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Design of Risk-Based Univariate Control Charts with Measurement Uncertainty

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ABSTRACT Control charts are widely used tools of statistical process control (SPC). They are effective tools for indicating process shifts, helping avoid an increased number of defective products in the manufacturing environment. However, their effectiveness is strongly dependent on the performance of the measurement system. In the scientific literature, numerous studies have examined the effect of measurement errors in statistical process control, but only a few of them have focused on developing risk-based control charts that consider the cost of decisions during process control; furthermore, they were developed only for the case of multivariate and adaptive control charts such as T^2 and VSSI (Variable sample Size and Sampling Interval) X-bar charts. This paper fills this gap by using the proposed risk-based concept for commonly used control charts such as X-bar, Moving Average (MA), Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) charts. The effectiveness of the developed risk-based control charts is demonstrated by simulations and real-life datasets.

INDEX TERMS Control charts, Consumer risk, Risk-based approach, Decision cost, EWMA chart

I. INTRODUCTION

Statistical quality control (SQC) is a widely used method in operation management. Conformity control and process control are important means of controlling a process or production method. The effectiveness of applied quality control methods depends on the performance of the measurement system. Measurement uncertainty in a system can lead to incorrect decisions such as unnecessary manufacturing stoppages or missed interventions [1]. Therefore, the rate of producer and customer risk is strongly impacted by the performance of the measurement process [2].

Traditional methods do not account for measurement uncertainty [3]–[5]. Several researchers have examined the effect of measurement uncertainty in conformity testing [6]–[10] and statistical process control [11]–[22]. Therefore, the development of new methods of decreasing the number of incorrect decisions is an important area of SQC research that requires further exploration.

As a solution to the abovementioned issues, a risk-based method was designed by [8] considering all four decision outcomes to minimize the risk of false decisions under parameter uncertainty. Risk-based control charts use simulation and optimization to select the optimal values of the chart parameters to minimize the risks arising from incorrect decisions related to process control. Contrary to Shewhart control charts, they consider the effect of measurement errors and count with the costs of each decision outcome during the control chart design. The results revealed that a risk-based approach was highly effective in reducing decision costs, even under a nonnormal measurement error distribution. Later, [7], [9] studied a risk-based approach in conformity testing, and [23]–[25] designed risk-based control charts for statistical process control. Using industrial datasets, [5] validated the applicability of risk-based methods in conformity testing and for control charts. The results of all these studies reveal the applicability of the risk-based approach in minimizing the decision cost. The aforementioned studies showed

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that risk-based control charts are important tools for statistical process control since they could achieve a 5-13% total decision cost reduction in the case of process control and a nearly 5% increase in total profit during industrial conformity control. Their importance is also supported by the ability to greatly reduce prestige loss due to the elimination of error type II.

Although the results of these studies provide great potential for significant cost and risk reduction in manufacturing processes, they focus on complex process structures only, such as multivariate and adaptive control charts. Nevertheless, the need for monitoring univariate processes with fixed sample sizes and intervals, such as the application of Shewhart $\bar{X}$, MA, EWMA and CUSUM charts, is common. Several studies have noted the common usage of Shewhart and memory-based univariate charts in multiple fields, e.g., healthcare [26], high-quality processes [27], wastewater treatment [28], electronics [29] and financial applications [30]. Although [31] proposed a joint optimization of control charts and preventive maintenance policies, this solution does not address the effect of measurement errors during the optimizations. Therefore, the lack of a univariate version of the risk-based control charts is a huge gap since it strongly weakens the practical applicability of the developed risk-based approach and prevents many practitioners from benefiting from the cost minimization potential.

This paper fills this gap by extending the risk-based approach to the commonly used univariate control charts.

There are two aims of this article:

- A_1 to design Shewhart $\bar{X}$, MA, EWMA and CUSUM charts in the presence of measurement uncertainty for monitoring the stability of a process in terms of the average (μ) and
- A_2 to validate the results via simulation and real-life data.

The main contributions of this paper can be determined as follows:

- C_1 The practical applicability of the risk-based concept is greatly increased due to the extension with univariate cases
- C_2 The cost consequences of decisions of traditional and risk-based charts are studied
- C_3 The proposed risk-based control charts are validated using simulated and real-life examples.

The rest of this article is organized as follows: Section II describes the methods designed in this work. The results are presented in Section III. Finally, the article ends with a summary and conclusion (Section IV).

II. METHODS

Following the process proposed by Ref. [25], risk-based control charts can be designed based on four steps: (1) calculation of the traditional control chart parameters, (2) estimation of the decision costs, (3) calculation of the total cost and (4) optimization of the control lines.

A. CALCULATION OF CONTROL CHART PARAMETERS

The first step is to calculate the traditional control chart parameters, such as the centerline, control lines and plotted data points.

$\bar{\mathbf{X}}$ chart

In the case of the $\bar{X}$ chart, the sample statistics for the i^{th} sample can be calculated as follows [32]:

$$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \tag{1}$$

where $\bar{X}_i$ is the sample mean, n is the sample size and X_{ij} denotes the j^{th} observation of sample i. The control lines can be expressed as follows:

$$UCL_{\bar{X}} = \mu + \frac{k_x\sigma}{\sqrt{n}} \tag{2}$$

$$LCL_{\bar{X}} = \mu - \frac{k_x\sigma}{\sqrt{n}} \tag{3}$$

where μ is the process mean (expected value), σ is the process standard deviation, $k_x\sigma$ is the shift regarding the process mean expressed in standard deviation units and $UCL_{\bar{X}}$ and $LCL_{\bar{X}}$ denote the upper and lower control lines, respectively.

MA chart

The MA chart statistic can be calculated as follows [33]:

$$MA_i = \begin{cases} \frac{\sum_{j=1}^{i}\bar{X}_j}{i}, & \text{for } i < w \\ \frac{\sum_{j=i-w+1}^{i}\bar{X}_j}{w}, & \text{for } i \geq w \end{cases} \tag{4}$$

where $\bar{X}_j$ $(i, j = 1, 2, ..., m)$ is the average of the j^{th} observation, m is the number of observations, w is the span parameter and MA_i is the (moving) average of the i^{th} sample as a control chart statistic. The control lines can be calculated based on the following equations:

$$UCL_{MA} = \begin{cases} \mu + \frac{k_m\sigma}{\sqrt{ni}}, & \text{for } i < w \\ \mu + \frac{k_m\sigma}{\sqrt{nw}}, & \text{for } i \geq w \end{cases} \tag{5}$$

$$LCL_{MA} = \begin{cases} \mu - \frac{k_m\sigma}{\sqrt{ni}}, & \text{for } i < w \\ \mu - \frac{k_m\sigma}{\sqrt{nw}}, & \text{for } i \geq w \end{cases} \tag{6}$$

where UCL_{MA} and LCL_{MA} are the upper and lower control lines of the MA chart, respectively.

EWMA chart

In the case of the EWMA chart, the exponentially smoothed moving average statistic can be calculated as follows [34], [35]:

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$$E_i = \lambda \bar{X}_i + (1-\lambda)E_{i-1} \tag{7}$$

where E_i is the sample statistic for sample i and $\lambda, 0 < \lambda \leq 1$ is the smoothing parameter. The control lines can be determined in the following way:

$$UCL_{EWMA} = \mu + k_e\sigma\sqrt{\frac{\lambda}{n(2-\lambda)}[1-(1-\lambda)^{2i}]} \tag{8}$$

$$LCL_{EWMA} = \mu - k_e\sigma\sqrt{\frac{\lambda}{n(2-\lambda)}[1-(1-\lambda)^{2i}]} \tag{9}$$

CUSUM chart

In the CUSUM scheme, the positive and negative changes in the process mean are calculated as follows [36]:

$$C_i^+ = \max\{0, C_{i-1}^+ + X_i - \mu - z\sigma\} \tag{10}$$

$$C_i^- = \min\{0, C_{i-1}^- + X_i - \mu + z\sigma\} \tag{11}$$

where C_i^+ and C_i^- denote the upper and lower cumulative sums, respectively, and z represents the detectable shift expressed relative to the process standard deviation. The upper and lower control lines are described in the following forms:

$$UCL_{CUSUM} = h_c\sigma \tag{12}$$

$$LCL_{CUSUM} = -h_c\sigma \tag{13}$$

where h_c is the number of standard deviations considered during the control line calculation; its value is usually set to 4 or 5.

B. ESTIMATION OF DECISION COSTS

To determine the cost of decisions, the decision outcomes must first be interpreted. Since the real value of the monitored product characteristic (x) is distorted by measurement error (e), each observed value (y) can be expressed as follows:

$$y_i = x_i + e_i \tag{14}$$

Due to the presence of measurement error, four different decision outcomes can be defined during process control: (1) correct acceptance, (2) type I error, (3) type II error and (4) correct control. Table 1 demonstrates the structure of decision outcomes according to the observed and real product characteristics.

TABLE 1. Decision outcomes of statistical process control (based on [9]).

	Observed characteristic	
Real characteristic	In-control state	Out-of-control state
In-control state	c_{11}	c_{10}
Out-of-control state	c_{01}	c_{00}

where c_{11} is the cost of correct acceptance, c_{10} is the cost associated with type I error or incorrect control, c_{01} is the cost of type II error or incorrect acceptance, and c_{00} denotes the cost of correct control when the out-of-control state is correctly detected.

Each decision cost can be estimated by considering the cost components from the ERP system (Enterprise Resource Planning system) of the manufacturer. For a more detailed description of the decision cost estimation, the reader is referred to [24], where the main components are described. In the following, a more detailed description of each cost component is also provided based on [24] to demonstrate the estimation related to the costs of the decision outcomes.

Cost of correct acceptance (c_{11}): The cost of correct acceptance consists of all the production and inspection costs. The production costs include labor, material costs, rent, operating and indirect costs as well. The inspection costs are the operating cost of the measurement device, labor and sampling costs. If c_{11} is considered the cost of the default case, its value is usually set to 1. In this way, the other three decision costs can be relative to c_{11}, which makes the interpretation easier.

Cost of correct control (c_{00}): The two main costs arising in this case are the cost of control and the cost of restart. In this case, the cost of control also includes the cost of root cause search and the maintenance cost of the manufacturing device. These can be estimated considering the mean stoppage- and maintenance times.

Cost of incorrect control (c_{10}): The cost of incorrect control contains the same cost components as it is observable by c_{00}. However, in this case the overregulation of the process can be considered an additional cost optionally to assign more weight to the type I error.

Cost of missed control (c_{01}): If the control chart does not detect the existing process shift (type II error), the manufacturing device produces scraps until the next stoppage. In this case, the mean cost of scraps should be estimated and added to the production and inspection costs.

As mentioned previously, the estimation of the different cost components, for example, the mean cost of scraps or the mean cost of root cause search, can be supported by the usage of an ERP system.

C. TOTAL COST CALCULATION

During risk-based control chart design, the total cost of decisions (TC) is the output of the objective function to be minimized. Opposed to the economic design of

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control charts [37] here, all the four states are considered in the total cost: letting the controlled process run, investigating for assignable cause when the process gets out of control and the two – type I and type II – decision errors. Based on [24], the total decision cost can be calculated by aggregating the cost of each decision outcome:

$$TC = C_{11}+C_{10}+C_{01}+C_{00} = q_{11}c_{11}+q_{10}c_{10}+q_{01}c_{01}+q_{00}c_{00} \tag{15}$$

where C_{ab} is the aggregate cost of a specific decision outcome and q_{ab} is the number of times each case occurred during control. $a = 1$ if the real product characteristic is in control and 0 otherwise, and $b = 1$ if the observed product characteristic is in control and 0 otherwise.

Type I. and type II. errors are identified by comparing the plotted control chart statistics (observed and real) to the control lines calculated considering the real and measurement error distorted processes. The cost of the decision outcome is calculated for each plotted control chart statistic.

D. OPTIMIZATION OF THE CONTROL LINES

To modify the control lines, the total cost of decisions (TC) is minimized by optimizing the control coefficient by each control chart. Let $\mathbf{k}$ represent a one-dimensional vector containing the correction coefficient for each control chart and $\mathbf{k} = [k_x, k_m, k_e, h_c]$. Then, the optimization problem is formulated as follows:

For each $\mathbf{k}(i)$:

$$\begin{aligned} \underset{\mathbf{k}(i)}{\text{minimize}} \quad & TC(\mathbf{k}(i)) \\ \text{subject to} \quad & \mathbf{k}(i) > 0 \\ & \forall \mathbf{k}(i) \in \mathbf{R} \end{aligned} \tag{16}$$

where $\mathbf{k}(i)$ represents the i^{th} element of vector $\mathbf{k}$. The Nelder–Mead simplex search is applied to solve the optimization problem. Only the control lines are optimized in this paper but as shown in [25], the method can be combined with the optimal setting of the sample size and sampling interval, as is done in economic design.

III. RESULTS

In this section, first, the simulation results are presented to examine the applicability of the risk-based approach related to the aforementioned additional univariate control chart types. The optimal parameter of any risk-based control chart has been determined using equation (16) via the Nelder-Mead simplex method. In this optimization, two different scenarios have been considered. First, a simulations study is conducted to obtain the optimal parameter of the risk-based univariate control chart in subsection (A). Second, real-life data are used to calculate the optimal parameter of the risk-based univariate control chart in subsection (B).

A. SIMULATION RESULTS

The simulation was conducted starting with random number generation. Figure 1 demonstrates the steps of one iteration during the simulation process. The equations used in each step are typed in red.

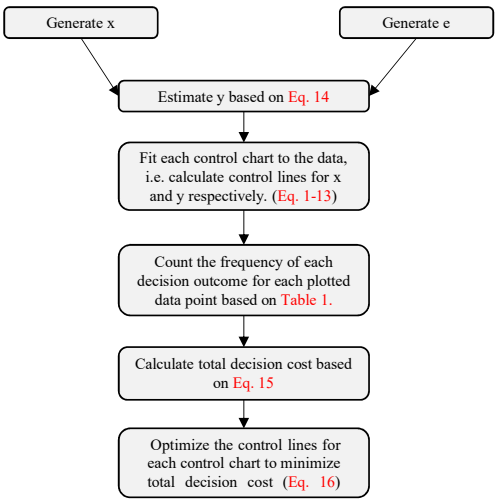


FIGURE 1. The steps conducted in each iteration during the simulation

A process consisting of 1000 samplings with a sample size $n = 1$ was generated following a normal distribution with expected value $\mu_x = 10$ and standard deviation $\sigma_x = 0.5$. Then, measurement errors were added to the process following a normal distribution with expected value $\mu_e = 0$ and standard deviation $\sigma_e = 0.05$ (i.e., two vectors are generated, one for the process and one for measurement errors.). A control chart was designed for the generated process, and the control lines were optimized to achieve the minimal total decision cost. The process and measurement error generation, control chart design and optimization were iterated one hundred times for each control chart type. Table 2 shows a detailed list of the input parameters of the simulation.

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TABLE 2. Simulation parameters

Sign	Definition	Value
μ_x	Process expected value	10
σ_x	Process standard deviation	0.5
σ_e	Measurement error standard deviation	0.05
m	Number of data observations generated	1000
n	Sample size	1
r	Number of simulations for each chart	100
l	Length to compute control lines	25
w	MA chart moving average lag parameter	3
c_{11}	Cost of correct acceptance	1
c_{10}	Cost of incorrect control	5
c_{01}	Cost of incorrect acceptance	60
c_{00}	Cost of correct control	5
λ	EWMA chart weighting parameter	0.2
z	Relative detectable shift by the CUSUM chart	1
s_{CUSUM}	Process trend simulated by CUSUM chart	0.01

As Table 2 shows, the lag parameter of the MA chart (w) was set to w=3 as the default value of widely used statistical software such as Minitab [38]. The EWMA weighting parameter λ is set to 0.2 based on the recommendation given by [39], namely that the EWMA chart is effective in the detection of small shifts when $0.05 < \lambda < 0.25$. The minimum detectable mean shift (z) by the CUSUM chart is set to 1 based on the recommendation of [40]. Regarding the decision costs (c_{11}, c_{10}, c_{01}, c_{00}), the cost structure proposed by [24] was applied in the simulation.

The results were visualized with boxplot diagrams for each control chart type, as shown in Fig. 2. The vertical axes denote the distribution and median of the total decision cost values for the generated processes before and after optimization (i.e., for the original and risk-based control charts).

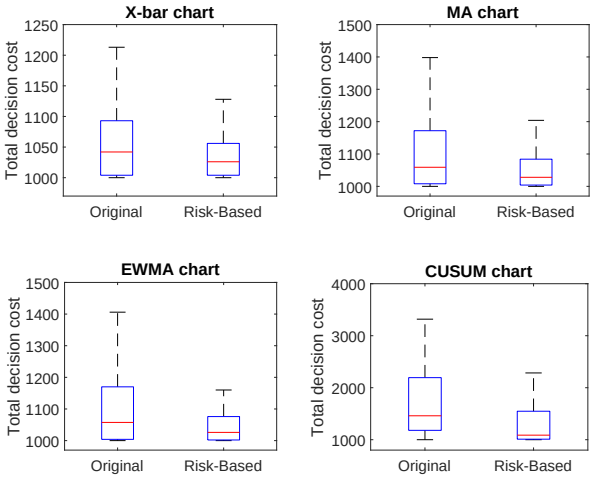


FIGURE 2. Simulation results

As Fig. 2 shows, a decrease in the total decision cost is achievable for all control chart types by applying the proposed optimization. Notably, higher total costs are observable in the case of the CUSUM control chart since to generate out-of-control situations, an increasing trend must be added to the process (the shift size is denoted by s_{CUSUM}). Therefore, the increasing trend causes more frequent out-of-control states, resulting in higher decision costs.

Table 3 shows the control chart schemes before and after the optimization of the control coefficients. Since the process generation and chart optimization were conducted one hundred times, UCL, LCL, CL and the coefficient parameters show average values. In the case of the other control chart parameters such as n, λ, z and w, the values remained constant during the simulation.

TABLE 3. Control chart schemes in the simulation

Chart	Type	n	$\overline{UCL}$	$\overline{LCL}$	$\overline{CL}$	λ	z	w	coeff.
X	O	1	11.49	8.53	10	-	-	-	$\overline{k_x}$=3.00
	RB	1	11.43	8.59	10	-	-	-	$\overline{k_x^*}$=2.88
MA	O	1	10.87	9.13	10	-	-	3	$\overline{k_m}$=3.00
	RB	1	10.84	9.17	10	-	-	3	$\overline{k_m^*}$=2.88
EWMA	O	1	10.51	9.47	10	0.2	-	-	$\overline{k_e}$=3.00
	RB	1	10.49	9.49	10	0.2	-	-	$\overline{k_e^*}$=2.89
CUSUM	O	1	2.01	-2.01	0	-	1	-	$\overline{h_c}$=5.00
	RB	1	2.62	-2.62	0	-	1	-	$\overline{h_c^*}$=5.21

O: original control chart
RB: risk-based (control chart after the optimization)

As Table 3 shows, the average values of the chart coefficients decreased after optimization for all control chart types, except for the CUSUM chart. The reason for this is as follows: in the case of the X-bar, MA and EWMA control charts, the average numbers of type I and type II errors are balanced, and since type II errors have significantly more serious consequences (meaning significantly higher decision costs), the method narrows the control region to avoid incorrect acceptance. On the other hand, since an increasing trend was added to the process in the CUSUM chart case, the number of out-of-control samples also increased, indicating a higher probability of committing a type I error. Therefore, the proposed method broadens the control interval to avoid type I errors. Table 4 shows the average count of each decision outcome and the average total decision costs before and after optimization.

TABLE 4. Decision outcomes and total decision cost in the simulation

Chart	Type	$\overline{q}_{11}$	$\overline{q}_{10}$	$\overline{q}_{01}$	$\overline{q}_{00}$	$\overline{TC}$
X	O	991.97	1.05	1.02	5.96	1088.22
	RB	990.12	2.90	0.04	6.94	1041.72
MA	O	987.68	1.23	1.62	9.47	1138.38
	RB	984.55	4.36	0.06	11.03	1065.10
EWMA	O	987.75	1.12	1.49	9.64	1130.95
	RB	985.32	3.55	0.04	11.09	1060.92
CUSUM	O	897.63	6.91	10.50	84.96	1986.98
	RB	933.05	3.81	5.47	57.67	1568.65

O: original control chart
RB: risk-based (control chart after the optimization)

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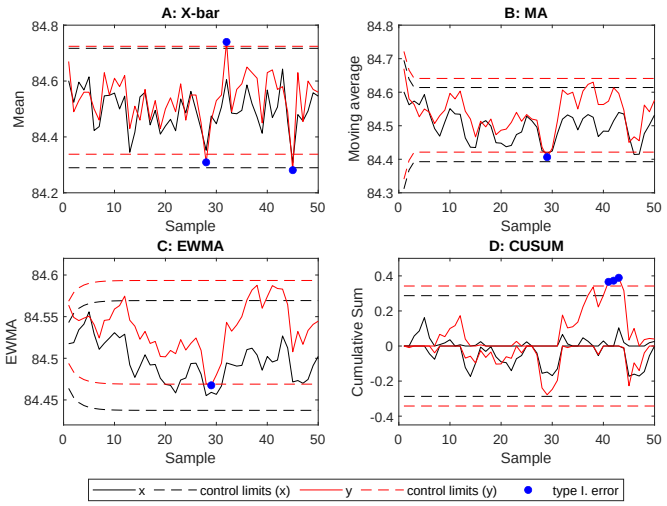


FIGURE 3. Control charts reflecting the real and observed processes

As Table 4 shows, the risk-based control charts outperform the traditional charts in terms of total decision cost. The numbers of type I and type II errors decreased after the optimization in every case.

The applicability of the proposed risk-based control charts was also tested on a real-life dataset.

B. RESULTS ON A REAL DATASET

The proposed risk-based univariate control charts were tested on a real dataset constructed by [5]. In the dataset, the cutting length of brake cylinders was monitored as a product characteristic. The dataset contains 50 measured values. Each part was measured twice, first with a manual height gauge (used in production) and then by a 3D high-precision device to obtain measurements representing the observed and "real" product characteristic values, respectively. Due to the short length of this process, a sample size n = 1 was applied in the case of all four charts, and the control lines were estimated taking the first 25 observations into consideration, since several software packages use 25 as the default value. Fig. 3 shows the control charts without optimization. The black lines represent the "real" process, and the red lines denote the "observed" sample values. The dashed lines denote the control lines based on the real and observed data. Finally, the occurrence of type I errors is indicated with blue dots.

As Fig. 3 shows, the application of a manual height gauge indicates measurement errors, and type I errors can be observed during the process monitoring in all four cases. Table 5 shows the applied control chart schemes before and after the optimization.

TABLE 5. Control chart schemes on real dataset

Chart	Type	n	UCL	LCL	CL	λ	z	w	coeff.
X	O	1	84.72	84.34	84.54	-	-	-	k_x=3.00
	RB	1	84.76	84.30	84.54	-	-	-	k_x^*=3.56
MA	O	1	84.64	84.42	84.54	-	-	3	k_m=3.00
	RB	1	84.66	84.40	84.54	-	-	3	k_m^*=3.53
EWMA	O	1	84.59	84.47	84.54	0.2	-	-	k_e=3.00
	RB	1	84.60	84.47	84.54	0.2	-	-	k_e^*=3.12
CUSUM	O	1	0.34	-0.34	0.00	-	1	-	h_c=4.00
	RB	1	0.43	-0.43	0.00	-	1	-	h_c^*=5.06

O: original control chart
RB: risk-based (control chart after the optimization)

As Table 5 demonstrates, the control coefficients are increased after optimization in all cases. In this practical example, the measurement gauge usually overmeasures the real product characteristic, which leads to type I errors. Therefore, the risk-based chart relaxes the control interval (through the increase in the coefficient) to eliminate type I errors. For the parameters n, λ, z and w, the same values were used as in the simulation. Table 6 depicts the decision outcomes and total decision cost on the real dataset.

TABLE 6. Decision outcomes and total decision cost on real dataset

Chart	Type	q_{11}	q_{10}	q_{01}	q_{00}	TC
X	O	47	3	0	0	62
	RB	49	1	0	0	54
MA	O	49	1	0	0	54
	RB	50	0	0	0	50
EWMA	O	49	1	0	0	54
	RB	50	0	0	0	50
CUSUM	O	47	3	0	0	62
	RB	50	0	0	0	50

O: original control chart
RB: risk-based (control chart after the optimization)

All the risk-based univariate control charts were able to eliminate type I errors by increasing the control interval (indicated by the increase of the control chart coefficients) in this real-life example. Furthermore, the total decision cost decreased in all cases.

IV. SUMMARY AND CONCLUSION

In an increasingly industrialized environment, the accuracy of measurements has become a key issue. It is important to detect and account for measurement uncertainty when making decisions. Therefore, it is very important to develop methods in process control that account for measurement uncertainty and the consequences of decisions. The main goal of this study was to propose a new family of univariate control charts. To do this, three goals were pursued and achieved.

A family of univariate risk-based control charts was proposed to handle measurement uncertainty (A_1). It was important to demonstrate that risk-based extension works not only in complex cases, such as multivariate

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and variable-sample-size control charts but also in simple univariate cases.

If the costs of decision errors, such as incorrect acceptance and incorrect rejection, are enumerated or at least their proportions are specified, the total decision cost can be significantly reduced (A_2) for both memory-based and nonmemory-based control charts.

The proposed extensions work not only in simulated but also in real-life situations (A_2). Therefore, the optimization of the risk-based univariate control chart is determined via simulation as well as using a real life dataset. Moreover, we believe that the reduction in total decision costs easily compensates for the cost of the implementation of risk-based control charts.

The contributions of this paper can be summarized as follows. First, the applicability of the proposed risk-based approach is significantly increased by the development of univariate RB control charts (C_1) The proposed univariate control charts opens new room for software development since they can be implemented in new software packages written in program languages such as R or Python. Both platforms offer easily accessible and free packages, which makes the application easier for practitioners. Second, demonstrating that the total decision cost can be decreased significantly in the case of the average control charts indicates that manufacturers working with univariate processes (and fixed samples) can benefit from the risk- and total decision cost reduction (C_2). Third, the validation of the proposed risk-based charts on a real dataset supports the necessity of the practical application of the proposed approach (C_3).

As a next step, we plan to implement the proposed methodology as an R package that provides all the functions needed to design risk-based control charts.

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