

ON THE INVERSE OF THE POMERANCHUK THEOREM

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In this note we prove the inverse of the Pomeranchuk theorem for logarithmically rising total cross sections.

Recent measurements on total cross sections and forward scattering amplitudes [1] reveal the following basic features in the 10 – 2000 GeV energy region:

- 1) The total cross sections rise with increasing energy and this rise is compatible with a logarithmic law;
- 2) the difference of the particle and anti-particle total cross sections decreases with increasing energy and this decrease is compatible with an inverse power law;
- 3) The forward scattering amplitudes are not dominantly real.

In 1958 POMERANCHUK proved [2] that for bounded total cross sections property 3) involves property 2). More precisely, if in the asymptotic region 3) is fulfilled and total cross sections tend to constant limits, then these limits are the same for a particle and its antiparticle hitting the same target. However, as mentioned, recent measurements do not indicate the boundedness of total cross sections at infinite energies, and for arbitrarily rising cross sections one cannot prove the zero asymptotic limit of the difference of the particle and antiparticle total cross sections. There exists a proof only on the ratio of the particle—antiparticle cross sections [3].

Therefore we consider the problem here from the inverse point of view. In this note we prove using dispersion relations that from the assumption that 1) and 2) hold asymptotically 3) follows also in the asymptotic energy region. More precisely, we repeat POMERANCHUK's line of thought in order to show that if

- i) the total cross sections rise with some power β of logarithm ($\beta \geq 1$) and if
- ii) the difference of the particle and antiparticle total cross sections remains finite, then

the real to imaginary ratios of both the particle and antiparticle forward scattering amplitudes are bounded.

These two amplitudes will be denoted by f and $\bar{f}$, whereas their sum and difference will be labelled by f_+ and f_- :

$$f_+ = \bar{f} + f; \quad f_- = \bar{f} - f. \quad (1)$$

The real and imaginary parts of any amplitudes will be denoted by D and A . We write the twice subtracted dispersion relations in the form [4]:

$$\begin{aligned} \frac{D(E)}{\bar{D}(E)} \Big\} = & \frac{1}{2} [\bar{D}(m) - D(m)] \mp \frac{E}{2m} [\bar{D}(m) - D(m)] + \\ & + \sum_i \frac{p^2}{E_i^2 - E^2} (E_i \mp E) \frac{2X_i}{E_i^2 - m^2} + \\ & + \frac{p^2}{\pi} \int_{E_0}^m \frac{\bar{A} \cdot (E' \mp E) dE'}{p'^2(E'^2 - E^2)} + \\ & + \frac{p^2}{4\pi^2} P \int_m^\infty \frac{E' \cdot (\bar{\sigma} + \sigma) \mp E \cdot (\bar{\sigma} - \sigma) dE'}{p'(E'^2 - E^2)}. \end{aligned} \quad (2)$$

Here p , E and m are the laboratory momentum, energy and the rest mass of the incoming particle, respectively. $\bar{D}(m)$ and $D(m)$ are the two subtraction constants, the subtraction being made at zero laboratory momentum. In the integral over the physical region we have substituted A by the total cross section using the optical theorem:

$$A = \frac{P}{4\pi} \sigma_i^{\dagger}. \quad (3)$$

The threshold of the unphysical region, E_0 , is the laboratory energy of the incoming particle corresponding to the CMS energy which is equal to the smallest mass of the physical continuum having the quantum numbers of the initial state.* The polynom represents the pole contributions, where E_i is the beam laboratory energy corresponding to the CMS energy equal to the rest mass m_i of the pole. X_i is proportional to the coupling constant square of the colliding particles to the pole [4]:

$$X_i = g_i^2 \frac{(m_i - m_t)^2 - m^2}{4m_t^2}. \quad (4)$$

* The laboratory energy corresponding to $\sqrt{s}$ energy in the CMS is:

$$E_{\sqrt{s}} = \frac{s - m_i^2 - m^2}{2m_t},$$

where m_t is the target mass.

Since we assume ii), it is enough to prove that

$$\lim_{E \rightarrow \infty} \left| \frac{D}{A_+} \right| < \infty \text{ and } \lim_{E \rightarrow \infty} \left| \frac{\bar{D}}{A_+} \right| < \infty \quad (5a)$$

or

$$\lim_{E \rightarrow \infty} \left| \frac{D_+}{A_+} \right| < \infty \text{ and } \lim_{E \rightarrow \infty} \left| \frac{D_-}{A_+} \right| < \infty. \quad (5b)$$

Hypothesis i) is equivalent to

$$|A_+| \sim cE(\ln E)^\beta \quad (6)$$

asymptotically, therefore in Eq. (2) all terms but the last one (which we denote by D^h) automatically fulfil the inequalities (5). Instead of (5) it is sufficient to prove the following inequalities:

$$\lim_{E \rightarrow \infty} \left| \frac{D_+^h}{A_+} \right| < \infty \quad (5c)$$

and

$$\lim_{E \rightarrow \infty} \left| \frac{D_-^h}{A_+} \right| < \infty. \quad (5d)$$

In Appendix 1 we calculate the integral D_+^h for $\beta = 0, 1, 2$ and show explicitly that the limit of (5c) is equal to zero. For arbitrary values of $\beta \geq 1$ we recall (see e.g. [5]) that analyticity and crossing require the following form of the amplitude in case of logarithmically rising cross sections:

$$\lim_{E \rightarrow \infty} f_+ = i \cdot c \cdot E \left(\ln E - \frac{i\pi}{2} \right)^\beta,$$

from which we get [5] the same result as for integer values of β .

Inequality (5c) can also be proved for a very general case. FISCHER et al have given namely [6] an upper bound for

$$\varrho_+ \equiv D_+/A_+$$

if σ_+ rises unboundedly:

$$\int^\infty \left(\varrho_+(E) - \frac{\pi}{\ln E} \right) \frac{dE}{E} < \infty.$$

This means that inequality (5c) is zero if ϱ_+ is positive. For this latter we recall the theorem of KHURI and KINOSHITA [7] who showed that for an unboundedly rising cross section D_+ cannot stay negative at all energies.

Turning now finally to inequality (5d) we reproduce in Appendix 2 the result of POMERANCHUK, which states that the crossing odd term, D_- approaches to

$$\lim_{E \rightarrow \infty} D_-^h = - \frac{\Delta\sigma}{2\pi^2} E \ln \frac{E}{E_1}.$$

if the difference, $\Delta\sigma$, of the antiparticle and particle cross sections remains constant beyond a certain value E_1 . Combining the limits (6) and (5) we arrive to prove (5d) and hence the whole statement.

We thus conclude that extrapolating the observed properties 1)–3) up to infinite energies they are compatible with dispersion relations in the sense that 3) follows from 1) and 2), although the inverse statement may not be proved.

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Appendix 1

The crossing even part of the high energy integral

1. σ and $\bar{\sigma}$ are constant beyond E_1

$$D_+^h \sim E^2 \int_{E_1}^{\infty} \frac{x \cdot (\sigma + \bar{\sigma})}{x(x^2 - E^2)} dx \sim E \ln \left| \frac{E_1 - E}{E_1 + E} \right| \sim E_1.$$

2. σ and $\bar{\sigma}$ are logarithmically rising: $\sigma \sim (\ln E)^\beta$; $\beta = 2$ (for $\beta = 1$ the procedure is essentially the same [8]).

$$\begin{aligned} D_+^h &\sim E^2 \int_{E_1}^{\infty} \frac{\ln^2 x dx}{x^2 - E^2} \sim E \int_{E_1/E}^{\infty} \frac{(\ln E + \ln x)^2}{x^2 - 1} dx \sim \\ &\sim E \ln^2 E \int_{E_1/E}^{\infty} \frac{dx}{x^2 - 1} + 2E \ln E \int_{E_1/E}^{\infty} \frac{\ln x dx}{x^2 - 1} + \int_{E_1/E}^{\infty} \frac{\ln^2 x dx}{x^2 - 1}. \end{aligned}$$

If $E \rightarrow \infty$

$$D_+^h \sim c_1 \ln^2 E + c_2 E \ln E \int_{-\infty}^{\infty} \frac{u du}{shu} + c_3 E \int_{-\infty}^{\infty} \frac{u^2}{shu} du.$$

The last term is zero, the second integral is finite and non-zero, therefore:

$$D_+^h \sim c_1 \ln^2 E + c_2 E \cdot \ln E,$$

where c_1 and c_2 are some constants.

Appendix 2

The crossing odd part of the high energy integral

$\Delta\sigma \equiv \sigma - \bar{\sigma}$ is constant beyond E_1

$$\begin{aligned} D_-^h &\sim \frac{E^3 \Delta\sigma}{4\pi^2} \int_{E_1}^{\infty} \frac{dx}{x(x^2 - E^2)} = \Delta\sigma \frac{E}{4\pi} \int_0^{(E/E_1)^2} \frac{du}{1-u} \sim \\ &\sim -\frac{\Delta\sigma}{2\pi^2} E \ln \frac{E}{E_1}. \end{aligned}$$

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