

Investigation of the vibration modes of a drone rotor using optical methods

Sz. Gungl ^{1,2}, T. Kimpián ¹, M. D. Masood ³, B. Istók ³, K. Tokaji ³, Cs. Horváth ³

¹ thyssenkrupp Components Technology Hungary Kft., System and Vehicle Dynamics Department, Acoustics and Signal Processing Group, Budafoki út 56, H-1117, Budapest, Hungary

² Budapest University of Technology and Economics, Faculty of Electrical Engineering and Informatics, Department of Artificial Intelligence and Systems Engineering, Magyar tudósok körútja 2. Building "I" Wing E IV. floor IE444, H-1117, Budapest, Hungary

³ Budapest University of Technology and Economics, Faculty of Mechanical Engineering, Department of Fluid Mechanics, Bertalan Lajos utca 4-6, H-1111, Budapest, Hungary
e-mail: szilard.gungl@thyssenkrupp-automotive.com

Abstract

This study presents the results of an ongoing collaboration to identify the vibration modes of drone propulsion systems (motor and rotor) using optical techniques. The research consortium first applied Phase-Based Motion Magnification (PBMM) and Digital Image Correlation (DIC) methods to investigate a stationary drone propulsion system subjected to rectilinear excitation via an electrodynamic shaker and torsional excitation via the propulsion system's brushless DC motor. The resulting vibration mode identification and classification have provided a baseline for the untensioned, non-rotating condition. Next, the PBMM and DIC methods have been applied to the same system while rotating. Both PBMM and DIC have constraints that make their application to rotating specimens difficult, with the experimental setup posing additional challenges. The paper presents the applied measurement methodologies and discusses the challenges, advantages, and disadvantages of each.

1 Introduction

Drone noise has caught the attention of both the scientific and environmental protection communities. As the number of drone applications has increased, so has the number of drones in the sky, and the resulting noise has become an increasingly significant problem [1,2]. Therefore, time and resources have been invested in better understanding the noise sources of drone propulsion systems to mitigate noise levels [1,2]. Many investigations focus on the dominant noise sources, studying the root causes and the effects of parameters such as Reynolds number [3], axial velocity [3], RPM [4], and the size and spacing of various elements [4].

A segment of the low Reynolds number acoustic investigations has focused on profile vortex shedding, i.e., the coherent structures that can be generated by airfoil profiles [5-9]. The classic description of profile vortex shedding, consistent with the works of Tam [10] and Wright [11], attributes it to a self-excited acoustic feedback loop. According to this description, an unstable laminar boundary layer on the airfoil surface produces Tollmien–Schlichting instability waves that propagate downstream toward the airfoil's trailing edge. There, sound scattering occurs, with acoustic waves propagating back upstream to the origin of the Tollmien–Schlichting instability waves in the unstable laminar boundary layer. These acoustic waves can drive the instabilities if the phase conditions are appropriate, thereby closing the feedback loop. Avoiding

profile vortex shedding is important because it can result in a periodically fluctuating force normal to the airfoil chord, which is cited as a primary source of aeroacoustic noise in the low-speed axial-flow fan literature [6,9]. Other investigations, such as the work of Ausoni et al. [12] on hydrofoils, state that the frequency of periodic vortex shedding may be influenced by periodic vibrations occurring at resonant frequencies, which could result in the periodic vortex shedding being coupled to the resonant frequencies of the given body, which could differ from those that would be calculated according to vortex shedding theories and models.

In the aeroacoustics community, many acoustic investigations of drones are conducted using single microphones (as in [13]) or linear microphone arrays (as in [4]). The microphone time signals are processed to provide Sound Pressure Level (SPL) frequency spectra. If linear arrays of microphones are processed, information can be extracted from the spectra regarding the directivity of the sound components. Using such investigative methods, one can determine the occurrence of profile vortex shedding and its influence on the sound field, based on models that predict the expected Reynolds number range and frequency content [5-8]. At the same time, investigations can be found in the literature that examine the vibrations of drones [14] and their propellers [15], with only a limited number of articles appearing that investigate the possible connections between the natural frequencies of the drone propulsion systems (motor and propeller pair) and the resulting acoustic field [16-18].

This investigation examines a drone propulsion system that has been utilized on a drone aeroacoustic and aerodynamic test bench at the Budapest University of Technology and Economics. This has been a T-Motor P15*5 Propeller and T-Motor MN4010 drone motor. Beyond the investigation of a single isolated drone motor and propeller, as presented herein, the test bench has been designed to investigate simplified drone configurations to better understand the noise sources resulting from basic modifications in drone layout and operating conditions, such as the effect of RPM, rotor position phase angle, axial distance between counter-rotating and co-rotating rotors, and in-plane spacing of distributed propulsion configurations, to name only a few. To truly understand the results of the aeroacoustic investigations and to draw general conclusions, a basic understanding of the periodic vibration properties resulting from the structural natural frequencies of the drone propulsion system must also be attained. In a first approach, as discussed in [17], rectilinear and torsional modal analysis methods [19,20], coupled with Phase-Based Motion Magnification (PBMM) optical modal analysis techniques, have been applied to identify the structural resonant frequencies and associated vibration modes of the drone propulsion system under investigation. The stationary drone propulsion system has been excited in the axial direction using a shaker, while the propulsion system's brushless DC motor has been used to excite it in the azimuthal direction. The investigation in [17] has provided a baseline dataset for studying the untensioned, non-rotating drone propulsion system and has demonstrated that the methods in [19] and [20] can be applied to drone propulsion systems. From another perspective, it has been deemed necessary to continue the investigation, as stationary drone propulsion systems are not in a prestressed state, as would naturally occur during rotation. Such a prestressed state can shift the resonant frequencies to higher values and even influence the mode shapes [21]. The literature provides methods for calculating the frequency shift, often based on Southwell's equation, assuming sufficient information is available about the system and the vibration modes under investigation [21]. From a measurement technique perspective, investigating a rotating system is inherently challenging, as examining a rotating drone propulsion system using optical modal analysis methods requires precise measurements and/or specialized equipment. This investigation presents the continuation of the measurement campaign in [17] by measuring the prestressed rotating drone propulsion system using optical modal analysis techniques. The paper discusses various aspects of the applied optical modal analysis investigation methodology, which have been examined to ensure that the resonant frequencies of rotating drone propulsion systems can be successfully identified. The paper compares PBMM to Digital Image Correlation (DIC) methods, both of which play relevant roles in the literature, for the baseline stationary drone propulsion system discussed in [17], and the rotating drone propulsion system introduced herein. The goal is to determine which combination of available measurement and post-processing methods should be used to investigate the propulsion systems of rotating drones.

2 Noncontact modal analysis methods

The literature on modal analysis methods is rather diverse. There are numerical and measurement methods described in detail and successfully utilized in the literature. This investigation considers only measurement-based methods, further narrowing the choice to noncontact optical modal analysis methods, as attaching accelerometers to the body of a thin, light structure, such as a drone rotor, would alter the body's structural dynamics and thus distort the results. Within these categories, PBMM and DIC are examined and utilized herein.

2.1 Phase-based motion magnification (PBMM)

Motion magnification can amplify subtle motion in video sequences, making them easier to examine. This is achieved by manipulating motion in the video, either by tracking the trajectories of moving objects (Lagrangian methods) or by manipulating motion at fixed positions (Eulerian method [22] and phase-based method [23]). Phase-based motion magnification [23] decomposes each frame of a video sequence into a multiscale, multi-orientation pyramid of spatial frequency bands. The local phase of each band, which contains information regarding sub-pixel positional information, is extracted and passed through a temporal band-pass filter tuned to the frequency of interest. By amplifying the filtered phase signal and reconstructing the frames, a magnified visualization of the motion at a specific frequency can be obtained. This approach enhances sensitivity to sub-pixel vibrations while attenuating uncorrelated noise and lighting changes, making it well-suited for structural dynamics applications. The extracted qualitative results often provide invaluable insights. The use of PBMM for quantitative results is not as common as it requires careful calibration and certain considerations regarding the underlying methods and mechanics. The PBMM code utilized herein is based on the literature and the publicly available implementation described in [23]. The code has been translated into Python and has undergone various minor modifications, none of which change its original behavior in any significant manner.

2.2 Digital Image Correlation (DIC)

Digital image correlation is a versatile, Lagrangian, non-contact, optical method used to measure full-field displacement and strain. DIC tracks the motion of random surface patterns, often resulting from the application of a speckle pattern, across the entire region of interest. The fundamental basis of DIC is the analysis of pixel brightness intensity (gray value) to recognize movement. It involves matching pixel subsets between a reference, undeformed image and a deformed image by minimizing the correlation coefficient. By analyzing changes in gray value intensity, DIC provides dense displacement fields for strain analysis in experimental mechanics [24]. Subpixel registration can be performed by fitting a curve to the integer pixel correlation coefficients and using the peak of this curve to determine the subpixel displacement [25]. This method, when combined with integer pixel correlation calculations in the Fourier domain, is relatively fast but does not provide the highest accuracy. Alternatively, subpixel displacements can be determined iteratively using a root-finding method such as Newton-Raphson. This approach involves interpolating the subset and calculating the correlation until the desired convergence is reached. Although the Newton-Raphson method enhances accuracy, it requires more processing time [26]. DIC has its own advantages. The basic method is used to determine the displacement between two frames. Hence, two consecutive images can be processed, and the solid-body rotation can be removed from the results. According to the literature, DIC is often applicable for rotations up to approximately 7° [27], which opens up multiple options for processing the data collected with a high-speed camera. The DIC code utilized herein is based on the literature and the publicly available implementation described in [28]. The code has been further modified in the version used herein to extract additional information from the data, but this has not changed the basic functionality of the code and therefore does not influence the outcome of the investigations.

2.3 Temporal Aliasing

High-speed cameras are often required for measuring a sufficient frequency range for certain optical modal analysis methods. To ensure a smooth temporal representation of the resonant frequency oscillations, certain methods, including PBMM, require that the camera's sampling rate exceed ten times the resonant frequency. In the drone propulsion system measurements, a high-speed Phantom VEO 640 camera has been used, which supports up to 20,000 frames per second (fps) at a resolution of 1920 x 200. Although this is a high-speed camera, this limitation still limits investigations to direct measurements of modes below 2,000 Hz. Techniques, such as controlled aliasing, have been provided in the literature for recording temporally aliased signals and reconstructing the original signals with minimal information loss [29]. These techniques extend the measurable bandwidth, enabling the investigation of resonant frequencies beyond the Nyquist limit, as the measured frequencies translate down to a lower baseband without overlap and with minimal information loss, provided the measurements are well designed.

3 Stationary rotor test case

The first investigation presented herein has already been introduced in [17]. In this investigation, rectilinear and torsional modal analysis measurement methods described in [19,20] have been applied to the stationary drone propulsion system (T-Motor P15*5 Propeller and T-Motor MN4010 drone motor). First, a three-phase Schröder multisine current [30] has been used to excite the drone propulsion system while recording the radiated nearfield sound pressure with a 1/4" measurement microphone (GRAS 40 PP 10 type) and a National Instruments DAQ card (NI-9234), as seen in Figure 1. The stationary drone propulsion system has been excited in the axial direction using an electrodynamic shaker and in the azimuthal direction using the propulsion system's own brushless DC motor. It should be noted that the measurements have been carried out in the nearfield, as being in close proximity to the rotor blade helped bring out the peaks in the spectrum.

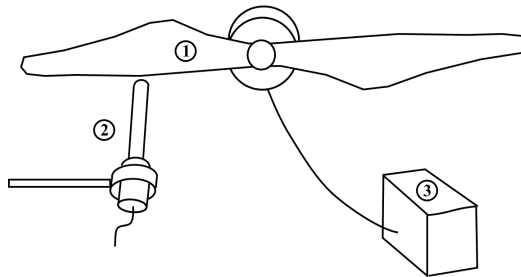


Figure 1: Stationary drone propulsion system nearfield noise measurement. Drone propulsion system (1), microphone (2), electrical excitation (3) [17].

After processing the nearfield microphone data, frequency peaks have been identified, indicating resonant frequencies that warrant further investigation. These frequencies have been individually excited, providing an opportunity to investigate their resonant frequencies using the Phantom VEO 640 high-speed camera and controlled aliasing methods. Figure 2 presents the measurement setup used to measure the resonant frequencies of the stationary drone propulsion system with a high-speed camera. As described in the section on temporal aliasing, controlled aliasing has been used to investigate the rotor at frame rates below the Nyquist limit. If the signal of interest is narrowband, the camera's frame rate can be taken as a fraction of the excitation frequency ($1/N$, with N being a whole number), as a result of which, the frame rate captures a repetition of the phenomena in the same phase for every N th repetition of the cycle. Then, deliberately offsetting this undersampled frame rate to approximately 0.9 times its frequency aliases the motion to a much lower baseband frequency while meeting the 10x oversampling requirements. This is effectively a form of undersampling, where the high-frequency oscillation is shifted ("folded") into the baseband. By ensuring a short exposure time and precise shutter control, each frame captures a sharp snapshot of the

motion in a different phase. Over many periods, this produces a slow-motion sequence that can be analyzed as if recorded at a much higher sampling rate.

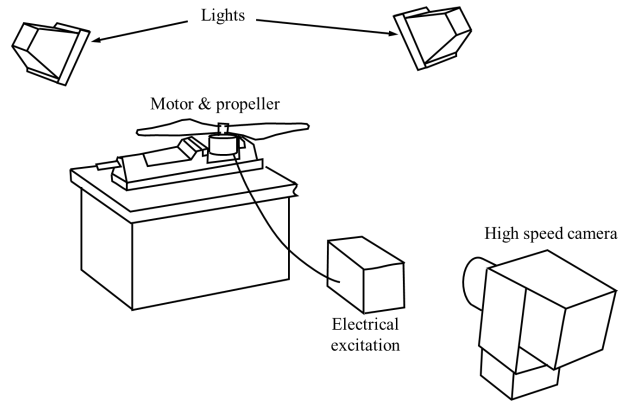


Figure 2: Stationary drone propulsion system high-speed camera measurement setup [17].

As can be seen herein, the project is running along two threads, with one portion of the team using PBMM and another using DIC. In [17], only PBMM results were presented; therefore, the test case has been revisited herein to highlight basic similarities and differences between the PBMM and DIC evaluations. The reason for using both methods is that we see relevant advantages in each, making it necessary to test them further. Figures 3-4 present the first out-of-plane bending mode of the propeller at 177.25 Hz. In Figure 3, the PBMM results are presented by simultaneously displaying three overlapping layers: the static propeller geometry and two displacement maxima, shown using two semi-transparent, differently colored layers. Although information can be extracted from the figure, it is relatively hard to see details, such as whether there are any nodes along the rotor span. On the other hand, the magnified videos created with PBMM provide a quick and intuitive way to understand the various components of rotor motion. It should be kept in mind that this method magnifies the rotor's motion without necessarily extracting quantitative displacement information. In comparison, the DIC results presented in Figure 4 show the rotor with a black reference (equilibrium) curve that extends along the span, along with the maximum (red) and minimum (blue) displacements of that curve. This information can be easily extracted from the quantitative data obtained for each point of the investigated object using the DIC method. In the case of the first out-of-plane bending moment, the advantage of this presentation mode is not immediately apparent. The second out-of-plane bending mode of the propeller is shown in Figures 5-6. Here, the advantage of the DIC presentation mode is evident, as the node positions are clearly identifiable. The presentation mode of the PBMM results can be ambiguous in such cases. It should be reemphasized that the magnified videos obtained with PBMM remain very useful for understanding the modes. This is especially true when the mode is complex. The usefulness of the DIC results is rather limited when objects move in and out of the investigation plane. In such cases, the magnified PBMM videos are more useful.

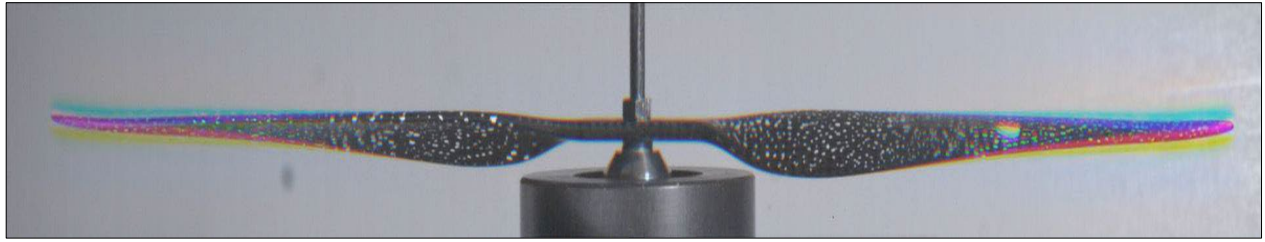


Figure 3: PBMM processing of the first out-of-plane bending mode of the propeller @ 177.25 Hz [17].

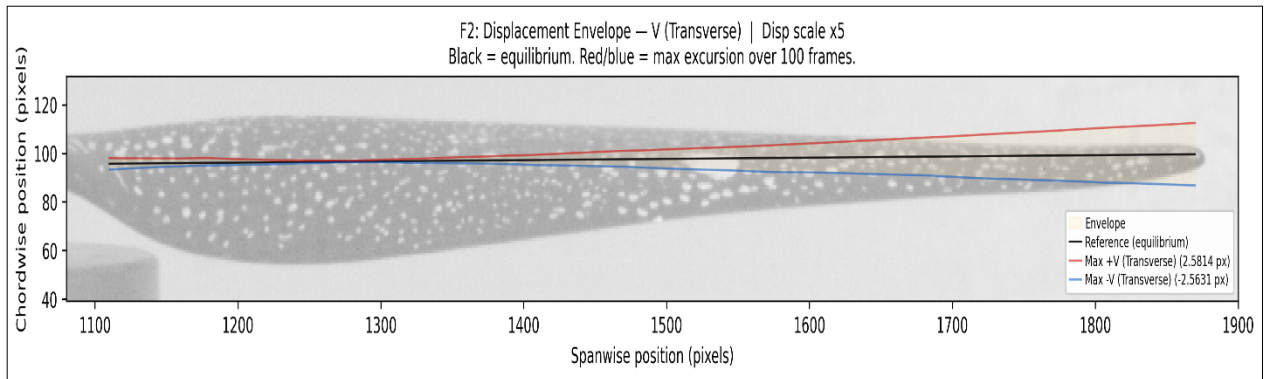


Figure 4: DIC processing of the first out-of-plane bending mode of the propeller @ 177.25 Hz.

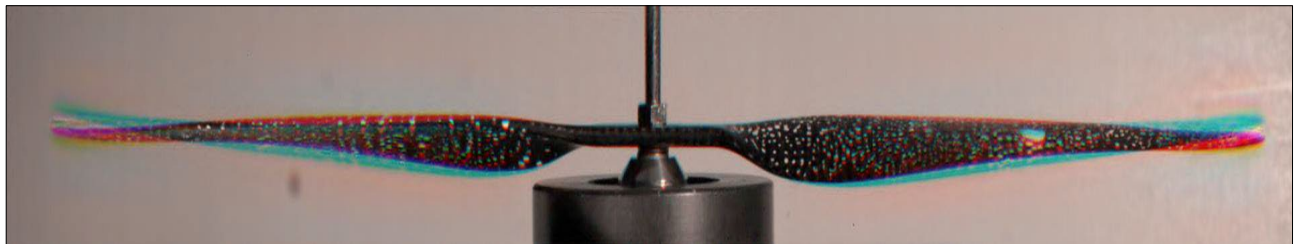


Figure 5: PBMM processing of the second out-of-plane bending mode of the propeller @ 614.5 Hz [17].

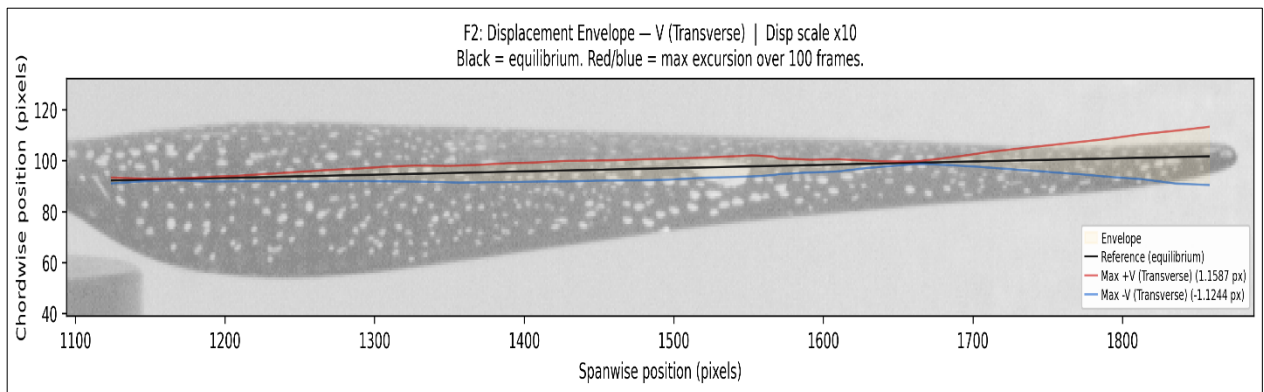


Figure 6: DIC processing of the second out-of-plane bending mode of the propeller @ 614.5 Hz.

4 Rotating rotor test case

As discussed in the introduction, the investigation of a stationary drone propulsion system does not account for the prestressed condition that naturally occurs under normal operating conditions. Therefore, the drone propulsion system has been investigated under rotation. The literature on the modal analysis of rotating objects is rather vast and diverse. Many methods exist for the experimental investigation of such systems, some of which require expensive equipment and expertise. Herein, the goal has been to utilize the already available and tested PBMM and DIC methods to see how successfully they can be applied in the investigation of drone propulsion systems.

The measurement setup utilized has been similar to that presented for the stationary drone propulsion system. The drone propulsion system has been mounted on a rigid arm to extend it away from obstacles, thereby ensuring the aerodynamics expected of an isolated drone propulsion system. During a speed sweep, nearfield sound pressure data have been collected using the same microphone and data acquisition system described above, consisting of a $\frac{1}{4}$ " measurement microphone (GRAS 40 PP 10 type) and a National Instruments DAQ card (NI-9234). Figure 7 depicts the measurement setup. It should be noted that the microphone had to be placed farther from the rotating rotor than the stationary rotor for safety reasons, and since wind noise can be significant in the close vicinity of the rotor. Being upstream of the rotor, the velocity gradient is rather steep. Therefore, the distance required to remove the microphone from the wind has been rather small. Multiple distances have been tested, and comparing results across distances indicates that a relatively short distance is sufficient for removing wind noise from the microphone data. After processing the data, spectrograms have been created that show how the radiated sound varies over time. A spectrogram for one of the microphone positions is shown in Figure 8. It has been generated using HEAD Acoustics' Artemis SUITE software. Here, the x-axis is time, the y-axis is frequency, and the color is the amplitude of the A-weighted SPL. The basic orders of the phenomena (once-per-rev and its multiples) can easily be identified as lines or bands that ramp up with RPM, but it is more difficult to localize and extract information regarding resonant frequencies, which would be peaks that are independent of the RPM, expected to appear as horizontal lines or bands.

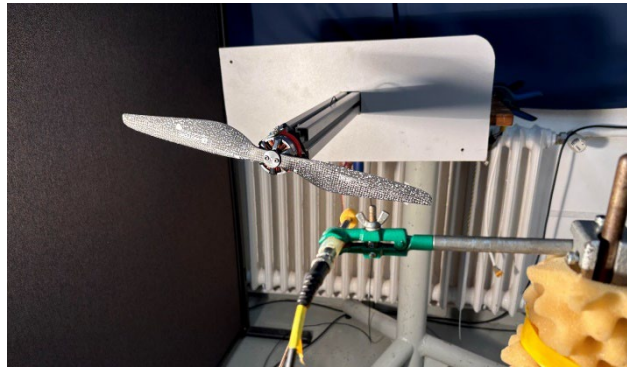


Figure 7: Microphone measurement setup for the rotating drone propulsion system.

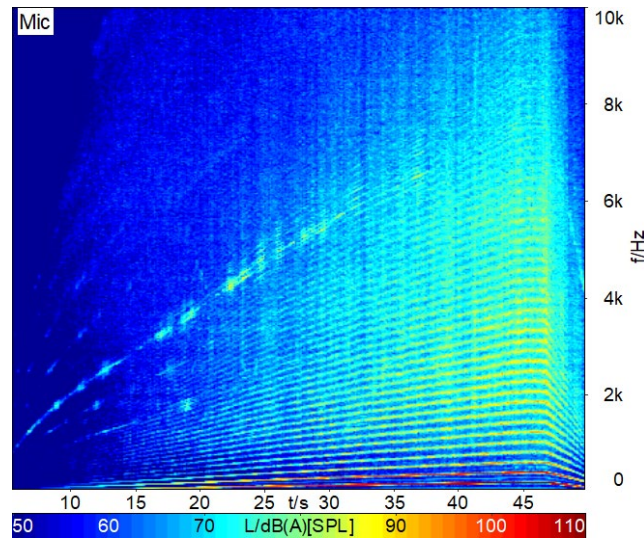


Figure 8: Spectrogram of the single microphone A-weighted SPL for the speed sweep.

At the same time, force and torque data have been collected for the drone propulsion system under investigation using the drone aeroacoustic and aerodynamic test bench of the Budapest University of Technology and Economics, shown in Figure 9. The goal has been to obtain spectrograms of the vibrations associated with the drone propulsion system. The drone propulsion system is mounted on a shaft housed in a pipe with bearings, allowing the shaft to move freely. Figure 10 presents a zoomed-in view of the force and torque measurement systems at the other end of the long shaft, which constrain the shaft's movement. The spectrograms derived from the force and torque data are shown in Figures 11 and 12, respectively. In the diagrams, the speed sweep is followed by a constant-speed section and then by a deceleration section. These spectrograms provide information regarding the vibration frequencies of the drone propulsion system.

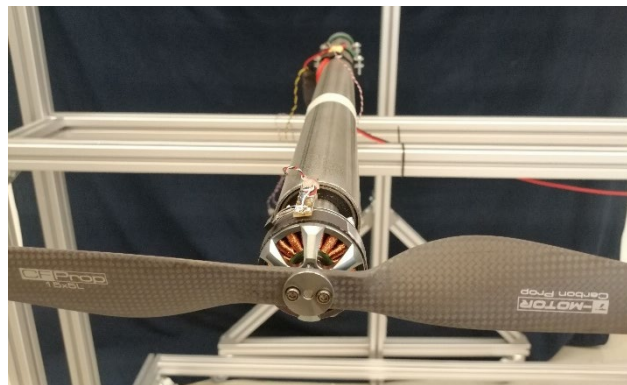


Figure 9: Drone aeroacoustic and aerodynamic test bench.

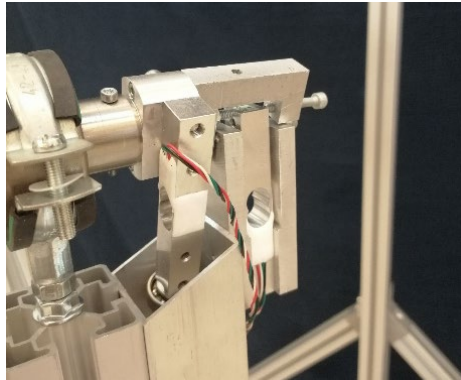


Figure 10: Force and torque measurement setup on the drone aeroacoustic and aerodynamic test bench.

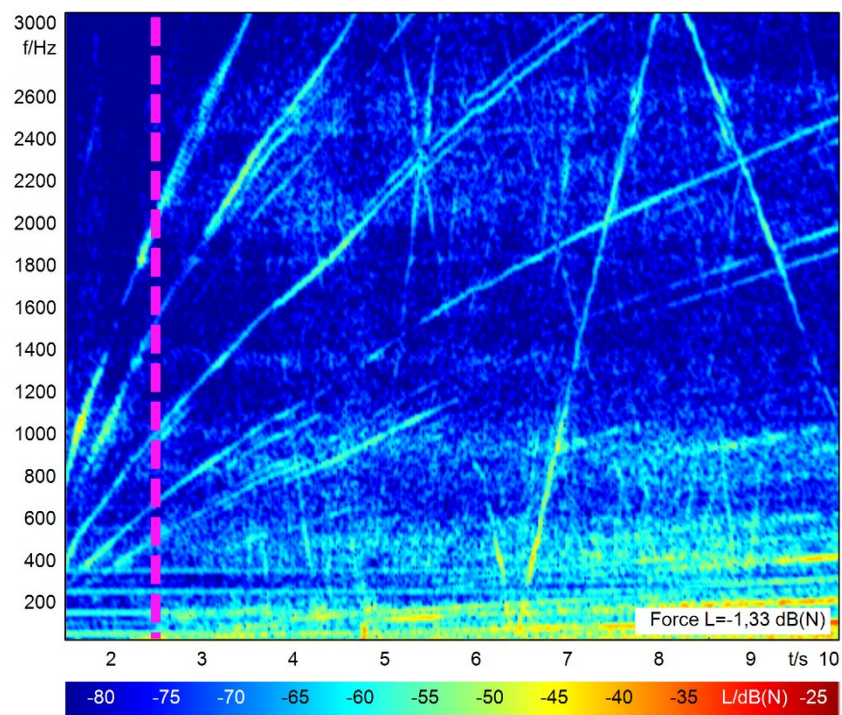


Figure 11: Force spectrogram for the speed sweep data.

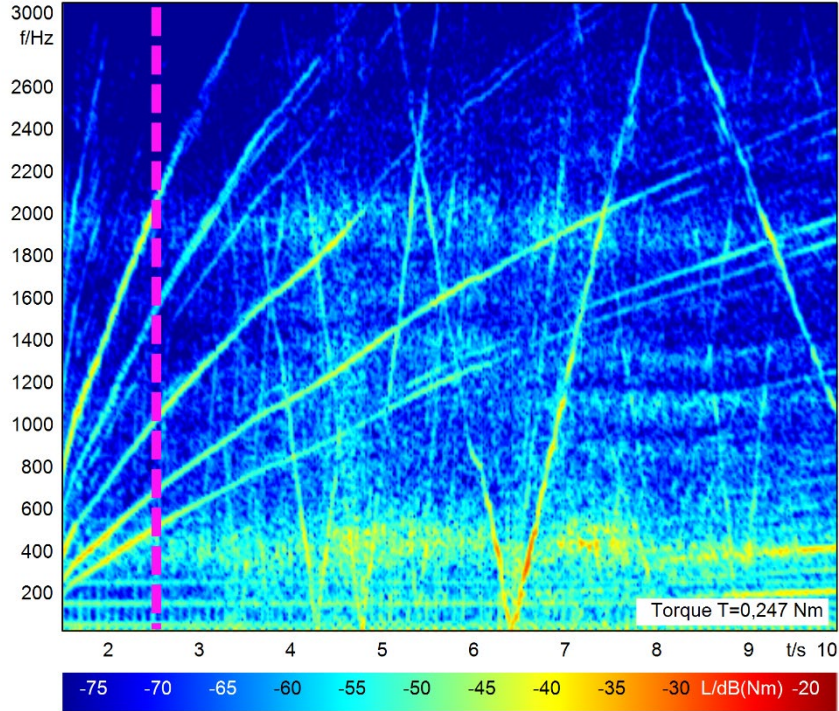


Figure 12: Torque spectrogram for the speed sweep data.

In this investigation, we aim to determine whether the PBMM and DIC methods can be used to examine rotating drone rotors. Therefore, the spectrograms have been used herein to identify an appropriate operating point for conducting the high-speed camera measurements. Looking at the force and torque diagrams, various orders related to the once-per-rev are visible. These are the lines that ramp up with RPM, then flatten out at constant RPM and ramp down as the motor decelerates. It can also be seen that Pulse-Width Modulation (PWM) produces upper and lower sidebands around the PWM carrier frequency. These sidebands appear in the order maps as positive and negative modulated orders, with the positive ones ramping upward and the negative ones ramping downward. The diagrams also contain horizontal lines/bands, which are the resonant frequencies of the drone propulsion system.

Before determining which operating point to investigate, the high-speed camera measurement setup needs to be discussed. The basic setup is similar to that of the high-speed camera measurement setup for the stationary drone propulsion system. A picture of the measurement setup used herein is shown in Figure 13. The drone propulsion system has been mounted on a shaft. The motor has been equipped with a once-per-rev sensor that provides a trigger signal for the Phantom VEO 640 high-speed camera. The rotor has been illuminated with non-flickering spotlights.



Figure 13: Rotating drone propulsion system high-speed camera measurement setup. (Left side: front view for in-plane motion, Right side: side view for out-of-plane motion)

Using the once-per-rev signal, the high-speed camera can be triggered once for each revolution. Within the drone propulsion system's operating range (up to approximately 6000 RPM), the camera can easily process and save this data. Therefore, measurements can be carried out by collecting multiple samples at one operating point (201 samples have been processed for each operating point herein). Therefore, one picture can be taken at the same position for each revolution. Considering the once-per-rev related orders observed in the spectrograms and the controlled aliasing technique applied herein, the choice of operating point has not been straightforward. Compared to the controlled aliasing used in the stationary measurements, it has not been possible to slightly offset the sampling frequency in a practical, usable manner. Therefore, a once-per-rev sampling frequency has been used. This results in a very low Nyquist frequency, and all once-per-rev related frequencies translate down to a baseband frequency of 0 Hz. In other words, the once-per-rev, its harmonics, and all modulated frequencies (i.e., all orders) translate down to 0 Hz. Therefore, the operating point has been chosen so that no orders pass through it, to avoid translation down to the 0 Hz baseband, since the components in one baseband cannot be separated. The choice of operating point is therefore highly limited, as shown in the force and torque spectrograms. Further limiting factors that can be read from the spectrograms include harmonics of the resonant frequency that could potentially fold into the same frequency bin. For reasons beyond the scope of this investigation, the operating point to be used herein has been further limited to low speeds.

The operating point presented herein has therefore been chosen as 1697 RPM. This operating point is marked by a vertical dashed magenta line in Figures 11 and 12. PBMM and DIC results for this operating point are presented in Figures 14 and 15. The data has been presented in the same manner as described above. As can be seen, the once-per-rev data appears to correctly capture an out-of-plane bending mode of the propeller. On the other hand, tests are still ongoing because the baseband frequencies introduce significant uncertainty and room for error.

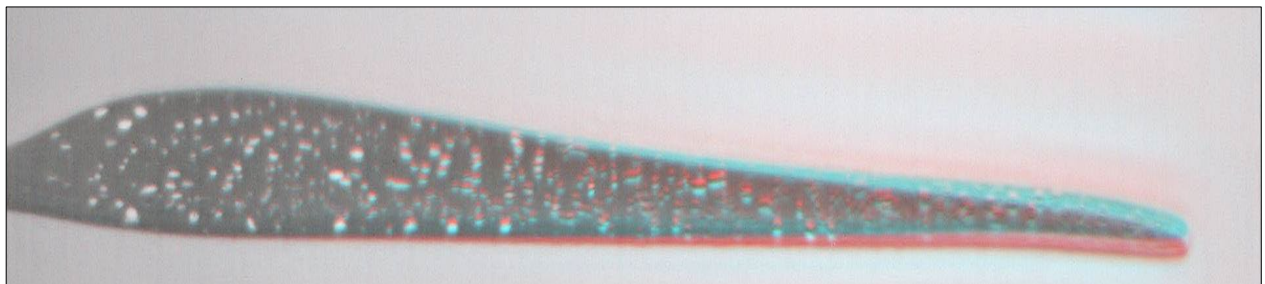


Figure 14: PBMM processing of an out-of-plane bending mode of the propeller.

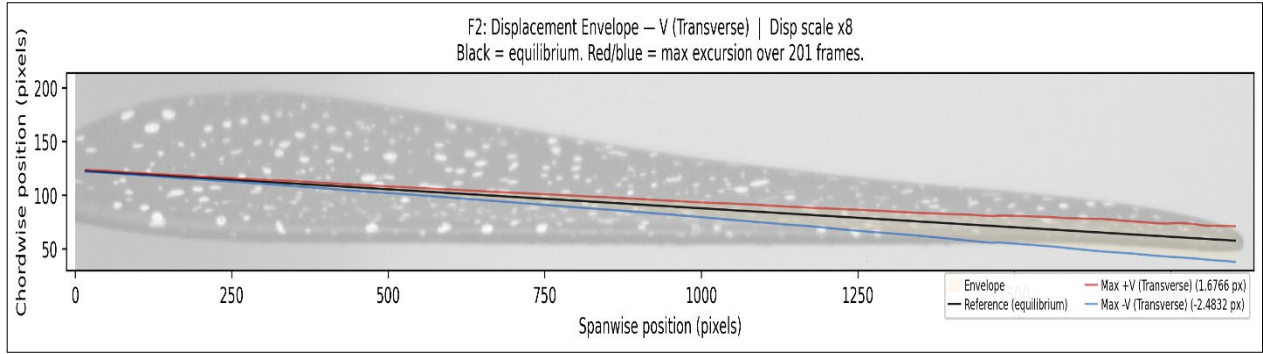


Figure 15: DIC processing of an out-of-plane bending mode of the propeller.

5 Conclusions and outlook

This paper is the next in a series of investigations aimed at applying optical modal analysis measurement methods to drone propulsion systems. The long-term goal of the investigation is two-fold. The first is to better understand the noise sources of drone propulsion systems by approaching the question from a vibroacoustic perspective. It is expected that the potential links between the aeroacoustic and vibroacoustic noise sources will be better understood and explained on behalf of the aeroacoustic community. The second is to introduce modal analysis measurement methods from other areas in drone propulsion system R+D, test and confirm their applicability, and/or further develop them for use in this application area.

Following successful measurements on a stationary drone propulsion system, processed using PBMM [17], this investigation has revisited the test case and successfully processed the data using DIC. Next, measurements have been carried out on a rotating drone propulsion system. The data has been analyzed with both PBMM and DIC. The results are encouraging, but further analysis is needed, as the many potential hurdles encountered during the measurement and processing of the drone propulsion system suggest that additional experience is needed before explicitly stating that the results can be trusted. During the implementation and testing of the methods, many ideas have been drawn from the literature and accumulated experience. The PBMM and DIC methods will therefore be further tested and developed to enable variations in the measurement and processing methods.

Acknowledgements

This collaboration has been a fruitful non-profit partnership for advancing knowledge and personal growth through mutual learning and the exchange of experience. We are grateful for the opportunity and support provided by Budapest University of Technology and Economics and thyssenkrupp Components Technology Hungary Ltd. The colleagues from the Department of Fluid Mechanics have been supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences, and project no. 143204, implemented with the support provided by the Ministry of Innovation and Technology of Hungary from the National Research, Development and Innovation Fund, financed under the OTKA K 22 funding scheme, while a portion of the equipment has been provided by the European Union and the Hungarian Government in the framework of Competitive Central-Hungary OP (Project ID: VEKOP-2.3.3-15-2017-00017). The first author gratefully acknowledges the support of the Doctoral Excellence Scholarship Program, funded by the Ministry of Culture and Innovation of Hungary and the National Research, Development and Innovation Fund of Hungary.

References

- [1] Y. Liua, M. Kreimeier, E. Stumpf, Y. Zhou, and H. Liu, "Overview of Recent Endeavors on Personal Aerial Vehicles: A Focus on the US and Europe led Research Activities," *Progress in Aerospace Sciences*, vol. 91, pp. 53-66, 2017, <https://doi.org/10.1016/j.paerosci.2017.03.001>.
- [2] C. Yang, R. J. Wallace, and C. A. Huang, "Review and Bibliometric Analysis of Unmanned Aerial System (UAS) Noise Studies Between 2015 and 2024," *Acoustics*, vol. 6, pp. 997-1020, 2024, <https://doi.org/10.3390/acoustics6040055>.
- [3] D. Casalino, E. Grande, G. Romani, D. Ragni, and F. Avallone, "Definition of a benchmark for low Reynolds number propeller aeroacoustics," *Aerospace Science and Technology*, vol. 113, 2021, <https://doi.org/10.1016/j.ast.2021.106707>.
- [4] R. S. McKay, M.J. Kingan, S.T. Go, and R. Jung, "Experimental and analytical investigation of contra-rotating multi-rotor UAV propeller noise," *Applied Acoustics*, vol. 177, 2021, <https://doi.org/10.1016/j.apacoust.2020.107850>.
- [5] G. Daku, and J. Vad, "Experiment-Based Preliminary Design Guidelines for Consideration of Profile Vortex Shedding From Low-Speed Axial Fan Blades," *Journal of Turbomachinery-Transactions of the ASME*, 143:6, Paper: 061014, 10 p., 2021, <https://doi.org/10.1115/1.4050392>.
- [6] G. Daku, and J. Vad, "Preliminary Design Guidelines for Considering the Vibration and Noise of Low-Speed Axial Fans Due to Profile Vortex Shedding," *International Journal of Turbomachinery, Propulsion, and Power*, 7:1, 2022, <https://doi.org/10.3390/ijtp7010002>.
- [7] E. Balla, and J. Vad, "Refinement of a semi-empirical method for the estimation of profile vortex shedding frequency from low-speed axial fan blade sections," In *The European Turbomachinery Society (Euroturbo) Proceedings of 14th European Conference on Turbomachinery Fluid dynamics & Thermodynamics ETC14*, 2021, Paper: ETC2021-591.
- [8] G. Daku, and J. Vad, "A Comprehensive Analytical Model for Vortex Shedding from Low-Speed Axial Fan Blades," *Journal of Turbomachinery- Transactions of the ASME*, 145:7, Paper: 071008, 11 p., 2023, <https://doi.org/10.1115/1.4056700>.
- [9] G. Daku, and J. Vad, "Analysis of shed vortices in the wake of an airfoil profile relevant to low-speed axial flow rotors," *International Journal of Heat and Fluid Flow*, vol. 121, 2026, [10.1016/j.ijheatfluidflow.2026.110499](https://doi.org/10.1016/j.ijheatfluidflow.2026.110499).
- [10] C.K.W. Tam, "Discrete tones of isolated airfoils," *J. Acoust. Soc. Am.*, vol. 55, pp. 1173–1177, 1974, <https://doi.org/10.1121/1.1914682>.
- [11] S. Wright, "The acoustic spectrum of axial flow machines," *J. Sound Vib.*, vol. 45, pp. 165–223, 1976, [https://doi.org/10.1016/0022-460X\(76\)90596-4](https://doi.org/10.1016/0022-460X(76)90596-4).
- [12] P. Ausoni, M. Farhat, Y. Bouziad, J.L. Kueny, and F. Avellan, "Kármán vortex shedding in the wake of a 2D hydrofoil: Measurement and numerical simulation," In *Proceedings of the IAHR International Meeting of WG on Cavitation and Dynamic Problems in Hydraulic Machinery and Systems*, 2006; pp. 14.
- [13] G. Yakhina, M. Roger, S. Moreau, L. Nguyen, and V. Golubev, "Experimental and Analytical Investigation of the Tonal Trailing-Edge Noise Radiated by Low Reynolds Number Aerofoils," *Acoustics*, vol. 2, pp. 293–329, 2020, <https://doi.org/10.3390/acoustics2020018>.
- [14] K. Chen, W. Meng, J. Wang, K. Liu, and Z. Lu, "An investigation on the structural vibrations of multi-rotor passenger drones," *International Journal of Micro Air Vehicles*, vol. 15, 2023, [doi:10.1177/17568293231199097](https://doi.org/10.1177/17568293231199097).
- [15] M.R. Ashory, F. Talebi, H. R. Ghadikolaei, and M. Karimpour, "Investigating the vibrational behaviour of a rotating two-blade propeller by using a self-tracking method," *Proceedings of the*

Institution of Mechanical Engineers, Part C, vol. 233 (3), pp. 835-847, 2018, doi:10.1177/0954406218764514.

- [16] S.K. Nt, P.K. Duba, N.P.B. Mannam, V.S. Mutnuri, and P. Rajalakshmi, "Aeroacoustics and Vibration Analysis of Multirotor eVTOL for Sustainable Urban Air Mobility (UAM)." *IEEE Sens. Lett.*, vol. 8, 2024.
- [17] Cs. Horváth, Sz. Gungl, B. Istók, and T. Kimpián, "Rectilinear and torsional modal analysis of a drone propulsion system using optical methods," *Journal of Physics: Conference Series*, vol. 3190, 012003, 2026, doi:10.1088/1742-6596/3190/1/012003.
- [18] J.D. Cluts, B.S. Henderson, and E. Pierson, "An Initial Electric Motor Rotor Vibration Model," In *AIAA SCITECH 2024 Forum*, 2024, AIAA 2024-2302, <https://doi.org/10.2514/6.2024-2302>.
- [19] T. Kimpián, and F. Augusztinovicz, "Universal broadband torsional exciter from a permanent magnet synchronous motor," In *Proceedings of ISMA2014-USD2014, International Conference on Noise and Vibration Engineering*, 2014, pp. 1101-1109, ISBN 978-16-343950-6-9.
- [20] T. Kimpián, and F. Augusztinovicz, "Torsional modal analysis made easy – Methodology and practical examples from electric motor testing," In *Proceedings of ISMA2022-USD2022, International Conference on Noise and Vibration Engineering*, 2022, pp. 4187-4201, ISBN 978-90-828931-5-1.
- [21] A. Algorfat, W. Wang, and A. Albarbar, "Study of Centrifugal Stiffening on the Free Vibrations and Dynamic Response of Offshore Wind Turbine Blades." *Energies*, vol. 15, 2022, <https://doi.org/10.3390/en15176120>.
- [22] H.Y. Wu, M. Rubinstein, E. Shih et al., "Eulerian video magnification for revealing subtle changes in the world," In *ACM Trans. Graph. (Proceedings SIGGRAPH 2012)*, vol. 31, no. 4, 2012.
- [23] N. Wadhwa, M. Rubinstein, F. Durand, and W.T. Freeman, "Phase-based video motion processing," *ACM Trans. Graph.*, vol. 32, no. 4, 2013, <https://doi.org/10.1145/2461912.246196>.
- [24] M.A. Sutton, J.J. Orteu, and H. Schreier, "Image correlation for shape, motion and deformation measurements: basic concepts, theory and applications." *Springer Science & Business Media*, 2009, ISBN 978-0-387-78747- 3.
- [25] J. Javh, J. Slavic, and M. Boltezar, "The subpixel resolution of optical-flow-based modal analysis," *Mechanical Systems and Signal Processing*, vol. 88, pp. 89–99, 2017, <http://dx.doi.org/10.1016/j.ymssp.2016.11.009>.
- [26] P. Bing, X. Hui-min, X. Bo-qin, and D. Fu-long, "Performance of sub-pixel registration algorithms in digital image correlation," *Measurement Science and Technology*, vol. 17, 2006, DOI 10.1088/0957-0233/17/6/045.
- [27] R. Huňady, P. Pavelka, P. Lengvarsky, "Vibration and modal analysis of a rotating disc using high-speed 3D digital image correlation," *Mechanical Systems and Signal Processing*, vol. 121, pp. 201–214, 2019, <https://doi.org/10.1016/j.ymssp.2018.11.024>.
- [28] S. Olufsen, M. E. Andersen, E. Fagerholt, "μDIC: An open-source toolkit for digital image correlation," *SoftwareX*, vol. 11, 2020, <https://doi.org/10.1016/j.softx.2019.100391>.
- [29] Y. Yang, C. Dorn, T. Mancini, Z. Talken, S. Nagarajaiah, G. Kenyon, C. Farrar, and D. Mascarenas, "Blind identification of full-field vibration modes of output-only structures from uniformly-sampled, possibly temporally-aliased (sub-nyquist), video measurements," *Journal of Sound and Vibration*, vol. 390, pp. 232–256, 2017, <https://doi.org/10.1016/j.jsv.2016.11.034>.
- [30] M.A. Schroder, "Synthesis of low peak-factor signals and binary sequences of low autocorrelation," *IEEE Transactions on Information Theory*, vol. 16, pp. 85–89, 1970, DOI: 10.1109/TIT.1970.1054411.