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Estimation of the Love numbers: k_2, k_3 using SLR data of the LAGEOS1, LAGEOS2, STELLA and STARLETTE satellites

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Abstract In this paper we present results for the determination of the global elastic parameter *k* (Love number *k*) for the first and the second degree tides: k_2, k_3. The observation data used for determining the parameters were the satellite laser observations conducted within the period of January 3, 2005 until July 1, 2007 by high satellites: LAGEOS1 and LAGEOS2 (H $\cong$ 6000 km) and low satellites: STELLA and STARLETTE (H $\cong$ 800 km). The purpose of selecting satellites of varied orbit altitudes was to indicate which of them provide a better solution and thus ought to be used for determining the Love number *k*. All computations were carried out by use of the GEODYN II NASA/GSFC software, and the obtained results were compared with determinations of other authors.

Keywords Love number *k* · Tides · Satellite laser ranging (SLR) technique

1 Introduction

This study is a continuation of our research on tidal parameters, previously published in Rutkowska and Jagoda (2010), Jagoda and Rutkowska (2013). In these publications we presented the results of determining tidal parameters related to radial and horizontal relocations of Earth masses to be generated by the activity of tidal forces—Love *h* and Shida *l* numbers. We based our calculations on the SLR data observed within the same period (from January 3, 2005 until July 1, 2007) and employing the same satellites (LAGEOS1, LAGEOS2, STELLA, STARLETTE).

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For more information about the theoretical treatment of the phenomenon of the Earth elasticity and elastic parameters the reader is refered to Rutkowska and Jagoda (2010); Jagoda and Rutkowska (2013).

Also the measurement methods, choice of force models and orbit computations, were described in detail in Rutkowska and Jagoda (2010), Jagoda and Rutkowska (2013) and therefore they are omitted in this paper.

All calculations related to determining the satellite orbits and the Love numbers were performed with the use of the GEODYN II NASA/GSFC software (McCarthy et al. 1993). The satellites orbits are computed using 11th order predictor–corrector Cowell's method for the numerical integration of the satellite equations of motion in rectangular coordinates (Maury and Brodsky 1969).

2 Research objectives and methods

The main objective of the research was to determine and analyze the global value of the parameter k for tides of the second and third degree—k_2, k_3. The observation data used to carry out computations are SLR data (compressed to so called normal points), conducted from January 3, 2005 until July 1, 2007 by satellites LAGEOS1 and LAGEOS2, STELLA and STARLETTE. The interval of 2.5 years, taken for the observation, seems sufficient for determining tidal parameters, as proved before in Rutkowska and Jagoda (2010), Jagoda and Rutkowska (2013). While choosing the satellites two main factors were decisive: an optimum ratio of their mass and cross-section (which minimizes the influence of non-gravitational effects upon satellite motion) and a large number of observations. Moreover, investigating data from satellites of varied altitude allow to distinguish which of them are more effective for determining the Love number k.

The SLR data were obtained from the global data bases maintained by the International Laser Ranging Service and next divided into orbital arcs: 30 days for satellites LAGEOS and 7 days for STELLA and STARLETTE (after Torrence et al. 1984). In total, 30 orbital arcs were obtained for LAGEOS1, 30 for LAGEOS2, 130 for STELLA and 130 for STARLETTE. In our calculations we used the data obtained from 18 SLR stations of the global network. The primary criterion of selecting them was the accuracy of measurement and the number of the obtained data. Additionally, considering the geometry of the solution, we tried to select the stations in such a way that they would be evenly located on the globe. The coordinates of stations were expressed in the ITRF2008 reference frame (Altamimi et al. 2011).

The total number of normal points used in the calculations were: 104582 for LAGEOS1, 101188 for LAGEOS2, 61757 for STELLA and 121668 for STARLETTE. The number of normal points with regard to individual stations are presented in Table 1.

After establishing the orbital arcs, we began to determine the orbits of satellites, and then the tidal parameters k_2, k_3. This process was carried out in two stages. In the first stage, for each orbital arc the $\Delta\varepsilon_j$ parameters were determined (Eq. (1))—satellite position and velocity, the direct solar radiation pressure scaling coefficient (adjusted one value for each orbital arc), atmospheric drag coefficients (adjusted five values per week only for STELLA and STARLETTE), empirical accelerations (for 7-day intervals) and range biases. The employed models of forces and the procedures of measurement reductions for satellites LAGEOS1, LAGEOS2, STELLA and STARLETTE are presented in Table 2.

Table 1 Number of normal points of LAGEOS1, LAGEOS2, STELLA and STARLETTE satellites obtained at each observatory station within the period from January 3, 2005 to July 1, 2007

No.	Station	Number ID for station	Number of normal points			
			LAGEOS1	LAGEOS2	STELLA	STARLETTE
1	Herstmonceux	78403501	17330	12625	3499	8290
2	Yarragadee	70900513	25630	31273	16339	27645
3	Simosato	78383602	6653	7306	2660	4005
4	McDonald	70802419	3618	4023	574	1332
5	Greenbelt	71050725	3947	3339	2646	5015
6	Wettzell	88341001	5787	6523	6886	12073
7	Monument Peak	71100412	7569	6987	4342	8390
8	Hartebeesthoek	75010602	4480	4599	1398	2700
9	Grasse	78353102	355	427	151	401
10	Riga	18844401	2006	1946	121	1028
11	Borowiec	78113802	1405	1353	118	1290
12	Changchun	72371901	4021	3109	2182	2650
13	Graz Lustbuehel	78393402	10513	7009	8492	14211
14	Shanghai	78372805	188	185	189	252
15	Solar Village	78325501	–	–	4075	7426
16	Mount Stromlo	78259001	7178	6602	4142	14072
17	Beijing	72496101	–	–	2032	6173
18	Potsdam	78418701	3902	3882	1911	4715
Sum of normal points			104582	101188	61757	121668

The values of the gravitational potential of the Earth diminish with satellite altitude. For low satellites such as STARLETTE and STELLA, when modeling the orbit with an accuracy of 2–3 cm we adopted Gravitational field EGM2008 (2159, 2159) model (Pavlis et al. 2008) using terms through degree and order 70, the same model we adopted for LAGEOS satellites using terms through degree and order 20 (consistent with the requirement for the LAGEOS satellites). The complete model EGM96 (Lemoine et al. 1998) was included for the computation of the solid Earth and ocean tide. The perturbations caused by the third bodies—Moon, Sun and the planets Venus, Mars, Jupiter, Saturn—on the satellite orbit are computed using the DE200. Empirical accelerations in along-track, cross-track and radial directions (for 7-day intervals) were estimated. The numerical values for the precession-nutation model IAU 2000 have been adopted to computations shown in IERS Conventions 2003 (McCarthy and Petit 2004). The Pole tide, polar motion (x_p, y_p) and UT1 {EOP05C04(IAU2000A)}, Ocean loading deformation and atmospheric pressure loading deformation model (McCarthy et al. 1993) were used in the solution. The formulation of the Mendes-Pavlis model for troposphere delay (Mendes and Pavlis 2004) and the center-of-mass correction equal to 25.1 cm for LAGEOS satellites and 7.5 cm for Stella and Starlette (McCarthy et al. 1993) were added to the laser ranging data, except for Herstmonceux station for which the best center-of-mass correction is 24.5 cm for LAGEOS and 6.9 cm for STELLA and STARLETTE satellites.

When stage 1 processing converges, stage 2 has been started, in which all the unknowns ($\Delta\varepsilon_j$, Δk_2, Δk_3) in Eq. (1) were determined in one solution. The weights of the observations

Table 2 Apriori force model employed in satellite orbit computations for LAGEOS1, LAGEOS2, STELLA and STARLETTE

Dynamic model
Gravitational field EGM2008 (2159, 2159), $a_e = 6378136.3$ m, $GM_E = 398600.4415$ km^3/s^2, (Pavlis et al. 2008)
Solid Earth and ocean tide model EGM96 (Lemoine et al. 1998)
The gravitational fields of the planets: Venus, Mars, Jupiter, Saturn. Planetary Ephemerides JPL DE200 (Standish 1990)
The atmospheric drag (Mass Spectrometer Incoherent Scatter) MSIS-86 (Hedin 1987)
Albedo and infrared Earth radiation (Melbourne et al. 1983)
Relativistic effects (McCarthy et al. 1993)
Accelerations in along-track, cross-track and radial directions (for 7-day intervals)
Reference frame
Precession according to IAU 2000 (McCarthy and Petit 2004)
Nutation according to IAU 2000 (McCarthy and Petit 2004)
Pole tide (McCarthy et al. 1993)
Ocean loading deformation, atmospheric pressure loading deformation (McCarthy et al. 1993)
Stations coordinates and stations velocities ITRF2008 system (Altamimi et al. 2011)
Processing model
Mendes-Pavlis model for troposphere delay (Mendes and Pavlis 2004)
Center of mass correction equal to 7.5 cm for STELLA and STARLETTE and 25.1 cm for LAGEOS1 and LAGEOS2 (McCarthy et al. 1993). Only for Herstmonceux station it was respectively: 6.9 and 24.5 cm (as advised by NASA Goddard Space Flight Center)

were computed according to: $W = 1/m_{ST}^2$, where m_{ST} is the measurement accuracy for each station. In our solution laser range standard deviations of normal points taken from CDDIS file of data were adopted as m_{ST}.

The values of partial derivatives $\frac{\partial C_i}{\partial \varepsilon_j}, \frac{\partial C_i}{\partial k_2}, \frac{\partial C_i}{\partial k_3}$ in Eq. (1) are computed by means of numerical integration of satellite orbit.

Knowing the partial derivatives allows to formulate observation equations:

$$(O_i - C_i) = -\left\{\sum_{j=1}^{n} \frac{\partial C_i}{\partial \varepsilon_j}\Delta\varepsilon_j + \frac{\partial C_i}{\partial k_2}\Delta k_2 + \frac{\partial C_i}{\partial k_3}\Delta k_3\right\} + dQ_i \tag{1}$$

and subsequently solving them and determining the unknowns by the least squares method.

Particular quantities in Eq. (1) denote:

$(O_i - C_i)$ SLR observations minus computed distance from station to satellite, n is the number of measurement, i is the number of estimated unknown, $\Delta\varepsilon_j$ is the corrections for satellite position and velocity, the direct solar radiation pressure scaling coefficient, atmospheric drag coefficients (only for the low satellites STELLA and STARLETTE), empirical accelerations and range biases, Δk_2, Δk_3, is the correction for the Love numbers k_2, k_3, dQ_i is the error of observation associated with the i-th measurement.

The process of estimating Love numbers k_2, k_3 was conducted by a sequential method. In the first phase k_2, k_3 parameters were calculated separately for each orbital arc. The further steps consisted in adding arcs, one by one. Each time k_2, k_3 parameters were calculated anew the stability of the solution has to be observed. As the apriori k_2, k_3 values we assumed the figures provided in IERS Technical Note No. 36 (Petit and Luzum 2010): $\boldsymbol{k_2} = 0.29525$, $\boldsymbol{k_3} = 0.0930$.

The adjustment was performed in an iterative process with convergence criterion: {RMS(m) – RMS(m − 1)} < 0.01 cm, where (m) is the number of iteration.

This solution allows the RMS of the post-fit residuals at the initial epoch of arc to be estimated. RMS of the post-fit residuals are computed from the following expression:

$$\text{RMS of the post} - \text{fit residuals} = \sqrt{\frac{\sum_{i=1}^{n}(O_i - C_i)^2}{n-1}}$$

In this analysis the following values of the RMS of the post-fit residuals for LAGEOS1, LAGEOS2, STELLA and STARLETE satellites were obtained: RMS LAGEOS1 = 1.55 cm, RMS LAGEOS2 = 1.59 cm, RMS STELLA = 3.11 cm, RMS STARLETTE = 2.40 cm.

The Love numbers k_2, k_3 determined in this research, were analyzed with respect to accuracy, stability and convergence of determination. The criterion of stability was the repeatability of the obtained values k_2, k_3 for subsequent orbital arcs, staying at the level of mean error of a determined parameter. And the criterion of the convergence of results obtained for two independent satellites was the assumption that they should not vary more than 15 % of the given parameter value. This number follows from the analysis of subject literature (Table 4): Takeuchi et al. (1962), Kaula (1963), Longman (1966), Farrell (1972), Melbourne et al. (1983), Dehant (1987), Mathews et al. (1995).

3 Results

The final results of the research were the global values of parameters k_2, k_3 with their mean errors. The obtained results are illustrated by Figs. 1, 2, 3, 4, separately for each parameter and listed in Table 3. To keep figures demonstrative we present only the values obtained for every fourth orbital arc.

To verify the correctness of the obtained results parameters k_2, k_3 were determined separately from observation data of satellites LAGEOS1 and LAGEOS2, and separately from satellites STELLA and STARLETTE. After confirming the convergence of the individual determinations, the next step was to determine combined values: LAGEOS1 data+LAGEOS2 data and STELLA data+STARLETTE ata.

The values of the Love numbers k_2, k_3 determined from a few arcs diverge considerably from their final values. Adding consecutive monthly (for LAGEOS1 and LAGEOS2) or weekly (for STELLA and STARLETTE) intervals allows to observe a slow convergence towards the final quantities. Also the mean errors of the determined quantity converge asymptotically towards their final quantities. Convergence is different for k_2 and k_3 for high and low satellites (Figs. 1, 2). The fastest stabilization of determining the unknown is obtained for Love number k_2, utilizing around 20 months data for LAGEOS1 and LAGEOS2 and 14 months for STELLA and STARLETTE. Furthermore, it can be noticed that since a certain period this parameter remains almost constant (does not alter after adding successive orbital arcs to calculations). This period amounts to 26 months for LAGEOS1 and STELLA, 27 months for LAGEOS2 and 21 months for STARLETTE respectively.

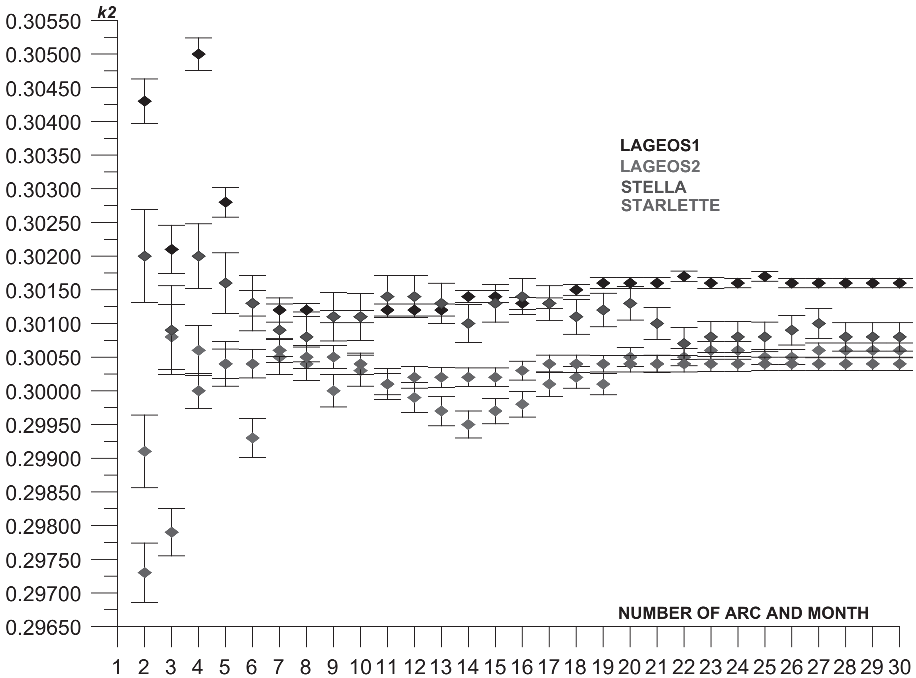

Fig. 1 The sequential solution for the Love number k_2 estimated in the individual analysis based on data for LAGEOS1 (*black squares*), LAGEOS2 (*red squares*), STELLA (*blue squares*) and STARLETTE (*green squares*). The final value of the Love number is equal to 0.30161 ± 0.00011 for LAGEOS1, 0.30060 ± 0.00011 for LAGEOS2, 0.30081 ± 0.00019 for STELLA and 0.30041 ± 0.00020 for STARLETTE data for a 2.5 years' time interval from January 3, 2005 to July 1, 2007

A slightly slower convergence is obtained for parameter k_3, which is acquired after about 22 months for the satellites LAGEOS1 and LAGEOS2, and about 26 months for STELLA and STARLETTE (see Fig. 4).

When focusing on the convergence, it may be concluded that the smallest discrepancies in final values can be observed for the number k_2, determined from the LAGEOS2 and from STARLETTE data (the difference amounts to 0.00019 and is of the order of value of k_2 mean error). A more worse convergence can be observed for parameter k_3. For satellites LAGEOS the difference in the final values of k_3 amounts to 0.0145, whereas for STELLA and STARLETTE 0.0026. As regards the accuracy of the determinations, the satellites LAGEOS seem to be more efficient, since lower values of k_2, k_3 mean errors were calculated for them than for low satellites STELLA and STARLETTE, that is respectively, 2-fold for k_2 and 1.5-fold for k_3. This is probably caused by the influence of atmospheric drag on low satellite motion. This is one of the reasons the orbits of low satellites are estimated with errors significantly larger than for high satellites, the larger orbit error of low satellites lead to increased tidal parameters errors.

In Fig. 1 we can see a slight bias for the k_2 LAGEOS1 solution compared to the LAGEOS2, STELLA and STARLETTE solutions. It is difficult to explain what it is due to. This effect is not visible for the k_3 parameter, also it is not observed in our previous publication where we presented the results of determining parameters h_2 and l_2 (Rutkowska

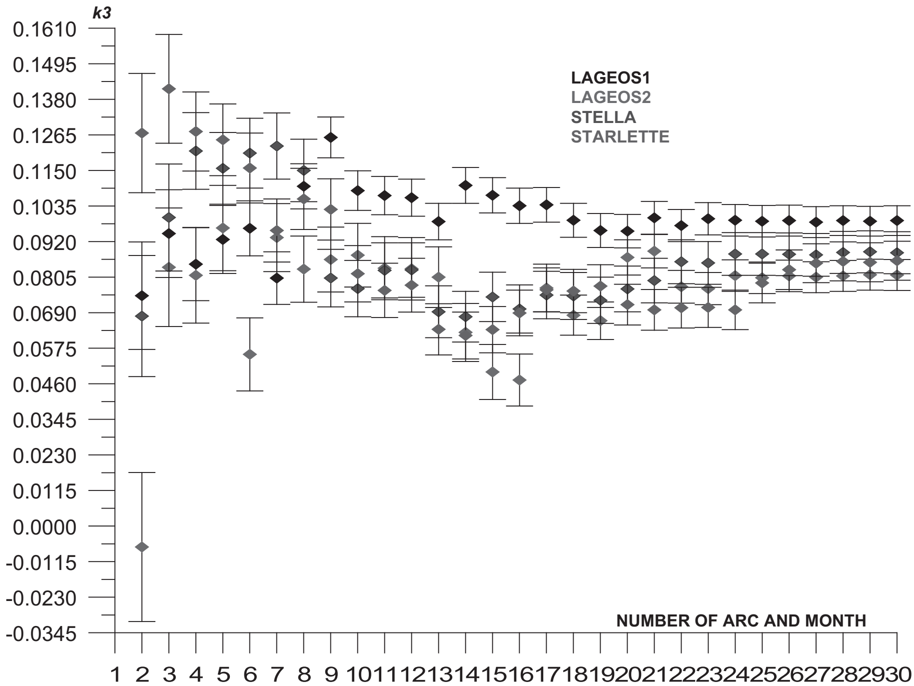

Fig. 2 The sequential solution for the Love number k_3 estimated in the individual analysis based on data for LAGEOS1 (*black squares*), LAGEOS2 (*red squares*), STELLA (*blue squares*) and STARLETTE (*green squares*). The final value of the Love number is equal to 0.0980 ± 0.0048 for LAGEOS1, 0.0835 ± 0.0050 for LAGEOS2, 0.0884 ± 0.0068 for STELLA and 0.0858 ± 0.0063 for STARLETTE data for a 2.5 years' time interval from January 3, 2005 to July 1, 2007

and Jagoda 2010). In the future we plan to determine k_2 and k_3 parameters using SLR data performed at different time interval, having two different determinations might be able to explain this effect.

After checking the accuracy of the obtained results we went on to combine the observations (LAGEOS1 + LAGEOS2 and STELLA + STARLETTE), and re-determine parameters k_2, k_3. The purpose of it was to increase stability and accuracy of determination. The results of this determination, are presented in Figs. 3, 4.

From the analysis of Figs. 3, 4 we learn that combining the observation data (LAGEOS1 + LAGEOS2, STELLA + STARLETTE) resulted in a faster stability of determining parameters k_2, k_3 and reduction of mean errors in relation to separate determinations for these satellites. Conclusively, the stabilization period of determining parameter k_2 is equal to 7 months for LAGEOS1+LAGEOS2 and 6 months for STELLA + STARLETTE, while for parameter k_3 19 and 16 months, respectively.

Starting from these periods the values of k_2 *and* k_3 obtained after adding to calculations of consecutive orbital arcs changes by an amount not greater than the formal errors for k_2 *and* k_3.

Starting with 22 months for LAGEOS1+LAGEOS2, 20 months for STELLA+STARLETTE for parameter k_2 and 27 months for LAGEOS1+LAGEOS2 for parameter k_3 the obtained values are constant.

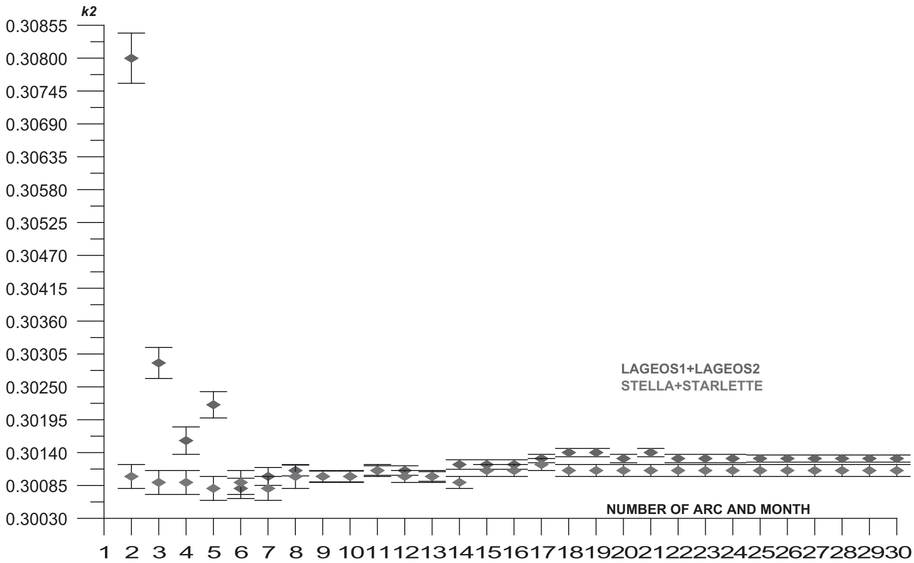

Fig. 3 The sequential solution for the Love number k_2 estimated in the combined analysis based on data for LAGEOS1 + LAGEOS2 and STELLA + STARLETTE. The final value of the k_2 number is equal to 0.30130 ± 0.00010 for LAGEOS1+LAGEOS2 and 0.30111 ± 0.00011 for STELLA + STARLETTE for a 2.5 years' time interval from January 3, 2005 to July 1, 2007

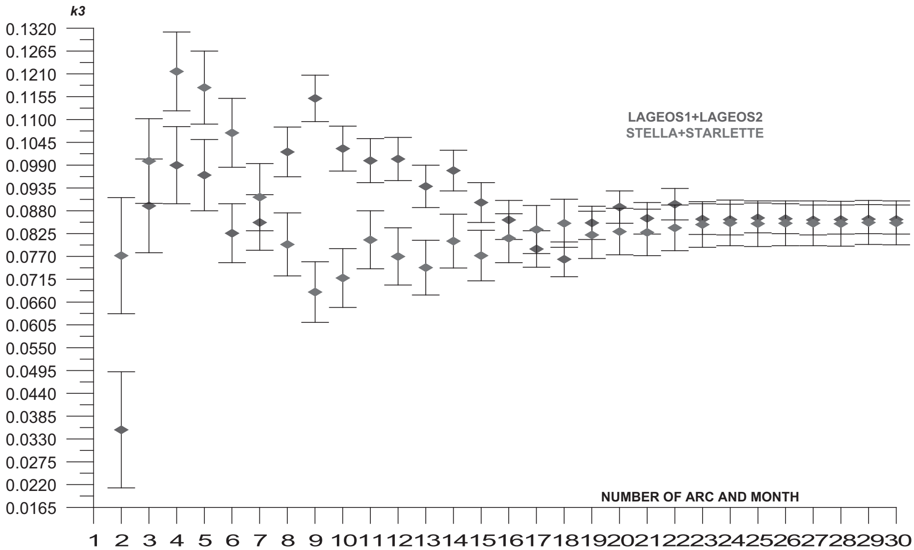

Fig. 4 The sequential solution for the Love number k_3 estimated in the combined analysis based on data for LAGEOS1 + LAGEOS2 and STELLA + STARLETTE. The final value of the Love number is equal to 0.0859 ± 0.0035 for LAGEOS1 + LAGEOS2 and 0.0851 ± 0.0053 for STELLA + STARLETTE for a 2.5 years' time interval from January 3, 2005 to July 1, 2007

Table 3 Values of k_2, k_3 parameters obtained from the SLR data of LAGEOS1, LAGEOS2, STELLA and STARLETTE, conducted in the 2.5 years time interval since initial epoch on January 3, 2005

Love number	LAGEOS1	LAGEOS2	LAGEOS1+ LAGEOS2	STELLA	STARLETTE	STELLA + STARLETTE
k_2	0.30161 ± 0.00011	0.30060 ± 0.00011	0.30130 ± 0.00010	0.30081 ± 0.00019	0.30041 ± 0.00020	0.30111 ± 0.00011
k_3	0.0980 ± 0.0048	0.0835 ± 0.0050	0.0859 ± 0.0035	0.0884 ± 0.0068	0.0858 ± 0.0063	0.0851 ± 0.0053

Table 4 Love numbers k_2, k_3

Author	k_2	k_3
H. Takeuchi (Takeuchi et al. 1962)	0.280	0.083
W. M. Kaula (Kaula 1963)	0.317	0.095
I. M. Longman (Longman 1966)	0.302	0.093
W. E. Farrell (Farrell 1972)		0.094
J. M. Wahr (Wahr 1981)	0.298	
W. Melbourne (Melbourne et al. 1983)	0.300	
V. Dehant (Dehant 1987)	0.2958	
P. M. Mathews (Mathews et al. 1995)	0.2962	
M. Jagoda and M. Rutkowska (LAGEOS1 + LAGEOS2 data)	0.30130 ± 0.00010	0.0859 ± 0.0035
Nominal values recommended in IERS Technical Note No. 36 (Petit and Luzum 2010)	0.29525	0.0930

Analyzing the obtained results as far as accuracy of calculation for both satellite groups is concerned, it is affirmed that the parameters determined based on the observation of LAGEOS1+LAGEOS2 are characterized by smaller mean errors. For this reason they were taken as the final parameters and will be compared to determinations of other authors. When studying the most recent literature it is hardy possible to find any more recent information on determining parameters k_2, k_3; most authors focus rather on parameters h_2, l_2, connected with tidal relocations of Earth and Ocean masses, which was thoroughly scrutinized in (Rutkowska and Jagoda 2010). In Table 4 we present the available determinations of parameters k_2, k_3. They were conducted by using geophysical methods, and those authors do not provide the errors with which parameters k_2, k_3 were determined. The last determination of parameter k_2 is the one made by P.M. Mathews (Mathews et al. 1995). In 1995 they obtained a k_2 value equal to 0.2962. The closest value to ours was obtained in 1983 by W. Melbourne (Melbourne et al. 1983) for parameter k_2 and in 1962 by H. Takeuchi (Takeuchi et al. 1962) for parameter k_3.

A combined multi-satellite solution (LAGEOS1 + LAGEOS2 + STELLA + STARLETTE) is currently under preparation. We expect that the multi-satellite solution will converge more quickly than any single-satellite solution. This multi-satellite solution will be a complete study of the elastic Earth parameter estimation.

4 Conclusion

1. Love numbers k_2, k_3 for the low satellites STELLA and STARLETTE are adjusted with errors greater than for the high satellites LAGEOS1 and LAGEOS2. This is probably caused by the influence of atmospheric drag on low satellite motion. The acceleration of satellite motion caused by atmospheric drag is a function of the atmospheric density model, which changes with time and depends on the modeled values of magnetic index, solar flux and the atmospheric drag coefficients. Due to this, the orbits of low satellites are estimated with errors significantly larger than for high satellites (as shown by the obtained values of RMS residuals). A second reason involves the geometric configuration between stations and the less convenient satellite

positions for low satellites. The larger orbit error of low satellites lead to increased tidal parameters errors.

2. A high internal compatibility of determining parameters k_2, k_3 from observations of high and low satellites can be noticed. However, because of the fact that the parameters determined from the data of the high satellites are characterized by smaller mean errors than those determined from the data of the low satellites, for their determination the high satellites are recommended.
3. As the final parameters the values obtained from 30 orbital arcs of combined data of the satellites LAGEOS1 and LAGEOS2 (LAGEOS1 + LAGEOS2) were assumed. And thus the estimated final Love numbers k_2 and k_3 are equal to 0.30130 ± 0.00010 and 0.0859 ± 0.0035. Stability in estimating k_2 and k_3 parameters can be noticed after passing 7 and 19 months within the assigned 2.5 years' time interval.
4. The 2.5-year' time interval, assumed in this research, seems long enough to obtain stability and convergence of determination of parameters k_2, k_3, which was shown in Figs. 1, 2, 3 and 4.
5. Lower stability of determining parameter k_3 and a bigger error with which the parameter was determined maybe linked to the fact that corrections of geopotential coefficients for tides of the third degree are smaller by an order of magnitude compared to corrections of the second degree. Likewise, the parameter k_3 has a smaller value and is much "harder" to determine than the parameter k_2.

References

Altamimi Z, Collilieux X, Métivier L (2011) ITRF2008: an improved solution of the international terrestrial reference frame. J Geodesy 85(8):457–473

Dehant V (1987) Tidal parameters for an elastic Earth. Phys Earth Planet Inter 49:97–116

Farrell WE (1972) Deformation of the Earth by surface loads. Rev Geophys Space Phys 10:761–797

Hedin AE (1987) MSIS-86 thermosferic model. J Geophys Res 92:4649–4662

Jagoda M, Rutkowska M (2013) Estimation of the Love and Shida numbers: h_2, l_2 using SLR data for the low satellites. Adv Space Res 52:633–638

Kaula WM (1963) Elastic models of the mantle corresponding to variations in the external gravity field. J Geophys Res 68:4967–4978

Lemoine FG, Kenyon SC, Factor JK, Trimmer RG, Pavlis NK, Chinn DS, Cox CM, Klosko SM, Luthcke SB, Torrence MH, Wang YM, Williamson RG, Pavlis EC, Rapp RH, Olson TR (1998) The development of the joint NASA GSFC and the National imagery and mapping agency (NIMA) Geopotential Model EGM96. NASA/TP 1998-206861, NASA Goddard Space Flight Center, Greenbelt

Longman IM (1966) Computation of Love numbers and load deformation coefficients for a model Earth. Geophys J Roy Astron Soc 11:133–137

Mathews PM, Buffett BA, Shapiro II (1995) Love numbers for a rotating spheroidal Earth: new definitions and numerical values. Geophys Res Lett 22:579–582

Maury JL, Brodsky GD (1969) Cowell type numerical integration as applied to satellite orbit computations. Goddard Space Flight Center X-533-69-46, Greenbelt

McCarthy DD, Petit G (2004) IERS Conventions (2003) IERS Technical Note No. 32. Verlag des Bundesamts fur Kartographie und Geodasie, Frankfurt am Main

McCarthy JJ, Rowton S, Moore D, Pavlis DE, Luthcke SB, Tsaoussi LS (1993) GEODYN II System Operation Manual, vol 1–5, STX System Corp. Lanham MD 20706, USA

Melbourne W, Anderle R, Feissel M, King R, McCarthy DD, Smith D, Tapley B, Vincente R (1983) Project MERIT Standards, USNO, Circ. No 167. U.S. Naval Observatory, Washington

Mendes VB, Pavlis EC (2004) High-accuracy zenith delay prediction at optical wavelengths. Geophys Res Lett 31:L14602. doi:10.1029/2004GL020308

Pavlis NK, Holmes SA, Kenyon SC, Factor JK (2008) An earth gravitational model to degree 2160: EGM2008. In: Presented at the 2008 general assembly of the European Geosciences Union, Vienna, 13–18 Apr 2008

Petit G, Luzum B (2010) IERS Conventions (2010), IERS Technical Note No. 36, Verlag des Bundesamts fur KArtographie und Geodasie, Frankfurt an Main

Rutkowska M, Jagoda M (2010) Estimation of the elastic Earth parameters (h_2, l_2) using SLR data. Adv Space Res 46:705–716

Standish EM (1990) The observational basis for JPL's DE200, the planetary ephemerides of the astronomical almanac. Astron Astrophys 233:252–271

Takeuchi H, Saito M, Kobayashi N (1962) Statistical deformations and free oscillations of a model Earth. J Geophys Res 67:1141–1154

Torrence MH, Klosko SM, Christodoulidis DC (1984) The construction and testing of normal points at Goddard Space Flight Center. In: 5th International workshop on laser ranging instrumentation, Herstmonceux, UK, published by Geodetic Institute, University, Bonn, pp 506–511

Wahr JM (1981) Body tides on an elliptical, rotating, elastic and oceanless Earth. Geophys J R Astron Soc 64:677–703