

CASH FLOW ESTIMATION FOR REAL OPTION ANALYSIS USING MARGRABE'S MODEL *

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This paper looks at how the parameters for real option analysis can be extracted from the general capital budget of a project and discusses how the estimated cash flows of a general project can be used for a real option analysis. A project is described where it is possible to stop the business operation in case of predicting a loss for the next year. Our model shows how the cash flow of the period influenced by the option-like decision has to be separated in order to get the exercise price and the price of the underlying asset for real option valuation. Besides providing arguments against the suitability of the general Black–Scholes formula for real option situations, the paper also shows how Margrabe's exchange option valuation formula may be used, and how the volatility as a parameter of this model can be calculated from the data available about the project.

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INTRODUCTION

Real option valuation is based on the valuation methodology for financial options. The quantitative origins of real options derive from the work of Black – Scholes (1973),

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and Merton (1973) in pricing financial options. In addition, Cox – Ross – Rubinstein's (1979) binomial approach enabled a more simplified valuation of options in discrete time, and Margrabe (1978) determined how to value an option to exchange one risky asset for another.

The limitations of *Net Present Value (NPV, or Discounted Cash Flow)* methodology arise from its assumptions. Firstly, it describes an immediate project initiate, a “now-or-never” situation. Secondly, decisions cannot be revised or changed during the life of the project since these potential changes cannot be valued.

Real option analysis includes aspects of financial options and investment opportunities. A number of seminal papers have focused on special cases of real options and provided closed form solutions. The valuation of projects and the decision to invest in case of the existence of sunk costs has been studied by Brennan – Schwartz (1985) and Dixit (1989). Brennan – Schwartz showed the effects of sunk costs on the decision to open and close (temporarily or permanently) a mine, when the price of the resource follows a geometric Brownian motion. Their model accounts for the costs of temporary shutdown, reopening and permanent closure. Brennan – Schwartz obtained an analytical solution for the case on an infinite resource stock. Their solution determined the value of the mine as a function of the resource price and the current state of the mine (i.e. open or closed). It also gave the decision rule for changing the state of the mine. Their work showed how sunk costs of opening and closing a mine may explain that during periods of low prices managers often continue to operate unprofitable mines that had been opened when prices were high, and at other times managers fail to reopen seemingly profitable ones that had been closed when prices were low. This insight is further developed by Dixit (1989).

The valuation process presented in this paper differs from the above since it bases real option analysis on a general cash flow series, where products and prices do not have to hold to specific conditions. To use the estimated cash flows for the real option analysis, we have to look at the cash flows from a somewhat different perspective, and specify them differently. Consequently, the point of our valuation process is not the real option analysis itself, but rather the classification of the cash flows of the project in a way that can be used for the real option valuation.

FUNDAMENTALS OF THE APPROACH

In order to illustrate the value of a decision that is modified later, the basic differences between the traditional *NPV* method and the real option approach are outlined by *Figure 1* below. The figure shows the risky future cash flows of a project by the expected values and the density functions.

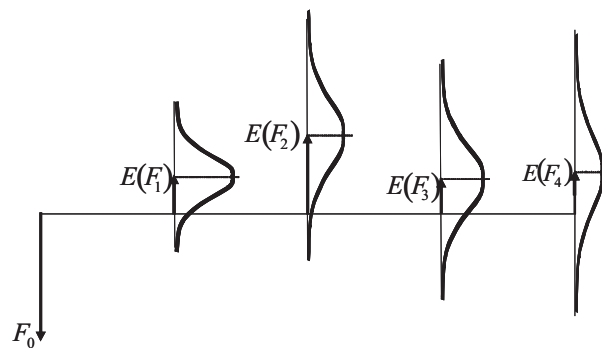


Figure 1. Risky cash flows of a project

The traditional *NPV* can be calculated by the equation below, where r denotes the opportunity cost of capital:

$$NPV = \sum_{n=0}^{\infty} \frac{E(F_n)}{(1+r)^n} = F_0 + \frac{E(F_1)}{(1+r)} + \frac{E(F_2)}{(1+r)^2} + \frac{E(F_3)}{(1+r)^3} + \frac{E(F_4)}{(1+r)^4} \quad (1)$$

In this calculation it is assumed that the future cash flows of the project will have to be undertaken, even, if for example, it is already predictable in the second year that the third and fourth years will probably result in a loss.

Let's assume that there is flexibility attached to the project, such as production may be stopped in any year. In many cases this is not an unrealistic assumption; in fact, it describes the real-life situation. The flexibility criterion may prove to be an important consideration, because it yields the opportunity to modify the initial decision; we can rethink and re-estimate the cash flows after part of the uncertainty connected to the project has disappeared.

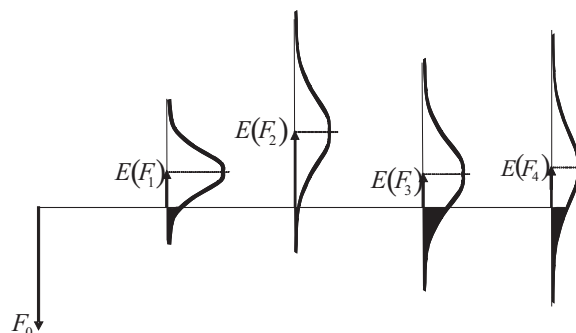


Figure 2. Effects of a later decision on the cash flows

The effect of flexibility can immediately be noted: negative cash flows may be avoided.

Note, however, that the flexibility described above does not simply imply the avoidance of negative cash flows. Even if we do not have to decide a future act (in our case continuing the business in the third and fourth years) in advance, but we may decide it later, there is still some risk left. However, by postponing the decision the risk may be decreased by receiving and utilising new information obtained in the interim.

First examine the expected cash flow in year 4. The expected value and the density function are based on current knowledge and information. Later, for example in year 3, our estimate will have smaller deviation. In year 0 the expected value of the year 3 estimation for year 4 cash flow is not yet known, but the deviation will certainly be smaller. *Figure 3* plots the year 0 estimate for F_4 and three more (A , B and C) possible estimations made in year 3.

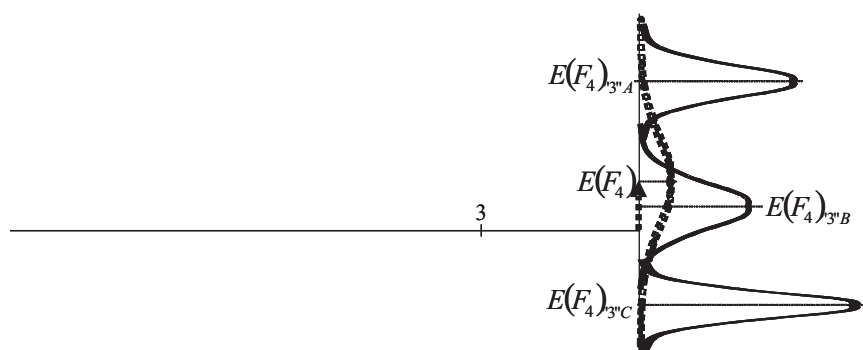


Figure 3. Possible estimations for F_4 in year 3

Although at this moment (in year 0) we do not know what we will think about the outlook for year 4 in year 3, it is expected that we will be able to predict the possible events more accurately than today. If we predict version A we will continue the business, if we predict version C , we will stop the operation and quit the business. If we predict version B for year 4 in year 3, the decision will have to be based on additional examination.

Having to decide at the start of the project, in year 0, about undertaking year 4 cash flow, the quality of our decision can not possibly be as good as it would be in year 3, when the possible loss may be avoided; that is why the later decision choice is better.

The figure also illustrates the difficulty of the question. Being aware at the initiation of the project that we will be in a decision situation in year 3 regarding its

continuation, we may not consider the whole density function, and will predict the negative part to be smaller. In case of a third year decision some risk of loss exists, the shaded part of the density function (on *Figure 2*) cannot be eliminated.

The possibility of reviewing our initial decision in year 3 reduces the negative tail of the density function of F_4 and increases the expected value:

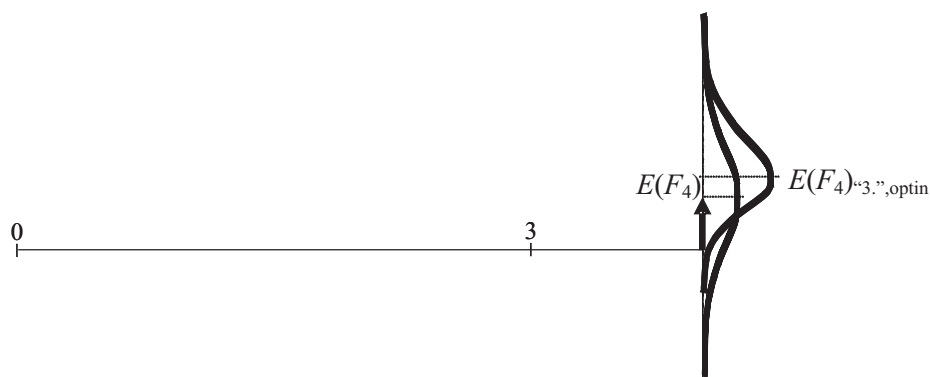


Figure 4. Density function based on our knowledge in year 3

The assumption of flexibility in year 3 increases the expected value of F_4 compared to the original situation. Because this flexibility will not increase the relevant risk (the beta of the CAPM), the flexibility results in higher project value.

The situation can be modelled as a call option for F_4 , $T = 3$ years maturity, so the flexibility can be valued by the option pricing theory.

CLASSIFICATION OF THE CASH FLOWS FOR REAL OPTION ANALYSIS

The next step is to identify the exact parameters of this real option situation and draw the parallel between these parameters and the parameters of financial options, through the disintegration of expected cash flow for year 4 according to option methodology. First, the need to consider the decision at the beginning of year 4 (at the end of year 3) arises; or rather, we have to clearly identify what we are going to decide about. The decision will be whether it is worth paying (or rather, promising to pay) additional expenses for year 4 in order to receive the expected revenues.

The expected cash flow of year 4 $E(F_4)$ can be divided into expected revenues and expected total costs:

$$E(F_4) = E(TR_4) - E(TC_4) \quad (2)$$

The expected total costs $E(TC)$ may be divided into three categories:

1. Fixed costs arising from past decisions. These costs cannot be influenced by the decision at the end of year 3 (at the beginning of year 4) so these are sunk costs. (When having decided about these costs earlier between year 0 and 3, these costs were considered as relevant fixed costs.) Let's denote these costs by $E(iFC_4)$ (expected irrelevant fixed costs).
2. At the end of year 3 (at the beginning of year 4) relevant fixed costs for year 4. We can decide about these costs at the end of year three, but then the decision is definite (for example investment, resources, employee contracts, etc.). Let's denote these costs by $E(rFC_4)$ (expected relevant fixed costs).
3. Variable costs of year 4. We take and pay these costs continuously during the year, and they vary with the quantity of output produced (for example costs of raw materials, energy, etc.). Let's denote the variable costs by $E(VC_4)$.

$$E(TC_4) = E(iFC_4) - E(rFC_4) + E(VC_4) \quad (3)$$

The revenues of year 4 may be divided into the following categories:

1. Revenues arising from earlier decisions. These revenues cannot be influenced at the end of year 3 (for example revenues from earlier sales arriving in year 4, revenues caused by earlier contracts). These revenues are sunk revenues. Let's denote the expected irrelevant revenues by $E(iR_4)$.
2. Revenues arise from the decisions made at the beginning of or during year 4. Let's denote these expected relevant revenues by $E(rR_4)$.

$$E(TR_4) = E(iR_4) + E(rR_4) \quad (4)$$

Various definitions of fixed and sunk costs exist in the prevailing microeconomic textbooks, the classification of total costs presented above is somewhat different from the usual methodology. These costs have to be defined clearly for our examination. In the present analysis the definitions of Andor (2005) were employed for a definition and classification of costs.

Sunk costs have been irrevocably committed. These costs exist by the accountancy; they were paid, or are going to be paid, and cannot be influenced by our decisions. Therefore, sunk costs do not have to be considered in our decision. (For example a wage paid according to contract, or equipment bought earlier, or a rental fee by contract.)

One commonly used definition of fixed costs in widely used microeconomics textbooks is that fixed costs are the "costs that do not vary with the quantity of out-

put produced” (Mankiw 1998). Other authors use fixed costs as a synonym for sunk costs. For instance Varian (1999) defines fixed costs as the “costs that are independent of the level of output, and, in particular, they must be paid whether or not the firm produces output”.

In our terminology two kinds of fixed costs exist: irrelevant fixed costs, which are identical with sunk costs described above, and relevant fixed costs that can be influenced by the project leaders’ decision, but their level is independent from the quantity of production. Varian (1999) defines these costs as quasi fixed costs: “costs that are also independent from the level of output, but only need to be paid if the firm produces a positive amount of output”. Wang – Yang (2001) refer to these costs as avoidable fixed costs.

Fixed and sunk costs result from two different classifications of total costs. Total costs of a given period may either be divided into variable costs and fixed costs, or into avoidable and sunk costs. The relationship between variable costs, fixed costs, avoidable costs, and sunk costs may be summarised as follows: Total costs can be categorised as the sum of sunk costs and avoidable costs, or sunk costs and avoidable (quasi) fixed costs, and (avoidable) variable costs; or fixed and variable costs (Wang – Yang 2001). Avoidable costs include avoidable fixed costs and variable costs. In the long run, there are no sunk costs, but there may easily be fixed costs. In the short run, at least some costs are sunk, and there may still be avoidable fixed costs.

The separation of irrelevant (or sunk) and relevant revenues is a new element of our approach. One general rule of the cash flow estimation is that cash flows have to be estimated on an incremental basis (Brealey – Myers 1996). According to this rule only the cash flows and revenues influenced by the decision can be taken into account. In the situation examined two decision possibilities exist, the initial decision in year 0 and the decision about the production of year 4 at the end of year 3.

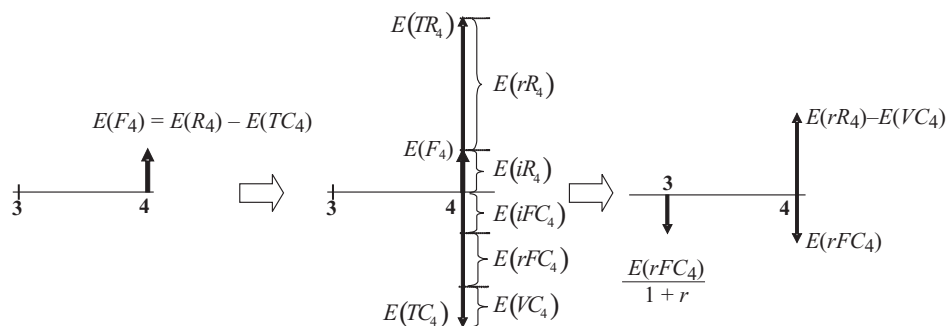


Figure 5. Components of expected cash flow for year 4, and the decision situation at the end of year 3

Following from this, revenues that result from the initial decision are considered irrelevant or sunk revenues while revenues influenced by the decision in year 4 are relevant ones.

It is obvious that irrelevant costs and revenues are not taken into account. We have to consider the cash flow of year 4 without sunk costs $E(rFC_4)$ and revenues $E(iR_4)$ which can not be modified by our decision.

OPTION ANALOGY

Let us now focus on the option analogy. At the end of year 3 we can decide whether to invest the relevant (quasi) fixed cost $E(rFC_4)$ or not. In case of an affirmative answer, the project (the production) will be continued in year 3, in case of a negative answer – apart from the irrelevant (or sunk) effects that result from past decisions – the project will be closed. By the option analogy it is a European style option which can be exercised at the end of year 3. The exercise price of the option is $E(rFC_4)$ or rather its discounted value for the end of year 3. In this approach the cash flows are interpreted for the end of the periods. The key period for the option is year 3, because that is the time when the decision about the exercise is made.

The exercise price of the option can be calculated as:

$$E(K_{T=3}) = \frac{E(rFC_{n=4})}{1+r} \quad (5)$$

The exercise price of traditional financial options is deterministic as it is the part of the option contract. In real (option) situations not only the cash flows of the project but also the future investment value (the exercise price of the option) are volatile, and so they can only be estimated. This means that the Black–Scholes option pricing formula cannot be used for valuing real options, because the formula considers the exercise price as exactly known and discounts its present value by risk free return.

It is possible to get around this problem by modelling real situations as exchange options. An exchange option means that we can exchange one risky asset (K) for another (P). Exchange options can be valued by Margrabe's model.

Margrabe's formula differs from Black–Scholes' formula in two aspects. First, it calculates the volatility in a different way; or rather it takes the volatility of the exercise price into account, as well.

The volatility of an exchange option is:

$$\sigma = \sqrt{\sigma_P^2 + \sigma_K^2 - 2k_{PK}\sigma_P\sigma_K} \quad (6)$$

where k_{PK} is the correlation coefficient between the underlying asset and the exercise price.

The theoretical background for the subjective estimation of correlation is that assuming correlated random variables K and P , the best estimate for P in the case of a known x value of K by the least squares method can be based on the regression line:

$$E(P|K=x) = E(P) + k_{KP} \frac{\sigma_P}{\sigma_K} (x - E(K)) \quad (7)$$

where $E(P|K=x)$ is the conditional expected value of P for $K=x$. The correlation coefficient from the formula above is:

$$k_{KP} = \frac{\sigma_K}{\sigma_P} \frac{E(P|K=x) - E(P)}{E(K) - E(K)} \quad (8)$$

The second difference between simple call options and exchange options is related to the exercise price. In case of a traditional call option, the future value of the exercise price is known, and its present value is to be discounted using the risk free return. In case of a financial exchange option, the present value of the exercise price is known from the spot market price of the risky asset which would have to be paid for the underlying asset in order to exercise the option. The Margrabe formula may be used for valuing exchange options, but in real option situations the exercise price (and its present value) is not known because there is no spot market price of the risky asset as was in financial exchange options. In order to get the present value of the exercise price one needs to estimate the investment cost of the future project, and calculate its present value, discounting the estimated future value by using the opportunity cost of capital (r).

After determining the exercise price of the option the price of the underlying asset of the option needs to be examined. If the decision at the end of year 3 is positive and the option is exercised we pay $E(rFC_4)/(1 + r_{alt})$ (we undertake the relevant fixed costs for year 4 at the beginning of the year) and receive the relevant revenues decreased by the variable costs $E(rR_4) - E(VC_4)$. The underlying asset of the option can be interpreted as a stock that pays a dividend at the end of year 4, and the dividend is the difference between relevant revenues and variable costs. The price of the stock at the end of year 4, and at the end of year 3 is:

$$\begin{aligned} E(P_4) &= E(rR_4) - E(VC_4) \\ E(P_{T=3}) &= \frac{E(rR_4) - E(VC_4)}{1 + r} \end{aligned} \quad (9)$$

The present value of the exercise price (K_0) and the present price of the underlying asset (P_0) are:

$$\begin{aligned} K_0 &= \frac{E(K_{T=3})}{(1+r)^3} \\ P_0 &= \frac{E(P_{T=3})}{(1+r)^3} \end{aligned} \quad (10)$$

The next parameter needed for option valuation is volatility. Volatility has to be calculated according to the Margrabe model since the exercise price (the relevant fixed cost for year 4) is not exactly known today. The volatility of the underlying asset (the stock) (σ_P) is the relative standard deviation for a time period. If we assume a random walk for price, which means the price movements of following time periods are unrelated, the relative standard deviation will increase by the square root of the time:

$$\begin{aligned} \sigma_P &= \frac{\sigma(P_1)}{E(P_1)} \\ \sigma_{P,T} &= \sigma_P \sqrt{T} \end{aligned} \quad (11)$$

where $\sigma_{P,T}$ is the relative standard deviation of the underlying asset for time T .

The volatility of the exercise price can be written as:

$$\begin{aligned} \sigma_K &= \frac{\sigma(K_1)}{E(K_1)} \\ \sigma_{K,T} &= \sigma_K \sqrt{T} \end{aligned} \quad (12)$$

In real option situations we have estimates for later values (for longer periods) and the volatility (the relative standard deviation for a time period) can be calculated from these data:

$$\begin{aligned} \sigma_P &= \sigma_{P,T} \frac{1}{\sqrt{T}} \\ \sigma_K &= \sigma_{K,T} \frac{1}{\sqrt{T}} \end{aligned} \quad (13)$$

In the modelled situation, the cash flows were estimated for the year after the decision. In the example we calculated the cash flows of year 4, but the option can

be exercised at the end of year 3. So from an option valuation view it is possible to predict the cash flows for time $T + 1$, and the volatility can be calculated as:

$$\begin{aligned}\sigma_P &= \sigma_{P,T+1} \frac{1}{\sqrt{T+1}} \\ \sigma_K &= \sigma_{K,T+1} \frac{1}{\sqrt{T+1}}\end{aligned}\quad (14)$$

The volatility using the Margrabe model is:

$$\begin{aligned}\sigma &= \sqrt{\sigma_P^2 + \sigma_K^2 - 2k_{PK}\sigma_P\sigma_K} \\ \sigma &= \sqrt{\sigma_{P,T+1}^2 \frac{1}{T+1} + \sigma_{K,T+1}^2 \frac{1}{T+1} - 2k_{PK} \frac{1}{T+1} \sigma_{P,T+1} \sigma_{K,T+1}} \\ \sigma &= \sqrt{\frac{1}{T+1} \sqrt{\sigma_{P,T+1}^2 + \sigma_{K,T+1}^2 - 2k_{PK}\sigma_{P,T+1}\sigma_{K,T+1}}}\end{aligned}\quad (15)$$

After determining the parameters the option may be valued for the relevant part of the expected cash flow for year 4 ($E(F_4)$). Let us denote this option value by $c_{4,T=3,n=0}$, where index 4 means it is for the cash flows of year 4, $T = 3$ shows the maturity of the option that we can decide to exercise the option at the end of year 3, and $n = 0$ denotes the time the option value is given for:

$$c_{4,T=3,n=0} = f(K_0, P_0, T, \sigma) \quad (16)$$

The value of the project can now be calculated as:

$$NPV_{option} = F_0 + \frac{E(F_1)}{(1+r)} + \frac{E(F_2)}{(1+r)^2} + \frac{E(F_3)}{(1+r)^3} + \frac{E(iR_4) - E(iFC_4)}{(1+r)^4} + c_{4,T=3,n=0}. \quad (17)$$

Although the initial decision on the project modelled and valued has to be made in year 0, this decision is not definite for the entire duration of the project, since there is an additional decision possibility at the end of year 3 for the project heads' to evaluate and make their minds up whether to continue the business or not. Thus, there is some flexibility to the project. The decision at the end of year 3 can influence only a part of the cash flow of year 4, since some sunk revenues and costs result from past decisions and cannot be changed. The model presented incorporates these critical elements.

CONCLUSION

The paper examined how the parameters for real option analysis can be extracted from the general capital budget of a project. The classification of cash flows of a project is a crucial step for the real option analysis. Various definitions for fixed and sunk costs exist in prevailing microeconomics literature which result from different classifications of total costs. The paper presented the relationship between variable costs, fixed costs, avoidable costs, and sunk costs and complemented it by the separation of irrelevant (or sunk) and relevant revenues.

The authors provided a real option model based on Margrabe's exchange option valuation formula and a project cash flow classification from which parameters of the valuation formula (including exercise price, price of the underlying asset and volatility) can be extracted. Real option valuation has generally been based on the valuation methodology for financial options. For special cases of real option situations numerous papers provided closed form solutions. The valuation process presented here differs from these since the real option analysis was based on a general cash flow series, where the products and prices do not have to hold to specific conditions.

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