of the experimental setup. The pressurization is performed by the pump with a fluid (the fluid source is a water tank), while the fluid flow is controlled by solenoid valves. The data acquisition of the pressure sensor and the control is performed by an Arduino microcontroller, connected to a computer.

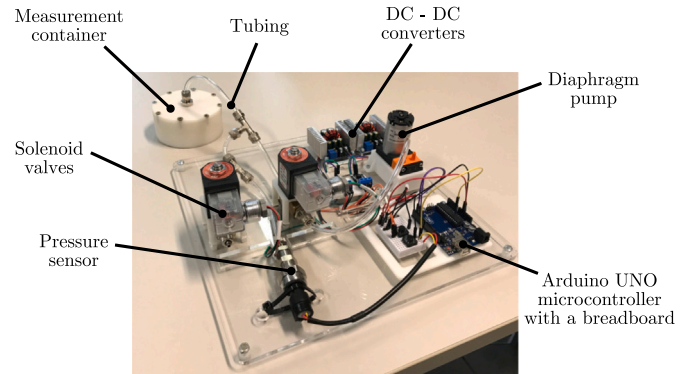


Fig. 15. The fluidic board. The measurement board with its main components: the Kamoer diaphragm pump is driven by an Arduino UNO R3 microcontroller. The fluid flow is controlled by M&M solenoid valves. The tubing consists of ø6/4 polyurethane tubes, connected with metal watertight tube fittings.

compared to the prescribed values. To follow the prescribed pressure curve, the solenoid valves are opened and closed by a PID controller, whose parameters were determined experimentally using the PID TUNER module of MATLAB, using measurement samples. The opening and closing sequence of the solenoids were recorded in timestamps, which were used to calculate the volume flow rate. During the experiments, linear volume and pressure curves were used, to ensure direct and efficient comparison with the simulation methodology.

6. Results and discussion

This section presents the numerical and experimental results obtained for the investigated axisymmetric elliptical shells. First, a broad parameter study is used to identify representative geometries and to examine how the main design parameters influence the released energy during snap-through. The selected geometries are then analyzed in detail by comparing axisymmetric simulations, 3D dynamic simulations, and experimental measurements under both volume- and pressure-controlled conditions.

The main focus is on the agreement between the different methods, the practical limitations of the experimental control strategies, and the influence of elliptical geometry on the snap-through response. It is also demonstrated how elliptical shell geometries can provide additional design flexibility compared to spherical shells.

6.1. Geometry selection

Preliminary parameter studies were performed using the axisymmetric simulations, to select interesting geometries. The major semi-axis ($A = 20$ mm), base radius ($R = 34$ mm), base thickness ($t_b = 2.0$ mm), shell thickness ($t = 3.5$ mm), container height ($H = 50$ mm) and container thickness ($v = 3.5$ mm) were kept constant during the analyses. To efficiently analyze the remaining system parameters, the following dimensionless parameters were defined:

$$\hat{B} = \frac{B}{A}, \quad \hat{t} = \frac{t}{A}, \quad \hat{\theta} = \frac{\theta}{\theta_{\text{ref}}}, \quad (32)$$

where $\theta_{\text{ref}} = 90^\circ$ is the reference opening angle. The parameter study involved varying these non-dimensional parameters in the ranges defined by Table 3. This resulted in a total of 1428 different geometries, which were run in a batch analysis. The snap-through behavior was characterized by the released energy during the instability, which is calculated by the area under the pressure–volume curve, as it was illustrated in Fig. 10. This energy was non-dimensionalized by the initial shear modulus of the material (μ_0) and the major semi-axis of the shell (A) as

$$\Delta \hat{E} = \frac{\Delta E}{\mu_0 A^3}. \quad (33)$$

A wide range of the analyzed shells did not experience snapping, or they could only be realized using pressure-controlled measurements and simulations. These results are not illustrated in the followings. The dimensionless released energy map, shown in Fig. 16 shows that the shell thickness has significant effect on the snap-through behavior, which correlates with the expectations: thick shells can store more strain energy, which is released during snapping. Results varying by orders of magnitude are found in the $\hat{t} = 0.175$ slice, so the more precise investigation takes place in this parameter range. The value of the $\hat{B}$ and the $\hat{\theta}$ parameters are also significant factors. For a fixed value of $\hat{t}$, the desired energy level $\Delta \hat{E}_d$ was introduced, with a correlation error of

$$\delta \hat{E} = \frac{\Delta \hat{E}_d}{10}. \quad (34)$$

With this, geometries whose energy level equals $\Delta \hat{E}_d$ within a $\delta \hat{E}$ range can be searched for. The extracted geometric points for multiple energy levels, and fixed $\hat{t} = 0.175$ values are illustrated in Fig. 17. The resultant

Table 3

Minimum, step, and maximum values for the non-dimensional parameters ($\hat{B}$, $\hat{t}$, $\hat{\theta}$) used in the parameter study.

Parameter	Minimum	Step	Maximum
$\hat{B}$ (1)	0.8	0.025	1.2
$\hat{t}$ (1)	0.1	0.025	0.175
$\hat{\theta}$ (1)	0.6	0.02	1.0

Table 4

Selected geometric parameters for the detailed analysis.

No.	$\hat{B}$ (1)	Type
1.	0.8	Elliptical segment
2.	1.0	Spherical segment
3.	1.2	Elliptical segment

points follow a linear trend, therefore linear fits were produced in the form of

$$\hat{\theta} = m\hat{B} + b, \quad (35)$$

where m and b are the parameters of the fitted linear equation. Fig. 17 shows that the slopes of the fitted lines are similar, which is assured by the standard deviation of m as

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^n (m_i - m_{\text{avg}})^2}{n}} = 3\%, \quad (36)$$

so the lines can be characterized by the average of the slopes as m_{avg} . With this assumption, the lines are parallel, which means that they can be translated into each other by varying parameter b . Namely, parameter m defines the linear relation between $\hat{B}$ and $\hat{\theta}$, while parameter b correlates to the released energy level $\Delta \hat{E}$ as

$$m \sim \hat{\theta} / \hat{B} \quad \text{and} \quad b \sim \Delta \hat{E}. \quad (37)$$

Note, that this property is also true for different $\hat{t}$ values. Geometries with parameters that satisfy Eq. (35), can release the same amount of energy during snapping. This property can be utilized during the design of a soft actuator, where geometric constraints are present. It is important to clearly define the validity limits of this proposed linear design rule. Currently, this relation is confirmed to be valid for the analyzed ELASTOSIL and RTV2 materials within the investigated geometric parameter ranges. However, it is expected to hold even beyond these specific bounds, as notable data scattering was not observed at the edges of the analyzed parameter space. It must be noted that this result is derived solely from the axisymmetric analyses, meaning the dynamics and potential asymmetric buckling modes are not captured and could alter the precise response.

Upon these findings, for future analyses and measurements, $\hat{t} = 0.175$ and $\hat{\theta} = 0.75$ were kept constant, while $\hat{B}$ was varied, resulting in three main types of geometries, summarized in Table 4.

6.2. Results of the experimental tests

The developed methods will be evaluated using qualitative and quantitative approaches. For this, the relative volume change $\Delta \hat{V}$ has already been defined, and the non-dimensional pressure is defined as

$$\hat{p} = \frac{p}{\mu_0}. \quad (38)$$

For every fabricated shell, 5 measurements were performed with both control methods. The recorded pressure and volume data were non-dimensionalized using the methods described above. The experimental results are summarized in Fig. 19. The curves shown on the graphs represent the average of the measurements, smoothed by a Savitzky–Golay filter from SciPy, PYTHON, with a window size of 7 data points, using second degree polynomials. The noise-to-signal ratio is the worst

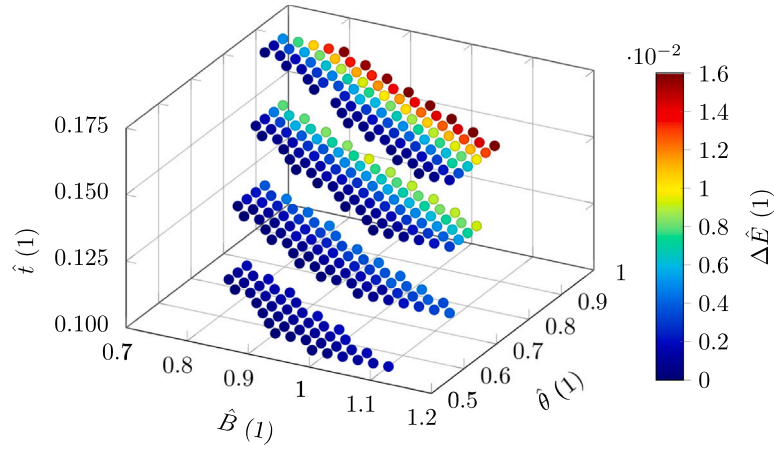


Fig. 16. Dimensionless released energy ($\Delta\hat{E}$) map from the quasi-static simulations. The released energy map for the set of input parameters: $\hat{B}$, $\hat{t}$, $\hat{\theta}$, defined in Eq. (32), for the *ELASTOSIL* material. The parameters were changed in a systematic way, summarized in Table 3, which resulted in a total of 1428 simulations. The fringe colors of the dots represent the magnitude of the dimensionless released energy. Note, that these dimensionless results are independent of the specific material choice. The results show that the thickness has a significant effect on the released dimensionless energy.

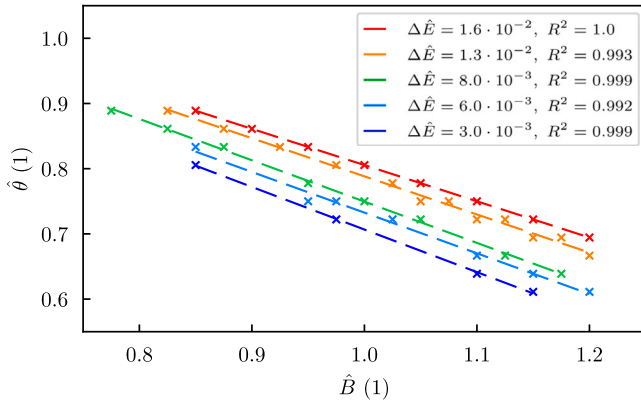


Fig. 17. Energy equivalent geometries. The results highlighted the presence energy equivalent geometries, with different energy levels and fixed $\hat{t} = 0.175$. Two geometry is said to be energy equivalent, if their dimensionless released energy matches with a calculated correlation error of $\delta\hat{E}$ using Eq. (34). The colors show the magnitude of the released dimensionless energy. The markers visualize the discrete geometric points and simulation results, while the dashed lines are fitted trend lines. The coefficient of determination (R^2) illustrates that the linear fits are excellent, confirming a strong linear relationship between the $\hat{B}$ and $\hat{\theta}$ parameters.

for the $\hat{B} = 0.8$, and best for the $\hat{B} = 1.2$ case. The volume-control method could capture the snap-through for every geometry. The initial part of the inflation follows a linear relation, which is captured correctly by both control methods. After reaching the critical pressure, the instantaneous snap-through was experienced in every case.

Interesting to note, that the $\hat{B} = 1.2$ geometry for the *ELASTOSIL* material, illustrated in Fig. 19(c), experienced a second snapping instability. We attribute this secondary snap-through behavior to the classical inflation instability that develops as the shell expands in its second stable state. The occurrence of this phenomenon appears to be strongly geometry-dependent and was only observed for the most elliptical configuration, suggesting that increasing ellipticity may promote the emergence of additional instability modes during continued inflation. The observed secondary inflation instability may also be influenced by dominant non-axisymmetric deformation modes (see Fig. 6); however, a detailed analysis of this phenomenon is beyond the scope of the present work. A comprehensive investigation of the interaction between the primary snap-through and this secondary instability, as well as their dependence on shell geometry, is therefore

left for future studies. It is also remarkable to notice, that the pressure-controlled simulations showed consistent discrepancy. The experienced critical pressure was lower than what was experienced with the volume-controlled method. Also, the instantaneous snap to the other stable state was not recorded, which can be attributed to control imperfections: the finite response times of the solenoid valves are comparable to the timescale of the snap-through. This is illustrated in Fig. 18, where a prescribed pressure history is compared to the actual measurement, with the actuation performed by the solenoid valves. It can be seen, that in the initial inflation, the desired curve is followed with fairly good precision, but upon the snap-through, the control loop tries to follow the sudden change, which results in large overshoots. To quantify the effect of the nonlinear inflation curve, simulations were performed with an ideal, and the measured pressure curve (see Fig. 18(b)).

It was also observed that the released energy is higher for the $\hat{B} = 1.0$ and $\hat{B} = 1.2$ shells. Their behavior is highly nonlinear, the distance between the two stable states are larger. This allows for fast and large displacements, even when the actuation is slow. Based on these results, the volume-controlled approach is a more stable and robust method to characterize the snapping behavior of the shell by qualitative means.

6.3. Results of the numerical simulations

The results of the numerical simulations are presented in Fig. 20. As it is expected, the FE simulations correlate with each other well. Small discrepancy can be found between the numerical methods around the limit points, but in general, they are well described by the axisymmetric simulations, therefore they can be applied confidently to study a wide range of geometric and material parameters, while remaining computationally efficient. The dynamic simulations experience a small numerical oscillation upon snapping to the other stable state. However, once the solution is settled, it remains stable (see Fig. 26(d)). Good agreement is found between the experiments and the simulations, with small differences in the limit points. The initial inflation part of the FE models follow the experiments very well, in general. The largest differences are noted on the $\hat{B} = 0.8$ case for the *RTV2* material graphed in Fig. 20(d), where the measured $\hat{p} - \Delta\hat{V}$ curve follows a different trend. The non-dimensional pressure-volume curve of this shell differs from all the others: the instability is not as drastic, due to its combination of geometric and material parameters. It is also important to note, that the second snap-through is not accessed with the simulations for the $\hat{B} = 1.2$ case for the *ELASTOSIL* material, in Fig. 20(f). Upon this, the stable solutions after the instability also follow different trends. However, good agreement is found in the stable

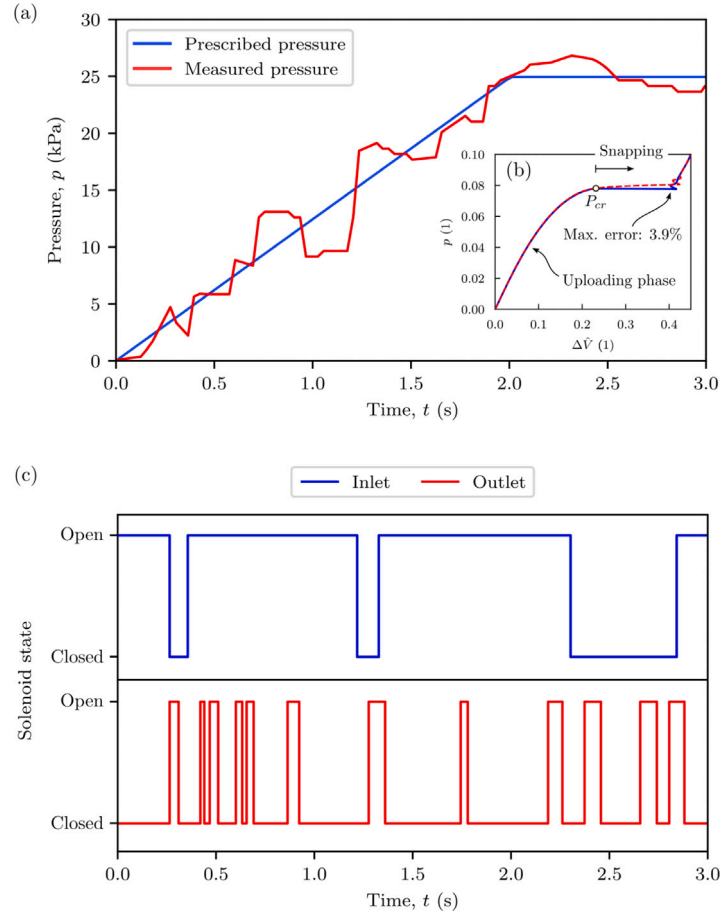


Fig. 18. Comparison of the prescribed and recorded pressure history. (a) Comparison between the prescribed and the recorded pressure history of an RTV2 shell, using the $\hat{B} = 1.2$ geometry. (b) Results of the 3D pressure-controlled simulation using the ideal (blue) and the real (red) pressure history. The results indicated that under ideal conditions, there is no deviation during the uploading path, and the differences observed after the P_{cr} critical pressure in the pressure–volume change curve are negligible. (c) The operation of the solenoid valves during the inflation.

branch after the snap-through for other geometries. The displacement fields were similar in all cases: an example for both control methods are presented in Fig. 21, where the numbers are corresponding to a designated point on the $\hat{p} - \Delta \hat{V}$ curve, showed in Fig. 20.

6.4. Comparison of methods

A quantitative analysis was performed by comparing all obtained results from the different methods with the result of the axisymmetric simulations. Exact values for the snapping pressure ($\hat{p}_s$) and the snapping volume change ($\Delta \hat{V}_s$), along with their standard deviation values from the measurements are summarized in Table 9 in Appendix A.4. The relative errors between the methods for the ELASTOSIL material are summarized in Fig. 22.

Excellent agreement can be seen between the numerical methods, while it can be seen that the volume-controlled measurement performs better for the $\hat{B} = 0.8$ case, whereas the pressure-controlled method performs better for more ‘elliptical’ shells. It was observed that the error of the pressure-controlled measurement method decreases as $\hat{B}$ increases, which occurs because these shells require a longer time to reach snap-through, allowing the control loop to operate more efficiently. However, it is important to note that these errors do not quantify the recorded pressure history, it only quantifies the error of the recorded pressure at the instant of the snapping. The computed errors of the volume-controlled method remains almost constant across the different geometries. It is important to note that the RTV2 material is

almost twice as hard as ELASTOSIL on the Shore scale, which is also reflected in the ratio of their initial shear moduli. However, the results indicate that their qualitative behavior is similar, which shows that the critical pressure is not determined solely by the shear modulus, but also depends on the nonlinear behavior.

The remaining discrepancies may be partly attributed to neglected physical effects in idealized FE models, such as residual stresses, geometric imperfections, local thickness variations, and non-axisymmetric deformation modes. These effects can influence both the critical pressure and post-snapping response, although the overall agreement indicates that the dominant snap-through mechanism is still captured with the proposed model.

By comparing the same relative error for shells made from the RTV2 material, the results shown in Fig. 23 are obtained. Similarities to the previous case can be seen; however, in this case, the volume-controlled measurement methods performed better.

It should be noted, that analytical and semi-analytical solutions exist for related spherical shell problems [97–99], but they are limited to simpler material models, such as neo-Hookean formulations or strain–energy functions with integer exponents. These methods use the well-developed shell theories [100], but their extension of such approaches to elliptical shells with fitted Ogden-type material models with real exponents lies outside the scope of the present work and is considered a possible direction for future research.

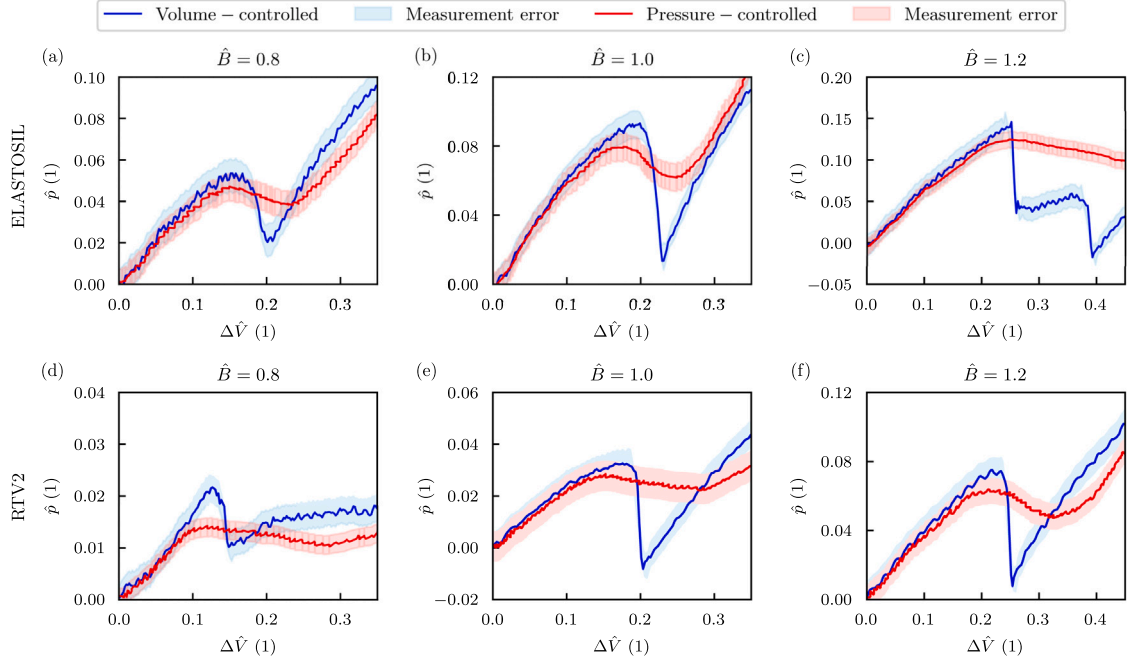


Fig. 19. Measurement results for the selected geometries. Dimensionless pressure–volume change curves from the measurements. The first row (a–c) corresponds to the *ELASTOSIL*, the second row (d–f) corresponds to the *RTV2* material. The columns represent the different values of $\hat{B} = B/A$, which is increases with the ‘ellipticity’ of the shell. Larger $\hat{B}$ parameter means that the shell is more elliptic. The solid curves (red and blue) illustrate the smoothed average of the measurements, while the dimmed ranges (light blue and light red) show the measurement error, which is calculated from the minimum and maximum envelope resulted from the distinct measurements.

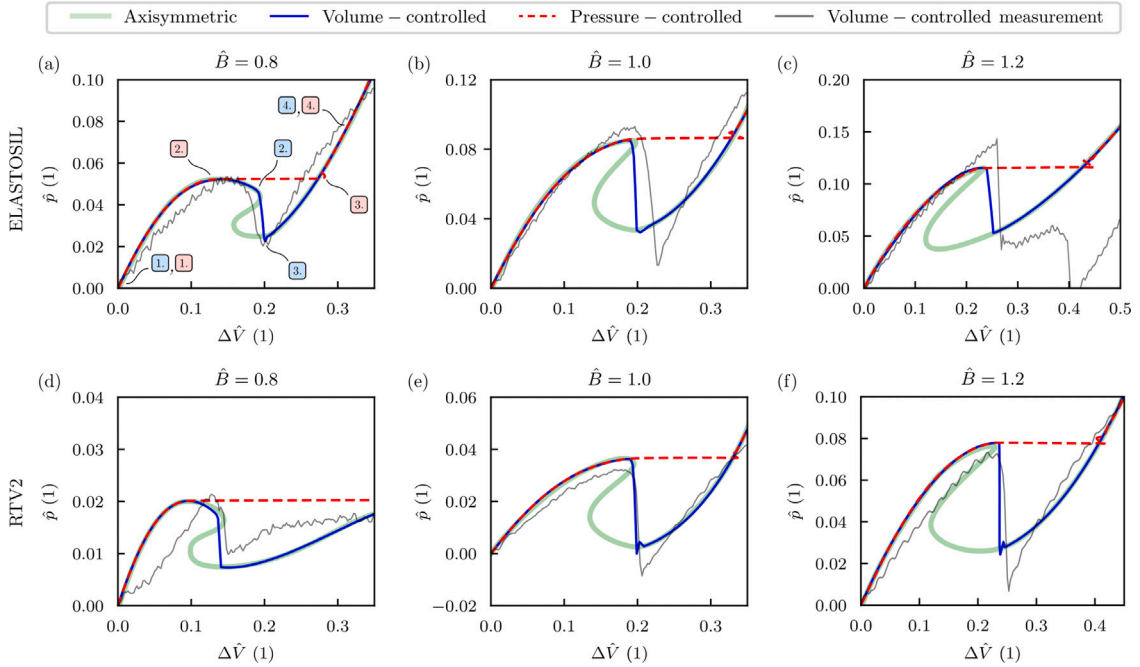


Fig. 20. Simulation results for the selected geometries. The simulation results are compared with the volume-controlled experiments (gray). The first row (a–c) corresponds to the *ELASTOSIL*, the second row (d–f) corresponds to the *RTV2* material. The columns represent the different values of $\hat{B} = B/A$. Larger $\hat{B}$ parameter means that the shell is more elliptic. The axisymmetric simulations (green) are used to capture the equilibrium path of the shell. The volume-controlled (blue) and the pressure-controlled (red) methods capture the instantaneous snap-through from one stable state to another stable state. The illustrated points on graph (a) are corresponding to the deformation map at that specific point, shown in Fig. 21.

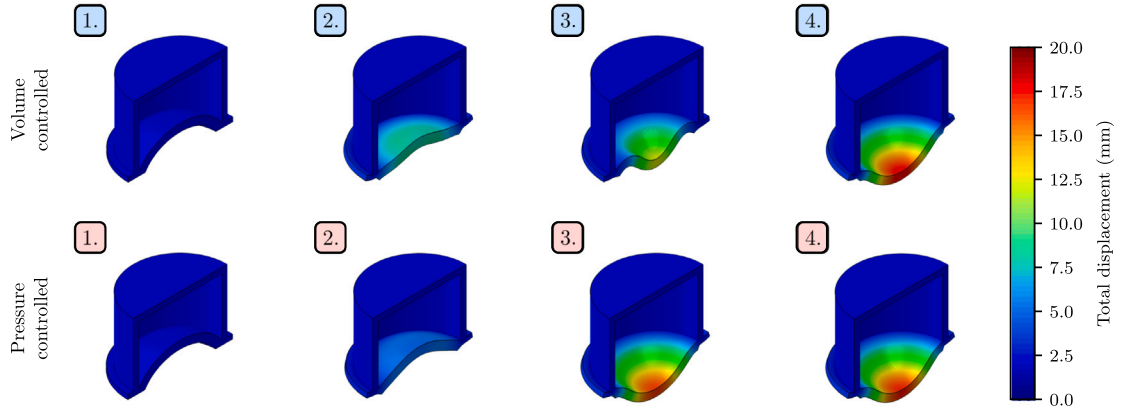


Fig. 21. Illustration of the 3D snap-through simulations. Visualization of the deformed shapes for the volume- and pressure-controlled simulation case, for $\hat{B} = 0.8$ geometry, made from the *ELASTOSIL* material. The fringe color represents the total displacement (magnitude of the displacement vectors at each material point). The numbers correspond to the marked points of the non-dimensional pressure - volume change curve ($\hat{p} - \Delta\hat{V}$) curve shown in Fig. 20/(a).

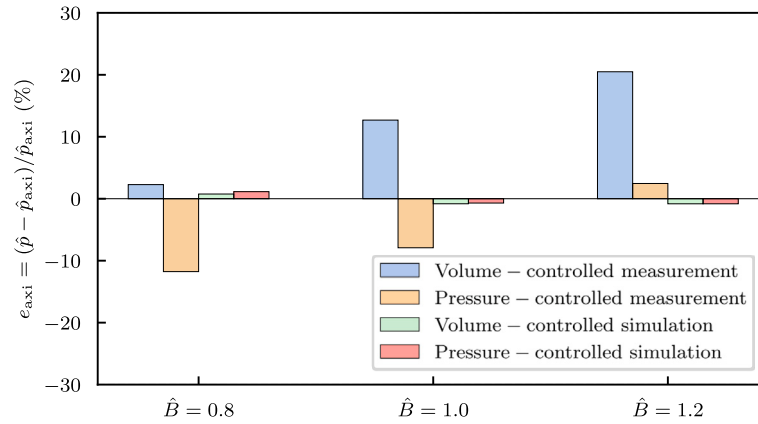


Fig. 22. Comparison of the acquired critical pressures for the *ELASTOSIL* material. The acquired relative pressures are compared in a quantitative way, by calculating e_{axi} . This quantity denoted the relative error of the critical pressure ($\hat{p}$) with respect to the critical pressure acquired with the quasi-static axisymmetric simulations ($\hat{p}_{axi}$). Larger $\hat{B}$ parameter means that the shell is more elliptic. The bar colors represent the different methods: volume- and pressure controlled scenarios, from both the simulations and the measurements.

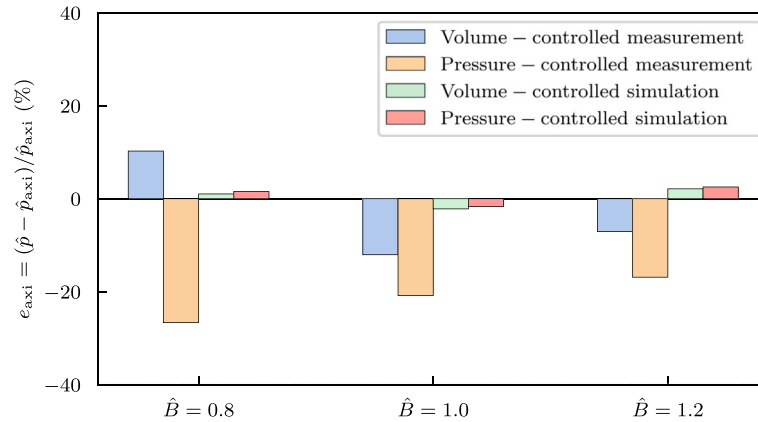


Fig. 23. Comparison of the acquired critical pressures for the *RTV2* material. The acquired relative pressures are compared in a quantitative way, by calculating e_{axi} . This quantity denoted the relative error of the critical pressure ($\hat{p}$) with respect to the critical pressure acquired with the quasi-static axisymmetric simulations ($\hat{p}_{axi}$). Larger $\hat{B}$ parameter means that the shell is more elliptic. The bar colors represent the different methods: volume- and pressure controlled scenarios, from both the simulations and the measurements.

6.5. Benefits of the elliptical geometry

The benefit of using an elliptical geometry can be emphasized if we look at sample geometries from Fig. 16. This parameter study includes

spherical shells as well, which are used for baseline comparison. The released energies are said to be equivalent, if the difference in their numerical value is less than $\leq 1\%$. With this, the following results can be seen, which are summarized in Table 5. The result show, that we can

Table 5

Equivalent geometries, where no. 1. is the baseline spherical geometry, while no. 2. and 3. are elliptical geometries, with different parameters.

No.	$\hat{B}$ (1)	$\hat{\theta}$ (1)	$\hat{i}$ (1)	$\Delta\hat{E}$ (1)
1.	1.0	0.56	0.125	$2.999 \cdot 10^{-4}$
2.	0.9	0.56	0.125	$2.992 \cdot 10^{-4}$
3.	1.1	0.56	0.10	$3.002 \cdot 10^{-4}$

achieve the same energy level with different actuators, compromising between the radial dimension and the thickness. Shells with higher geometric ellipticity require a smaller thickness to achieve the same actuation level.

The results demonstrate that elliptical shells are capable of reproducing the same energetic response observed in spherical shells when the overall form factor and material properties are kept constant. In particular, the strain energy stored during the snap-through process reaches comparable levels for both geometries. From a design perspective, this result confirms that elliptical shells can serve as viable alternatives to spherical shells in applications relying on snap-through driven energy release. Specifically, in traditional spherical shells, the geometry is defined by a single curvature, which inherently limits the accessible design space by strictly coupling the height of the actuator to its base width. Elliptical geometries overcome this limitation by introducing an additional geometric parameter that decouples these spatial dimensions, enabling a more detailed tuning of the structural response. This increased geometric freedom allows the critical snap-through load and total energy release to be independently adjusted through the ratio of the semi-axes ($\hat{B}$) and thickness ($\hat{i}$). Consequently, if a soft robotic application imposes strict vertical space constraints but demands a specific energy release threshold, the elliptical design space enables more compact and efficient structural solutions compared to equivalent spherical shells. Ultimately, this multi-parameter tuning capability provides a direct engineering advantage for integrating predictable snap-through actuators into complex, space-limited mechanical systems.

7. Conclusions

The present work investigated the snap-through behavior of soft, elliptical shells. A comprehensive framework was presented, spanning geometric description, numerical simulations, and experimental validation, that can be applied to characterize the instability behavior of a wide range of fluidic actuators. The simulation methodology showed that good agreement can be found between the different control methods, which enables us to characterize the phenomena based on the desired application. A relation between the geometric parameters was expressed, which can be used to tune the behavior of the instability. The numerical models are validated through experiments, with qualitative and quantitative analysis. The 3D simulations accurately capture the underlying mechanics but are computationally demanding, while 2D models offer a favorable balance between accuracy and efficiency, making them suitable for extensive parameter studies. While the numerical models successfully capture the dominant mechanics, the results indicate that precise critical pressure predictions are sensitive to material parameter uncertainties and unmodeled residual stresses from the fabrication process, highlighting an area for future model refinement. The volume-controlled case is easy to replicate in the experiments, while the pressure-controlled method is difficult, due to the need for a precise control loop, which should be tuned for every geometry. It was found that the numerical and experimental results correlate well, even for high $\Delta\hat{E}$ energy levels, which opens the possibility of using the presented methods to design fluidic actuators that can benefit from the instability. The results showed that the phenomenon is highly nonlinear, and it is advisable to use multiple methods to characterize

the snap-through instability to get an accurate sense of the behavior. The main findings are summarized in the followings.

Equivalent released energy level geometries were found using axisymmetric simulations. Upon this, different geometries can be translated into each other, using the linear relation which was found between the design parameters B and θ . The dynamic numerical simulations follow the equilibrium path acquired from the quasi-static simulations, in its stable regions. Small oscillations can be detected upon snapping, but after their decay, the solutions remain smooth. The proposed experimental setup is found to be more accurate with the volume-control method. The pressure-driven measurements are qualitatively worse, due to the finite response time of the control loop. The nonlinear material behavior contributes to the effect of snap-through significantly, especially in cases where $\hat{B} > 1$.

The elliptical shell design provides additional geometric flexibility while preserving the energetic performance characteristic of spherical shells. The results show that comparable levels of released strain energy during snap-through can be achieved by adjusting the shell ellipticity and thickness, without changing the overall form factor. This enables designers to tailor the structural response through geometric parameters rather than relying solely on size or material modifications. Consequently, elliptical shells represent a promising alternative for applications where spatial constraints require adaptable yet energetically equivalent snap-through actuators.

CRedit authorship contribution statement

Barnabás Piri: Writing – original draft, Visualization, Software, Methodology, Formal analysis, Conceptualization. **Szabolcs Berezvai:** Writing – review & editing, Supervision, Methodology, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

A.1. Material parameter sensitivity study

While residual stresses (which may influence the material behavior) are not explicitly considered in the present work, uncertainties in the identified material parameters may also affect the predicted snap-through response. To evaluate the robustness of the numerical results, a sensitivity analysis of the constitutive material parameters was performed. The baseline parameters were obtained from the experimental fitting procedure described in Section 3.1. This study was performed with the *ELASTOSIL* material, and the

$$\hat{B} = 0.8, \quad \hat{\theta} = 0.75, \quad \hat{i} = 0.175 \quad (39)$$

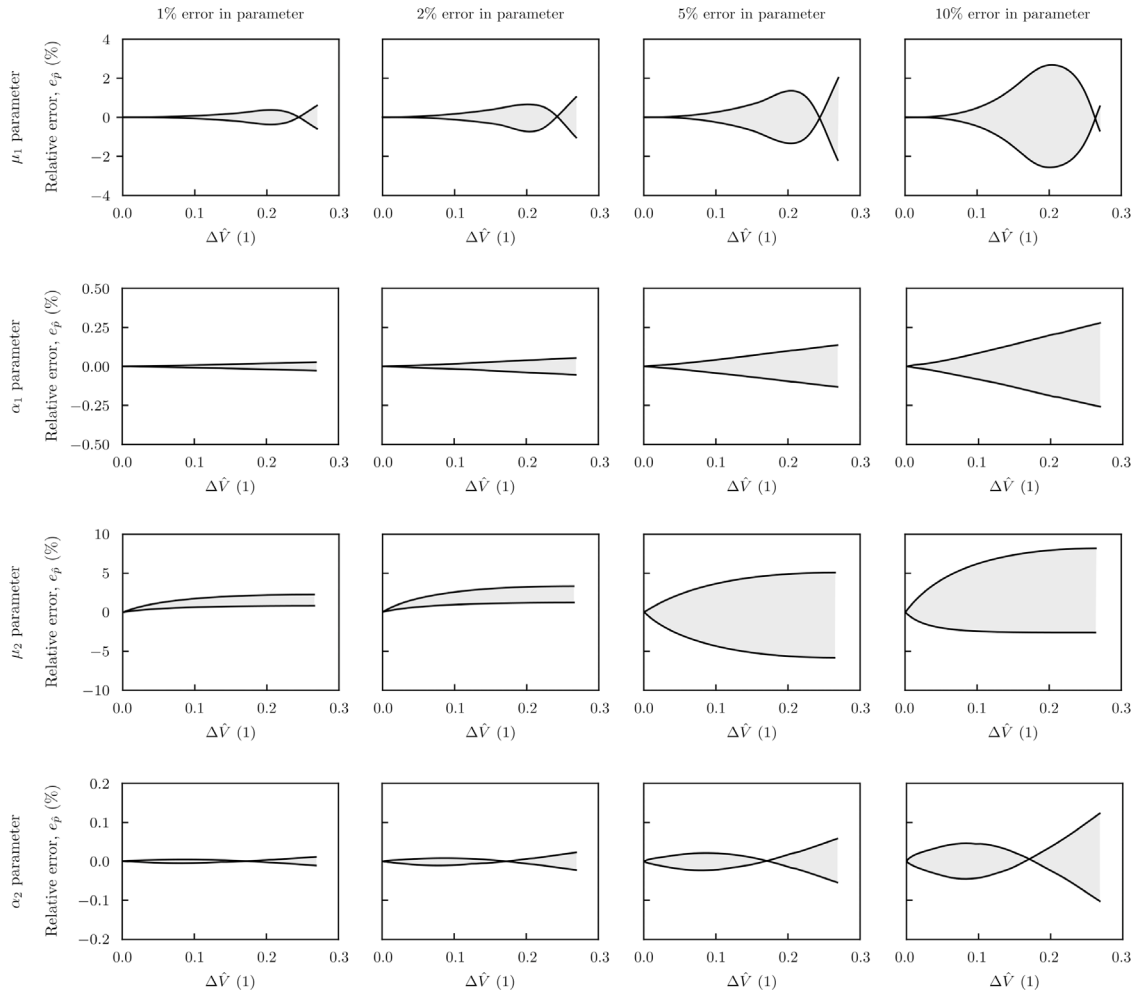


Fig. 24. Results of the material sensitivity study. A sensitivity study is performed in case of the *ELASTOSIL* material, for the material parameters of the second-order incompressible Ogden model (μ_1, α_1, μ_2 and α_2). The graphs illustrate the resultant relative error (e_p , according to Eq. (41)) in the obtained pressures ($\hat{p}$) as a function of the dimensionless volume change ($\Delta \hat{V}$), due to an introduced error in a single material model parameter. The shaded (gray) regions represent the uncertainty ranges, which are bounded by the black curves, the resultant curves from the simulation cases. The rows are corresponding to a single Ogden material parameter, while the columns correspond to an introduced error in that parameter.

geometric case. Each material model parameter was varied individually while keeping the remaining parameters fixed at their fitted values. The variations considered were

$$\delta_{\text{par}} = \pm 1\%, \pm 2\%, \pm 5\%, \pm 10\%. \quad (40)$$

For each perturbed parameter set, the presented quasi-static simulation was performed. This approach allows the isolated influence of each parameter on the predicted snap-through response to be evaluated. The resulting pressure–volume curves were non-dimensionalized and a relative error was calculated as

$$e_{\hat{p}} = \frac{\hat{p}_{\delta} - \hat{p}_{\text{org}}}{\hat{p}_{\text{org}}}, \quad (41)$$

where $\hat{p}_{\delta}$ and $\hat{p}_{\text{org}}$ are the non-dimensional pressure results using the perturbed and the original material model, respectively. The results are summarized in Fig. 24. These analyses showed that the snap-through response is more strongly influenced by variations in μ_1 than by changes in α_1 . Furthermore, the captured results are significantly influenced by the precision of the μ_2 parameter. The sensitivity of α_2 parameter is more pronounced than the influence of α_1 parameter, but it is still negligible compared to the μ_1 and μ_2 parameters. The critical pressure was also calculated for each set of simulations. Relative errors were calculated between the critical pressures obtained from the disturbed and the original material parameters, providing information

Table 6

The relative change in the critical pressure ($\delta_{\hat{p}_{\text{cr}}}$) due to the material parameter disturbance (δ_{par}). The quantitative comparison showed that the small perturbations in the α parameters lead to negligible differences in the recorded pressures.

δ_{par} (%)	$\delta_{\hat{p}_{\text{cr}}, \mu_1}$ (%)	$\delta_{\hat{p}_{\text{cr}}, \alpha_1}$ (%)	$\delta_{\hat{p}_{\text{cr}}, \mu_2}$ (%)	$\delta_{\hat{p}_{\text{cr}}, \alpha_2}$ (%)
± 1	+0.28 −0.29	± 0.03	+2.17 +0.72	± 0.01
± 2	+0.56 −0.57	± 0.06	+2.89 +1.44	± 0.03
± 5	+1.41 −1.42	± 0.14	+5.07 −5.81	+0.07 −0.06
± 10	+2.83 −2.84	+0.29 −0.27	+8.69 −2.18	+0.14 −0.12

about the sensitivity of the critical point to material imperfections. These results are summarized in Table 6.

In general, the snap-through response shows a higher sensitivity to the μ parameters, which are related to the material stiffness. The influence of the α parameters are almost negligible. However, it is important to note, that these material parameters provide the acquired fits as a system, so a small correction in one parameter would lead to a whole new set of parameters, which provide the best fit based on the proposed quality function.

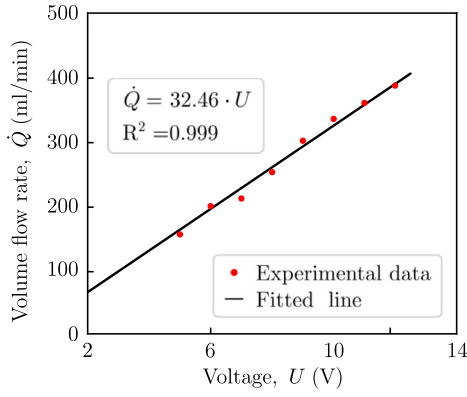


Fig. 25. Diaphragm pump characteristics. The measured flow-rates ($\dot{Q}$) of the pump at different input voltages (U). The red markers denote the discrete measurement points, while the black line illustrated the fitted linear trend line. Based on the coefficient of determination (R^2), a very good linear relationship is formed between the applied input voltage and the resulting fluid flow-rate.

A.2. Mesh convergence study

A mesh convergence study was performed to assess the sensitivity of the results to the discretization. The study was conducted on both the quasi-static and the dynamic finite element models proposed in Section 4. A thin ($\hat{t} = 0.1$) and a thick ($\hat{t} = 0.175$) shell were analyzed, with the *ELASTOSIL* material, while $\hat{B} = 1.2$ and $\hat{\theta} = 0.75$. The shell thickness was discretized using 2, 4, and 8 elements, and the simulations were repeated for each mesh density. The results showed negligible variation beyond the mesh with 4 elements through the thickness, indicating that the solution is mesh-independent. Small numerical oscillations can be seen in the dynamic models, due to the inertial effects. These oscillations did not change the response significantly. The instability initiates from the same stable path for each mesh density. Following the transient phase of these oscillations, the second stable branch is

Table 7

Number of elements (N_{ϕ_i}) for every generated mesh.

Model type	2D (quasi-static)			3D (dynamic)		
	N_{ϕ_2}	N_{ϕ_4}	N_{ϕ_8}	N_{ϕ_2}	N_{ϕ_4}	N_{ϕ_8}
0.1	74	288	1051	7052	60 776	487 072
0.175	83	274	1090	6292	68618	502 644

Table 8

Root mean square errors between the $\phi_4 - \phi_8$ models.

Model type	2D (quasi-static)	3D (dynamic)
	$e_{\phi_4-\phi_8}$ (%)	$e_{\phi_4-\phi_8}$ (%)
0.1	1.98	2.13
0.175	1.86	2.06

found, which remains stable. The results are showcased in Fig. 26. The resultant element numbers for each model are summarized in Table 7.

The root mean square error along the whole solved domain were calculated between the $\phi_4 - \phi_8$ case with

$$e_{\phi_4-\phi_8} = \sqrt{\frac{1}{M} \sum_{i=1}^M \left(\frac{\hat{p}_i^{\phi_4} - \hat{p}_i^{\phi_8}}{\hat{p}_i^{\phi_8}} \right)^2}, \quad (42)$$

where M is the number of the data points, $\hat{p}_i^{\phi_4}$ and $\hat{p}_i^{\phi_8}$ are the non-dimensional pressures. This shows that the gained numerical precision is negligible compared to the change in computational cost (see Table 8).

Based on these, a discretization with 5 elements along the shell thickness was adopted for subsequent simulations, providing a good compromise between numerical accuracy and computational cost.

A.3. Pump characteristics

The characteristics of the diaphragm pump is defined by a voltage-volume flow rate curve. In order to deduct this relationship, several

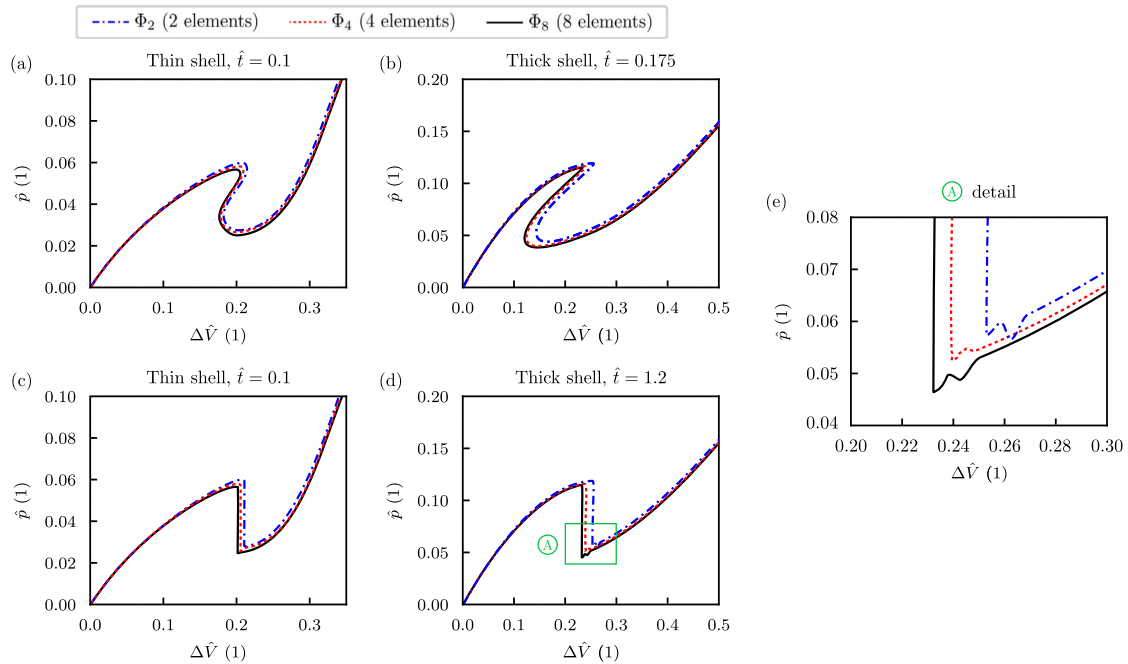


Fig. 26. Results of the mesh convergence study. For all geometries and simulation types, simulation results are graphed using 2 (blue dashed curve), 4 (red dotted curve) and 8 (black curve) elements along the thickness. (a) Thin shell, quasi-static simulation. (b) Thick shell, quasi-static simulation. (c) Thin shell, dynamic (volume-controlled) simulation. (d) Thick shell, dynamic (volume-controlled) simulation. (e) The A detail illustrates a smaller section of the (d) subfigure. The magnitude and duration of the numerical oscillations during the dynamic snap-through remain largely independent of the chosen discretization.

Table 9

Numerical values obtained for the snapping pressure ($\hat{p}_s$), snapping volume change ($\Delta\hat{V}_s$) and the corresponding standard deviation value for the measurements for the *ELASTOSIL* and *RTV2* materials.

Material	Method	$\hat{B} = 0.8$		$\hat{B} = 1.0$		$\hat{B} = 1.2$	
		$\hat{p}_s$ (10^{-2})	$\Delta\hat{V}_s$ (1)	$\hat{p}_s$ (10^{-2})	$\Delta\hat{V}_s$ (1)	$\hat{p}_s$ (10^{-2})	$\Delta\hat{V}_s$ (1)
ELASTOSIL	Volume-controlled sim.	4.4	0.19	8.3	0.19	12.0	0.24
	Pressure-controlled sim.	5.2	0.13	8.3	0.18	12.0	0.23
	Volume-controlled meas.	4.3 ± 0.5	0.17 ± 0.01	9.1 ± 0.5	0.20 ± 0.02	14.0 ± 0.5	0.27 ± 0.02
	Pressure-controlled meas.	4.5 ± 0.4	0.17 ± 0.01	7.9 ± 0.6	0.18 ± 0.02	12.0 ± 0.5	0.25 ± 0.02
RTV2	Volume-controlled sim.	1.7	0.13	3.5	0.19	7.7	0.24
	Pressure-controlled sim.	2.0	0.09	3.5	0.19	7.7	0.24
	Volume-controlled meas.	2.1 ± 0.2	0.13 ± 0.01	3.1 ± 0.3	0.18 ± 0.01	7.1 ± 0.4	0.24 ± 0.02
	Pressure-controlled meas.	1.3 ± 0.1	0.12 ± 0.01	2.7 ± 0.2	0.16 ± 0.01	6.2 ± 0.6	0.21 ± 0.05

voltage inputs were applied ($U = 5, 6, \dots, 12$ V) and the pump was operated for a minute, in order to transport water from a source to a destination vessel. The volume of the fluid at the destination vessel was measured by the weight of the water using a precision scale. The measured flow-rate values and the fitted linear line is shown in Fig. 25. It is important to note, that the pump cannot operate below 5 V input voltage.

A.4. Quantitative results

The critical points for each shell were determined, including their numerical values and associated measurement deviations. These results are tabulated (see Table 9) to provide a quantitative comparison of the different methods.

Data availability

Data will be made available on request.

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