

DUCTILITY OF STEEL STRUCTURES: THE MODEL OF INTERACTIVE HINGE

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Abstract: Before the 1960s the ductility notion was used only for characterizing the material behaviour. After Baker's research on plastic design, this concept has been extended to the structural level. This paper provides a general overview of ductility. The paper examines the behaviour of material ductility, cross-section ductility and member ductility separately, then it studies how the sum of the effects of all these ductilities affect the structural ductility. The effect of the different ductilities can be built into the structural behaviour through an interactive hinge model. The model of the interactive hinge also takes into consideration the effects of the residual stresses and deformations, the strain hardening and the plate buckling.

Keywords: Ductility, Steel frame, Interactive hinge, Gable frame structure, Multi-storey frame structure

1. Introduction

1.1. Limit state design philosophy

The increasingly powerful experimental and computational tools of structural design require well-defined design philosophies. As the basis of the current design philosophy the concept of limit states [1] is seemingly accepted in many countries. The current design philosophy requires the estimation of the (fairly small) risk that the given structure is brought to its ultimate state (failure) and the (somewhat bigger) risk of the occurrence of a phenomenon restricting its regular use (serviceability). All this (excluding now brittle fracture and fatigue) necessitates the analysis of the structural

response over a broad range of load levels, from working loads up to exceptionally high loads.

As it is pointed out in the literature [2], in different periods of the engineering practice different importance was attributed to the two classes of limit states. In the earliest periods (e.g. in the works of Coulomb) – possibly inspired by the experience of collapsing vaults and breakdown of earthworks – interest was focused on the ultimate state and accordingly the applied methods of analysis could describe only the last phase of structural response. This methodology is still in use, for example in some branches of soil mechanics.

A second period can be connected with the activity of the brilliant French scientist Navier, who seems to have been more interested in the second class of limit states. Quoting the preface of his 1826 book, which is of enormous influence on engineering practice [3]: “Knowing the cohesion, the ultimate load to be carried by a body can be determined. For the structural engineer, however, it is not sufficient to know the force great enough to cause breakdown of the body, but rather the load to be carried by the structure without causing in it changes progressing with time”. The concept of allowable stresses and the corresponding methods of analysis originates from this viewpoint: the “accurate” and simplified methods based on the theory of elasticity which were good enough to describe structural response at relatively low (working) load levels.

The subsequent period is associated with the work of the Hungarian scientist Kazinczy, who is regarded as the initiator of plastic design of steel structures. In his early – and because of its language hardly accessible – paper (1914) on his tests with fixed-end beams he states [4]: “In the case of statically indeterminate steel structures there is a higher load than the load that is determined by the allowable stress (first yielding) and therefore the lower load value gives no information whatsoever about the margin of safety”. This indicates that the main interest was shifting again towards the first class of limit states, towards failure. Accordingly research was directed to complement the methods of analysis with new ones (based on the theory of plasticity among others) describing structural behaviour in the vicinity of and at the peak load, and often in post-failure phase as well.

Thus the recent concept of limit states can be regarded as a balanced synthesis of the previous design philosophies complemented with the ductility phenomenon.

1.2. Leading role of ductility in limit state design

1.2.1. The definition of ductility

Before the 1960s the ductility notion was used only for characterizing the material behaviour. After the Housner's studies of earthquake problems [5] and Baker's research works on plastic design [6], this concept has been extended to the structural level. Following Baker's work, in this paper the effect of the quasi-monotonic static loading system will be examined.

In the practice of plastic design of structures, ductility defines the ability of a structure to undergo deformations after its initial yield, without any significant

reduction in the ultimate strength. The ductility of a structure allows us to predict the ultimate capacity of a structure, which is the most important criteria for designing structures under conventional loads.

The following ductility types are widely used in the literature [7]:

- material ductility, or deformation ductility, which characterizes the material plastic deformations for different loading types;
- cross-section ductility, or curvature ductility, which refers to the plastic deformations of the cross-section, considering the interaction between the parts composing the cross-section itself;
- member ductility, or rotation curvature, when the properties of members are considered;
- structural ductility, or displacement ductility, which consider the overall behaviour of the structure.

A correlation among these types of ductility also exists.

1.2.2. Ductility for plastic design

The plastic behaviour of a structure depends upon moment redistribution. The attainment of the predicted collapse load is related to the position of plastic hinges, where sections reach the full plastic moment, and to the plastic rotation which other hinges can develop elsewhere. Hence, a good behaviour of a plastic hinge requires a certain amount of ductility, in addition to its strength requirement. The plastic rotation capacity is the more rational measure of this ductility.

The basic requirement for plastic analysis of statically indeterminate structures is that large rotations (theoretically infinite) are possible without significant changes in the resistant moment. But these theoretical large plastic rotations may not be achieved because some secondary effects occur. The plastic rotation is usually limited by flexural-torsional instability, local buckling or brittle fracture of members. Due to the reduction of the plastic rotation, *cross-section* behavioural classes are used in the design practice (Fig. 1a). EUROCODE 3 defines the following classes:

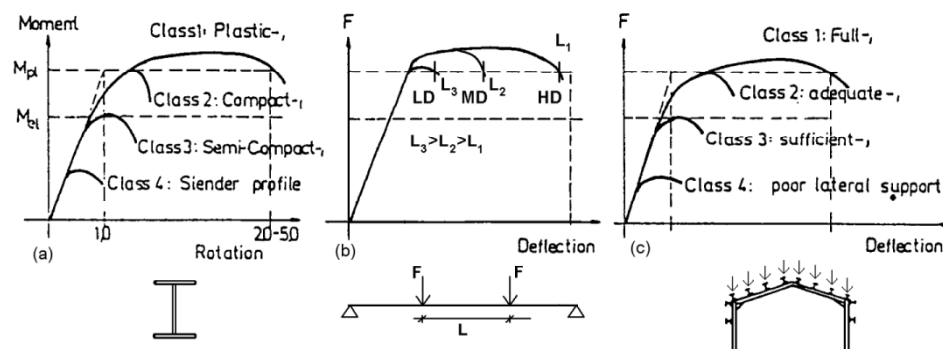


Fig. 1. Cross sectional, member and frame behaviour classes

- class 1 (plastic sections); sections belonging to class 1 are characterized by the capability to develop a plastic hinge with high rotation capacity;
- class 2 (compact sections); class 2 sections are able to provide their maximum plastic flexural strength, but they have a limited rotation capacity, due to some local effects;
- class 3 (semi-compact sections); sections fall into this class when the bending moment capacity for the first yielding can be achieved without reaching the plastic moment;
- class 4 (slender sections); sections belonging to this class are not able to develop their total flexural resistance due to the premature occurrence of local buckling in the compressed parts.

Evidently, only the first two classes have sufficient ductility to assure the plastic redistribution of moments. This classification is limited to the cross-section level only, so it has many deficiencies.

Another more effective classification at the level of a *member* has been proposed by Galambos and Lay [8] as shown in *Fig. 1b*:

- ductility class HD (high ductility) corresponds to a member for which the design, dimensioning and detailing provisions are such that they ensure the development of large plastic rotations;
- ductility class MD (medium ductility) corresponds to a member designed, dimensioned and detailed to assure moderate plastic rotations;
- ductility class LD (low ductility) corresponds to a member designed and dimensioned according to general code rules which assures low plastic rotations only.

An effective classification at the level of a *frame* considering the spacing and efficiency of lateral support has been proposed by Ivanyi [9] as shown in *Fig. 1c*:

- ductility class 1: *full support* corresponds to a frame for which the design, dimensioning and detailing provisions are such that they ensure the development of the plastic carrying capacity with large plastic rotations;
- ductility class 2: *adequate support* that ensures the development of the plastic carrying capacity without plastic rotations;
- ductility class 3: *sufficient support*, which ensures the development of the elastic carrying capacity;
- ductility class 4: *poor lateral support* under the elastic carrying capacity.

1.3. Difficulties in predicting failure

In contrast to the expectation of the initiators of plastic design, the analysis of structural response in the vicinity of the peak load proved to be extremely complicated, due not only (and even not mainly) to the inelastic behaviour, but to the fact that in the vicinity of the peak load

- changes in the geometry (geometrical non-linearity) gain importance over other factors because the effect of initial geometrical imperfections (often negligible at lower load levels) is magnified;
- residual stresses (remaining latent at lower loads) interact with growing active stresses resulting in premature plastic zones; and last but not least;
- the usual and widely accepted tools of analysis, e.g. the beam theory based on the Bernoulli–Navier theorem or the small-deflection theory of plates, restricting the actual degree of freedom of the structure, cannot describe exactly the real response of the structure at failure.

These difficulties can be overcome in the case of simple structural elements (separated compression members, parts of plate girders, etc.) by using more refined methods, for example the finite element method or considering degrees of freedom (e.g. distortion of cross-sections) that are excluded from traditional analysis. The difficulties can also be avoided in the case of statically determinant structures, where the above-indicated complex behaviour is usually confined to a limited section of the whole structure.

If the simplified model is not elaborate enough to reflect real structural behaviour, a secondary, more detailed, local model or “*target*” model is inserted to depict the mostly critical part of the structure. In this way more realistic quality parameters (and limit surface) can be deduced from the primary parameters that are already known [10].

Because of the interaction of the local and the global behaviour, there is a larger problem in the case of hyper-static structures, as the additional information gained by the secondary local model cannot be fed back to the computation of the primary parameters. Furthermore, if – as happens very often – the secondary model can be analysed by numerical methods or only experimentally, but the results have to be either re-interpreted to gain mathematically treatable, sufficiently simple rules, or the secondary model has to be simplified to furnish digestible results. In both cases the validity or accuracy has to be proved by failure tests with full-scale structures (which are usually very expensive). The same difficulties apply to quantities which cannot be measured directly or which are hardly measurable, such as residual stresses.

1.4. The role of the softening phenomenon

When studying the effect of the softening phenomenon it should be kept in mind that the load–displacement diagram of the structure may be of an ascending type even if the given member section or semi-rigid connections are of a descending type.

In the theory of plasticity, when deriving the condition of plasticity or some other physical relationships, Drucker’s postulate for stability is applied, by assuming stable materials [11].

It should be noted that Drucker’s postulate is not a natural law but a criterion for classification [12]. Materials very often do not correspond to the assumptions of stable materials, or structural elements may behave in an unstable way, while at the same time their material is of a stable state.

Maier [13] was one of the first to treat the problem of the effect of the unstable state of certain members when he studied the behaviour of a triangular shaped structure.

Later it was Maier again who in 1966 re-introduced the subject and investigated a structure consisting of compressed members and rigid beams where load–displacement diagram of individual members contained stable and unstable parts.

Maier and Drucker [14] re-examined the original Drucker's postulate since the original postulate is suitable for the determination of the convexity and normality of the condition of plasticity in the case of stable materials only.

When studying the load bearing capacity of steel structures, the problem of unstable material or softening material, according to Drucker's postulate does not appear since the strain–hardening of the steel material may increase in a major way over the plastic load bearing capacity of steel structure. However, as it has been known for a long time, the final collapse of steel structures is caused – in a high percentage of cases – by instability (plate buckling, flexural–torsional buckling), or semi-rigid phenomena that may occur in the cross section or in a structural joint (*Fig. 2*).

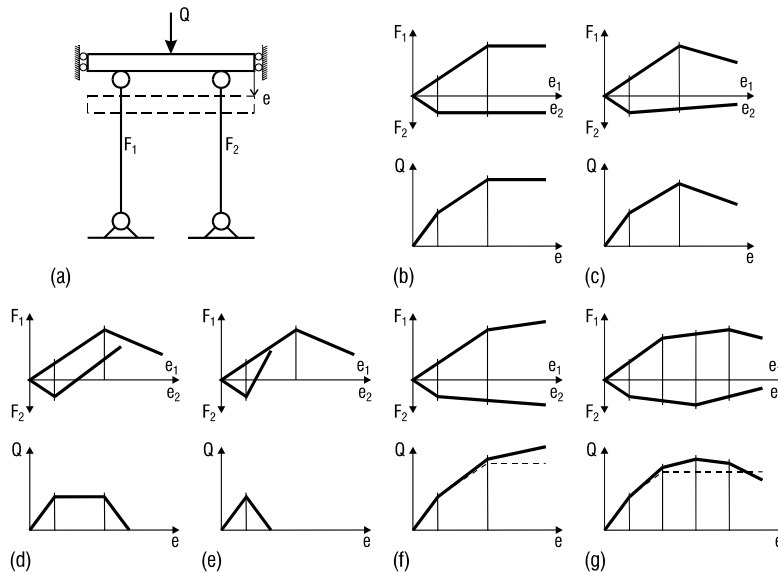


Fig. 2. Behaviour of a simple structure

Concerning steel structures the properties of plastic hinges over and above the usual elastic–ideally plastic–hardening behaviour may be complemented with the effect of instability (flexural–torsional buckling) developing in the given structural unit (environment of the plastic hinge), or in the periphery of semi-rigid connections.

This type of inelastic or interactive hinge describes the behaviour of the structural unit and at the same time, also satisfies the criteria of unstable or softening structural unit, according to the Maier–Drucker's postulate.

When determining the plastic load bearing capacity of steel structures the softening has not been considered or applied so far. The effects of the stability phenomena

causing the softening character (flexural–torsional buckling, plate buckling, semi–rigid connections) can be taken into account indirectly with the aid of construction rules. In principle, mathematical programming allows the investigation of more complex steel structures too, however, it is less suitable for designing practice. Ivanyi [15] has suggested a procedure that takes into account the softening character of the inelastic hinge in the form of an interactive zone. The softening character of the interactive zone is caused by the buckling of the component plates, a phenomenon that can be studied with the help of the yield mechanism.

The purpose of this article is to describe 1) how the load–deflection curve of frames may be constructed in as exact a manner as possible and 2) to describe approximate methods whereby the load–deflection curve, and particularly the limit point load, can be estimated with the softening phenomenon.

2. Investigation of plate buckling with the aid of yield mechanism

In the course of plate experiments, if the thickness/width ratio is small the plate does not lose its load–bearing capacity with the development of plastic deformations but is able to take further (small increase in) load up to the point where the deformation capability is exhausted. In the course of the loading process “crumplings” (buckling) can be observed on the plate surface. These “crumplings” are formed by a yield mechanism, with the plastic moments acting in the plastic hinges (peaks of waves) not constant but ever increasing due to strain–hardening. The yield mechanism performed by “crumplings” can be extended to the component plates of the bar. The description of the behaviour of the yield mechanism is obtained from the extreme–value theorems of plasticity with the aid of the theorem of kinematics.

Thus, in the course of our investigations, an upper limit of load bearing capacity can be determined. However, to be able to assess the results, the following have to be considered: the yield mechanisms are taken into account through the “crumpling” forms determined experimentally; and on the other hand, the results of the theoretical investigations are compared with the experimental ones.

2.1. Yield mechanism forms based on experimental results [16, 17]

The different forms of yield mechanisms can be determined on the basis of experimental results. The yield mechanism forms of an I–section bar can be classified according to the following criteria.

- a) According to the way of loading;
- b) According to the positions of the intersecting lines of the web and the flanges, the so–called “throat–lines”; thus;
 - i) the evolving formation is called a *planar yield mechanism* if the two “throat–lines” are in the same plane after the development of the yield mechanism;

- ii) the evolving formation is called a *spatial yield mechanism* if the two “throat-lines” are not in the same plane after the development of the yield mechanism.

2.1.1. Constant bending moment along the bar axis

a) *Planar yield mechanism*: The buckled form of the bent specimen and the chosen yield mechanism formation are shown in Fig. 3a. As an effect of moment M , a rotation θ develops furthermore tension and compression regions appear. The symbol of the yield mechanism is $(MC)_p$, where C stands for the constant bending moment.

b) *Spatial yield mechanism*: The form of the spatial yield mechanism in the case of a bent beam is shown in Fig. 3b. The beam ends are assumed to be hinge-supported in both main inertial directions. The yield mechanism models the buckling of the component plates of the bent members, the lateral buckling of the beams as well as their interaction. The symbol of this yield mechanism is $(MC)_s$.

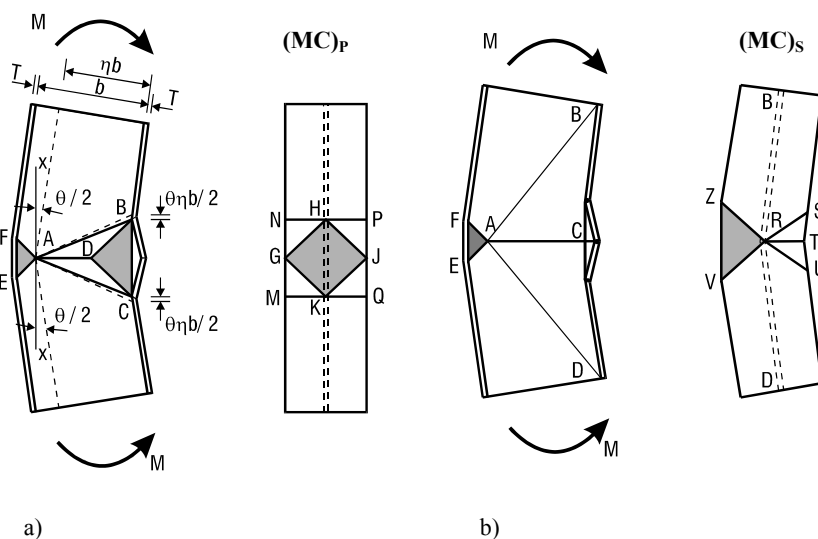


Fig. 3. Planar a) and spatial b) yield mechanisms of an I-beam with constant bending moment along the member axis

2.1.2. Varying bending moment along the bar axis

In the case of a varying bending moment along the member axis, it is assumed that the “crumplings” of the web plate of the I-section in the cross-section of the concentrated force is hindered by the thickness of the web plate or by the ribs.

Climenhaga and Johnson [18] assumed yield mechanism forms for the steel beam part of a composite steel-concrete construction, which are similar to those introduced in this paragraph.

a) *Planar yield mechanism*: The buckled form of a bent specimen and the selected yield mechanism are shown in Fig. 4a. As an effect of the moment, a rotation θ develops and because of the clamping of the cross-section EC, the yield mechanism loses its symmetric character. The symbol of the yield mechanism is $(MV)_P$ where V stands for the varying moment.

b) *Spatial yield mechanism*: The form of the spatial yield mechanism in the case of a varying bending moment along the beam axis is shown in Fig. 4b. As an effect of the moment, a rotation θ develops. The symbol of the yield mechanism is $(MV)_S$.

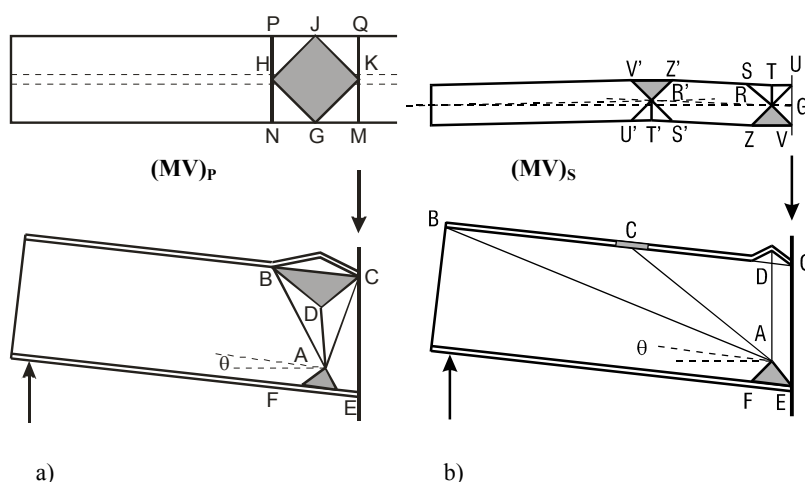


Fig. 4. Planar a) and spatial b) yield mechanism of an I-beam with varying bending moment along the member axis

2.1.3. Yield mechanism of the component plates of an I-section member

Yield mechanism formations have been determined for different stresses. On the basis of the experimental results it is expedient to decompose these yield mechanism formations into the yield mechanism formations of the component plates of an I-section beam, as certain yield formations are common considering all yield mechanisms.

Fig. 5 shows the yield mechanisms of the component plates where F is the flange plate, W is the web plate; the odd numbers refer to the planar yield mechanisms and the even numbers to the spatial yield mechanisms.

To classify the yield mechanisms of component plates, the following groups has been identified:

- a) Flange plate, if the plate is supported along one line;
- b) Web plate, if the plate is supported at the unloaded ends;
 - bi) axial forces and bending ($W-1$)–($W-6$);
 - bii) transverse forces transmitted directly through the web ($W-11-12-13$);
 - bii) transverse forces only on one side of the web panel ($W-21-22-23$);
 - biii) tension fields on the web panel ($W-30$; $W-40$).

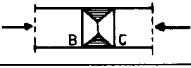
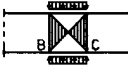
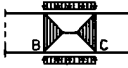
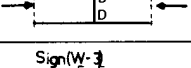
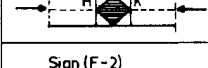
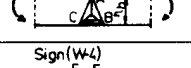
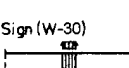
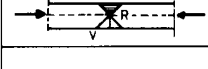
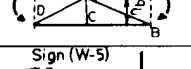
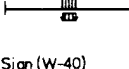
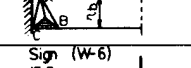
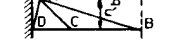
Flange plate yield mechanism		Web plate yield mechanism			
Tension flange plate	Axial forces and bending	Transverse forces			
Sign (F- H) 	Sign (W-1) 	Sign (W-11) 	Sign (W-12) 	Sign (W-13) 	
Compressed flange plate	Sign (W-2) 	Sign (W-21) 	Sign (W-22) 	Sign (W-23) 	
Sign (F-1) 	Sign (W-3) 	Sign (W-30) 			
Sign (F-2) 	Sign (W-4) 	Sign (W-40) 			
	Sign (W-5) 				
	Sign (W-6) 				

Fig. 5. Yield mechanisms of the component plate elements of an I-section member and beam-to-column joints

2.1.4. Yield mechanism of joint configurations

Yield Mechanism of Single-Sided Joint Configurations. The main sources of deformability of joint configurations which must be contemplated in a beam-to-column joint (Fig. 6a) are:

- the connection deformability $M_b-\theta_c$ characteristic;
- the column web panel shear deformability $V_{wp}-\gamma$ characteristic;
- the local buckling of the column web panel.

In the case of the yield mechanism formations shown in Fig. 6a, the effect of the local buckling of the beam cross-section, the local yield of the column web panel caused by shear and the effect of patch loading has also been taken into account. The symbol of this yield mechanism is (SSJ)_p.

Yield Mechanism of Double-Sided Joint Configurations. The main sources of deformability of joint configuration which must be contemplated in a beam-to-column joint (Fig. 6b) are:

- the left hand side connection deformability $M_{b1}-\theta_{c1}$ characteristic;
- the right hand side connection deformability $M_{b2}-\theta_{c2}$ characteristic;
- the column web panel shear deformability $V_{wp}-\gamma$ characteristic;
- the local buckling of the column web panel.

The yield mechanism formation is shown in *Fig. 6b*. The symbol of this yield mechanism is $(DSJ)_p$.

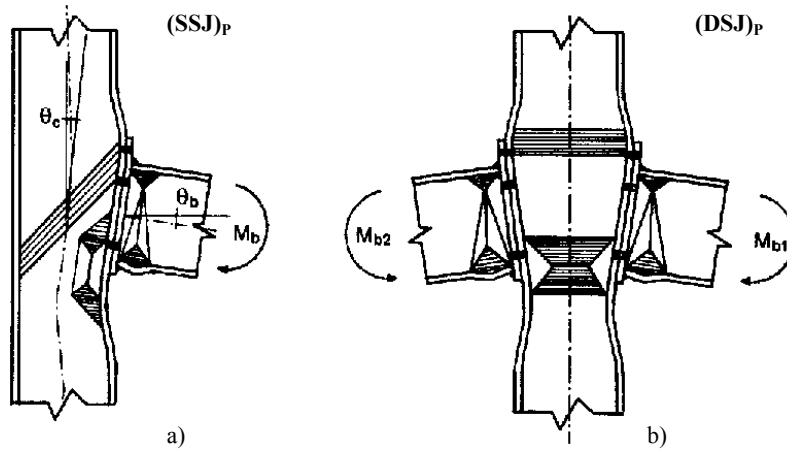


Fig. 6. Single-sided a), and double-sided b) joint configuration

2.1.5. “Joining” the yield mechanisms of component plates

It can be noticed that the joint configurations shown in *Fig. 6* can also be constructed from the yield mechanisms of the component plates. The process of selecting the appropriate yield mechanisms based on experimental results is called “joining”. The “joining” process depends on the current positions of the “throat-lines” compared to their initial position. The relationships between the component plate yield mechanisms and the “joining” of the component plates have been given by Ivanyi [16] who gave the basic relationships of partial cases [19, 20, 21] and Gioncu and Petcu [22] prepared a software to calculate the moment–rotation curve.

2.2. Model of the interactive hinge

The plastic load–bearing investigation assumes the use of rigid–ideally plastic hinges. This hinge describes the inelastic behaviour of steel structures but with major constraints and approximations. However another type of hinge can be developed which considers some further effects and in this way the behaviour of the steel material, the I–section members and the beam–to–column joints can be modelled in a more realistic way. The characteristic curve of the hinge is composed of the following sections (*Fig. 7c*):

- the so–called “rigid” state which is denoted by symbol (R);
- the effect of residual stress and deformation is characterised by a straight line for ease of handling. The symbol used when taking the residual stress and deformation into consideration is (O);

- strain-hardening is one of the important features of the steel material, therefore the characteristic curve contains a section denoted by (S);
- the effect of “crumpling” of different plate elements is taken into account by a section indicated by (L).

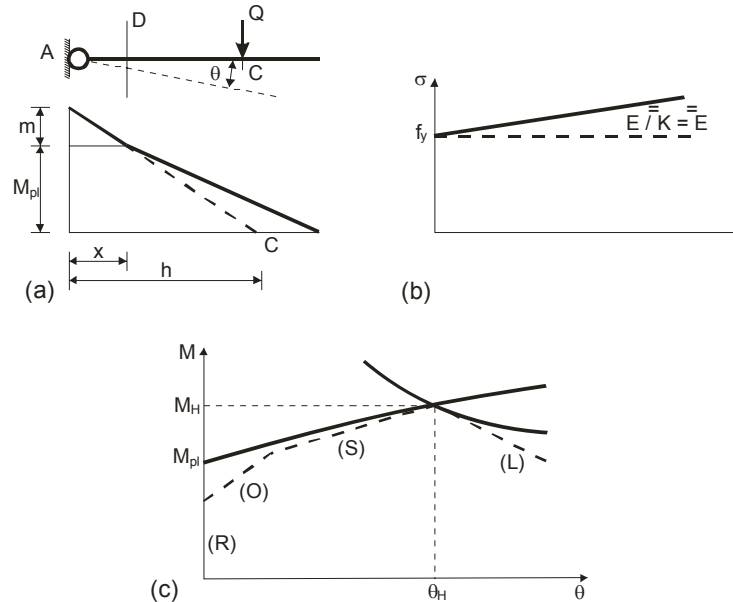


Fig. 7. Model of interactive hinge

The models that take the above effects into consideration in the investigation of load–displacement (relative displacement) relationships are called “interactive” models.

Some alternative empirical models, such as the exponential model [23], the Ramberg–Osgood model [24] and the Richard–Abbott model [25, 26] have been widely used to represent the nonlinear moment–rotation of members and semi-rigid connections. The ability of the Richard–Abbott function to model characteristics, as shown in Fig. 8, which exhibit strain hardening (a), strain softening (b), as well as strain stiffening (c) behaviours, makes it very useful in the analysis of steel structures however the present study introduces the piecewise linear model to describe the moment rotation.

The model of the interactive hinge taking into consideration the effect of the rigid state, the residual stress, the strain-hardening and the plate crumplings can be described with the aid of the “equivalent beam length” suggested by Horne [27] (Fig. 7a). The material model employed in the investigations is shown in Fig. 7b. The effect of the residual stresses and deformations is substituted by a straight line. The effect of strain-hardening can be determined with the help of the rigid-hardening (R–S) model. The buckling of the I-section member component plates is described by the yield mechanism curve, which is substituted by a straight line.

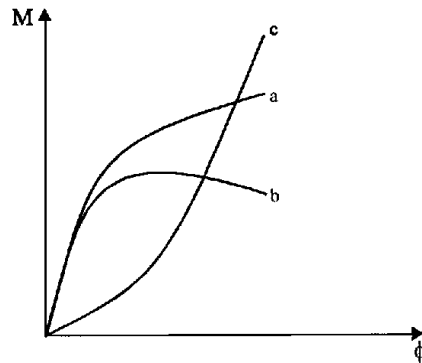


Fig. 8. Richard-Abbott curve shapes

Fig. 7c indicates the load-displacement relationship of the (R-O-S-L) interactive hinge. The substitution by straight lines is justified to simplify the investigations. In the (R-O-S) sections the intersections are connected while in section (L) the moment-rotation relationship is substituted by a tangent line that can be drawn at the apex.

3. Analysis of steel frames with global bar elements

3.1. Matrix method

Matrix methods are available to determine the stresses in plane bar systems by computers [28]. These methods are relying either on the force or on the displacement method. The displacement method has been used in the present study.

Simpler cases involve the bar element shown in Fig. 9a permitting fast, easy computation mainly using an elastic material model. The stiffness matrix, K is common knowledge; stiffness values are obtained by solving basic problems of hyperstatic beams.

Our goal seemed to be better achieved by applying a complex bar element as shown in Fig. 9b.

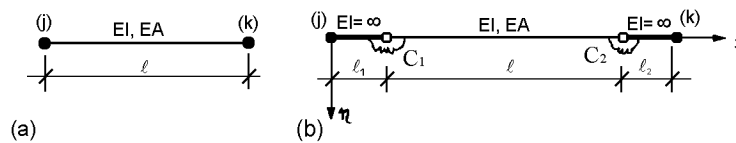


Fig. 9. Simple (a) and complex (b) bar element

The two end parts of the bar, of lengths l_1 and l_2 , are infinitely rigid (maybe $l_1 = l_2 = 0$); the middle part is elastic. Rigid and elastic parts of the bar are connected by rotational springs. Each of the springs is able to rotate only in the plane of the structure. Stiffnesses, i.e. spring constants are c_1 and c_2 [29].

Details of the method that gives the connection between the unit end deformations and the relevant stress resultant can be found in the literature [17].

Spring characteristics have the general form as shown in *Fig. 7* and it models an interactive hinge. Sections have different spring constants $c = \Delta M / \Delta \theta$ indicating the given section of the elasto-plastic behaviour or of the stability condition of the bar. The characteristics are strictly monotonous for θ but not for M . Namely there is a peak followed by a descending path of the curve.

3.2. Simple approximate method

Numerous approximate engineering methods are introduced in the literature [30], from which as one of the possibilities we are going to deal with the extension of the Mechanism Curve Method. The Mechanism Curve Method – besides the determination of the plastic load bearing capacity – can take into consideration the effect of finite deformations and strain hardening of steel.

3.2.1. Mechanism curve method

Horne [27] proposed the use of the simple rigid-plastic-rigid relationship in order to take into account the effect of strain-hardening on the collapse load of a structure.

The change of the geometry due to elasto-plastic deformations tends to decrease the ultimate load bearing capacity of steel frames in comparison with the plastic collapse load. This tendency is counteracted by the strain-hardening properties of steel. The rigid-plastic-rigid theory of structural behaviour is found to be an adequate mean to assess the stiffness of a structure immediately when the last hinge is formed in a plastic hinge mechanism.

Different strain-hardening theories can be used during the analysis (*Fig. 10*):

- rigid-plastic-rigid (RPR) model [27];
- rigid-plastic-hardening (RPH) model [31];
- rigid-hardening (RH) model.

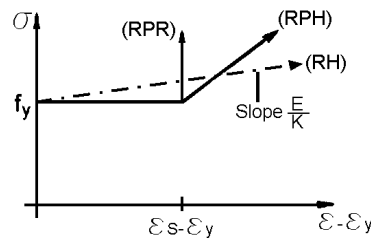


Fig. 10. Strain-hardening models

This study uses the rigid-hardening (RH) hinge model and in this case the rigid-plastic work equation [16] becomes

$$\lambda \left(\sum_i Q_i u_i + \sum_k N_k L_k \phi_k^2 \right) = \sum_j (M_{pj} + m_j) \theta_j. \quad (1)$$

3.2.2. Approximate engineering method to take into consideration the effect of plate buckling [16]

We extend the category of hardening plastic (RH) hinges by taking the effect of plate buckling into consideration. Such hinge model can be the basis of an Approximate Engineering Method that – without analysing the full load history – with simple methods can directly take the effect of plate buckling into consideration.

Fig. 7c shows the linear interaction of moment-rotation of the interactive hinge that contains the effects of strain-hardening and plate buckling. The essence of the Approximate Engineering Method is that the two effects are separated and the interactive hinge of the structure is put together from two separate components as shown in Fig. 11:

- Strain-hardening component: (S);
- Plate buckling component: (L).

With the assumed two hinge components the values of the load parameter for the chosen mechanism of the framework can be determined as a function of finite deformations.

The strain-hardening component of Eq. (1) is

$$\lambda_{(S)} (\sum Q u + \sum N L \phi^2) = \sum M_p \theta + \sum m_S \theta, \quad (2)$$

$$\lambda_{(S)} = \frac{\sum M_p \theta + \sum m_S \theta}{\sum Q u + \sum N L \phi^2}. \quad (3)$$

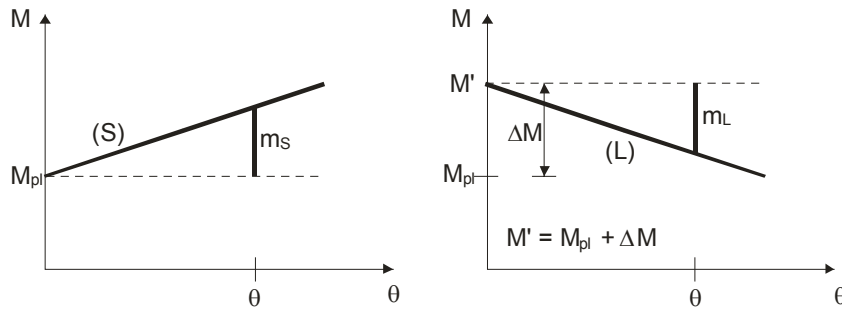


Fig. 11. Separated components of interactive hinge

To write down Eq. (1) for the plate-buckling component it should be assumed that the interactive hinge characteristic curve contains the rigid and the plate buckling effects.

The rigid curve section goes up to the value of $M' = M_p + \Delta M$ first, then a linearly decreasing change is taken into consideration due to the effect of plate buckling. Because of the section of the characteristic curve belonging to the plate-buckling component, external and internal capacities and works are written similarly to Eq. (2), except the sign of the increment $m \theta$

$$\lambda_{(L)} (\sum Qu + \sum NL \phi^2) = \sum M' \theta - \sum m_L \theta, \quad (4)$$

$$\lambda_{(L)} = \frac{\sum M' \theta - \sum m_L \theta}{\sum Qu + \sum NL \phi^2}. \quad (5)$$

Load parameter $\lambda_{(S)}$ takes into consideration the effect of strain-hardening, while load parameter $\lambda_{(L)}$ includes the effect of plate buckling.

From the displacement given by the intersection of the two curves; the reductionlike change of state is due to the effect of plate buckling as shown in *Fig. 12*. In connection with the results it should be emphasized, that – similarly to the plastic load bearing capacity analysis – the expression-taking the two separate components into consideration-assumes the structure motionless till the moments M_p and M' in the hinges form.

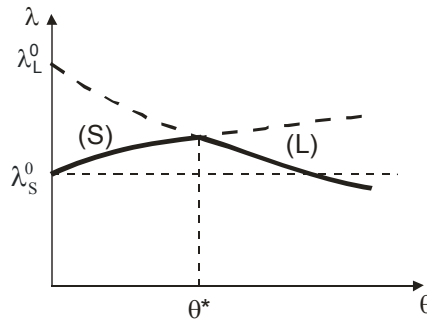


Fig. 12. Load parameter and displacement curve

3.3. The case of the “direct method of design”

Halasz [34] used previously the “direct method of design” for the frame with elastic-ideally plastic hinges and ideal rigid or hinge connection model.

3.3.1. First and second order approach

The analysis of the behaviour of elasto-plastic frames neglecting the change in the geometry of the structures while setting up the equations of equilibrium is referred to as *first order approach*.

The steel frame has normal plastic hinges, semi-rigid connections and column bases at the respective locations. The load-deflection diagram of the frame shown in *Fig. 13* according to the first order approach $F_0^{(I)}$ can be expressed as

$$F_0^{(I)} = f(M_{pl}; M_{Rd}), \quad (6)$$

where M_{pl} is the plastic moment of the cross section and M_{Rd} is the moment resistance of the semi-rigid connection or column base.

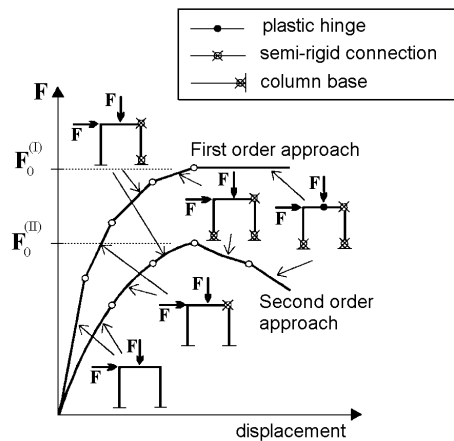


Fig. 13. First and second order approach

In the second order approach the equilibrium equations are set up in such a way that they take into account the deflections of the structure. A typical load–deflection diagram according to the second order approach $F_0^{(II)}$ is illustrated in Fig. 13.

$$F_0^{(II)} = f(M_{pl}; EI; M_{Rd}; S_j), \quad (7)$$

where M_{pl} is the plastic moment of the cross section, EI is the elastic stiffness of the cross sections, M_{Rd} is the moment resistance and S_j is the rotational stiffness of the semi-rigid connection or column base.

The second order load–deflection curve differs basically from the curve based on the first order approach as follows: i) the branches are curvilinear; ii) the failure load (or peak load) is lower than in the case of the simple plastic (first order) theory; iii) the failure may occur before the complete yield mechanism has developed and is followed by unstable behaviour. In addition, the location and sequence of occurrence of the generalized hinges do not necessarily coincide with those determined in the case of first order theory.

3.3.2. Direct method of design

Let a frame – such as the one shown in Fig. 13 – be subject to monotonously increasing loads proportional to a single load factor F . For simplicity we confine our

investigation to cases when the axial forces can be expressed in the form of $N_k = \beta_k \cdot F$, where parameter β_k is constant.

The frame may be built up of perfectly elastic members and generalized hinges at certain locations. The load–deflection curve can be characterised by the diagram shown in *Fig. 14*. The subsequent branches of the curve indicate the behaviour of the frame containing an increasing number of generalized hinges [32]. Each branch continued beyond its range of validity (dashed lines in *Fig. 14*) approaches asymptotically a certain value $F_{cr,n}$. These values are referred to in the literature as “deteriorated critical loads” (Horne and Merchant [33], Halasz [34]). They represent the load factor causing buckling of a frame in a completely elastic state. During the analysis, the generalized hinges developed in the frame have been replaced with real hinges.

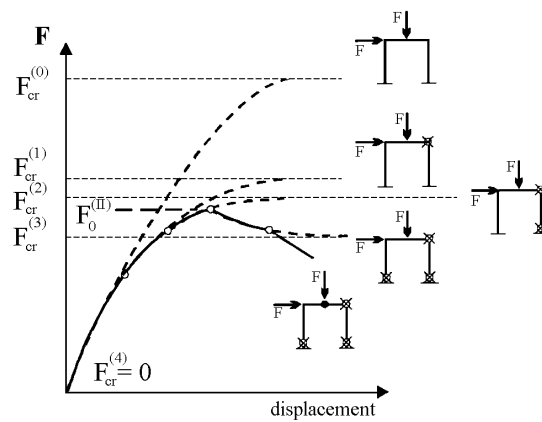


Fig. 14. Load displacement curve

According to the first order approach, the failure load depends upon the value of the generalized moment only. In a second order approach, however, failure load depends upon two types of quantities: the moment resistance of the cross sections or the semi-rigid connections (i.e. the “strength” of the structure), and the flexural stiffness of the cross sections (EI) or the rotational stiffness of the semi-rigid connections or column bases (i.e. the “rigidity” of the structure).

Assuming EI and S_{joint} to increase infinitely, the concept of rigid-plastic material can be derived. In this case failure will only occur after the formation of a complete yield mechanism, however by studying *Fig. 15* two frame types can be distinguished:

- “*relatively rigid*” frame: the failure should take place when the number of generalized hinges has reached the number of hinges necessary for the complete mechanism, thus $i = n$;
- “*relatively flexible*” frame: the failure load is reached in the presence of a lower number of generalized hinges than the number of hinges that transforms the structure into a complete mechanism ($i < n$).

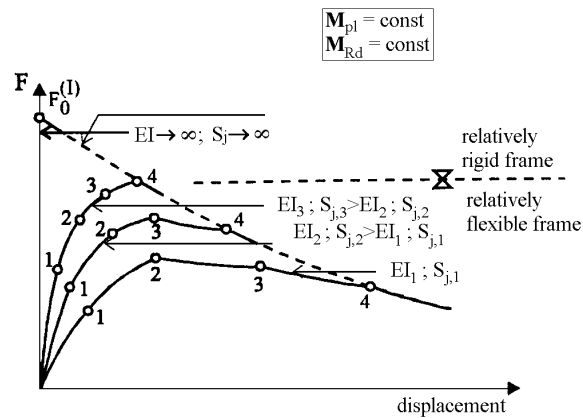


Fig. 15. Relatively rigid and relatively flexible frames

A special but easy to handle case of the “Direct Method of Design” is to design a structure where the predetermined failure load F_F coincides with one of the “deteriorated critical loads” $F_{cr,n}$. Fig. 16 represents a case with $n = 3$, i.e. the structure fails as soon as the third generalized hinge has developed.

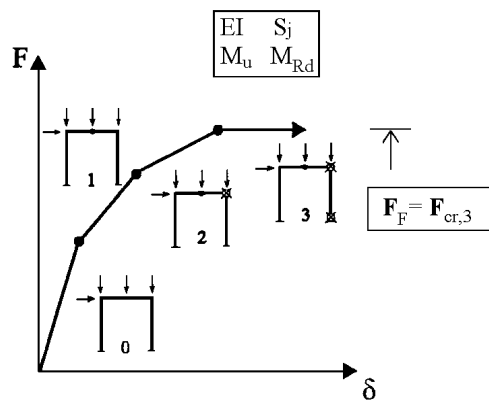


Fig. 16. Illustration of the “Direct Method of Design”

4. Evaluation of load bearing and ductility capacity of steel frames

4.1. Test program for gable-frame structures

The experimental research project was carried out in the Laboratory of the Department of Steel Structures, Technical University of Budapest. Two main series of experiments were covered by the test program.

The *first part* of the program contained the additional tests on stub columns, frame corners, plate elements, simple beams [35]. A brief summary of this series is given in Fig. 17.

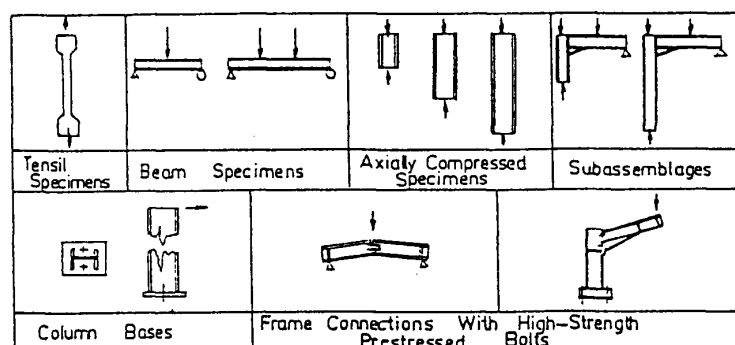


Fig. 17. First part of the test program

4.1.1. Full-scale tests of frames

In the *second part* of the program the full-scale tests of frames have been carried out. Fig. 18 gives a brief summary of the full-scale tests and dimensions of the specimens, indicating the loads and the characteristics of the loading process [35].

Test frames had rafters with a slope of 30 % (16.7°), welded column sections. Rafter-to-column and mid-span connections have been end-plated ones with high-strength prestressed bolts. Different types of lateral supports were applied to the frames to prevent or decrease out-of-plane displacements and/or rotations of selected sections.

Vertical loads at the purlin supports were applied to the upper flange of rafter, so web and bottom flange were not restrained laterally. To make horizontal displacement (sidesway) unrestricted, jacks were fastened not directly to the floor-slab, but through a so-called gravity load simulator (Fig. 19). The simulator consists of three elements: two bars and a rigid triangle. The two bars had pin-joints at both ends resulting in a one-degree-of-freedom mechanism. Hydraulic jacks were joined to the rigid triangle.

This mechanism produced a vertical load acting upon the intersection of the two bar axes. Characteristics curve of the simulator illustrating the ratio of horizontal and vertical load component as a function of the horizontal displacement is shown in Fig. 19c.

A remark in connection with the characteristics of the loading seems to be rather important. During the mathematical (theoretical) investigations, virtual disturbances have been assumed for the equilibrium state analysis of the structure so that these disturbances do not influence loading [36]. However, in the case of experimental investigations, these disturbances are, quite naturally, real ones thus their effect does not only manifest itself on the structure but also in the loading system. Therefore the

interaction of the structure and the loading system has also to be determined during the experimental investigations.

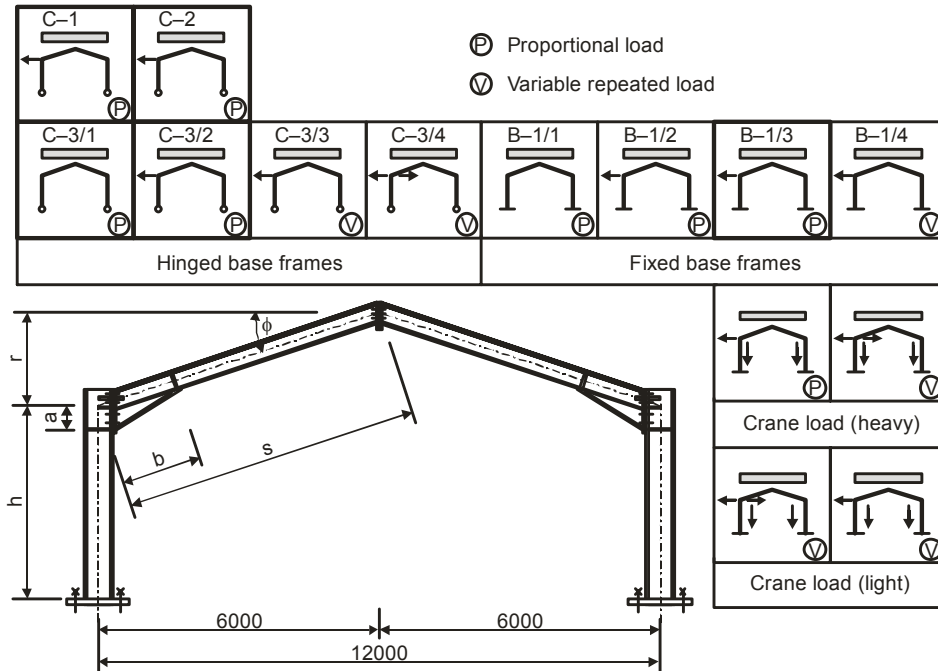


Fig. 18. Second part of test program

First of all it should be noted, that a large amount of steel structures in the engineering practice are loaded with dead or gravity load. The characteristic curves of gravity loading are horizontal lines. The highest characteristic line corresponding to the highest gravity load is tangential to the load-displacement curve and it touches the load-displacement curve at the peak of the curve. This peak point also indicates the loss of stable equilibrium state.

However, this general observation has become a hindrance to the cognitive process, since, on the basis of the above observation, not only the complete structure but also individual structural elements have been analysed experimentally mainly for gravity load. These analyses may have “given” basis to the statement that no stability loss of the structural elements could take place prior to the development of global stability loss. However should the stability loss of the structural element occur it means the collapse, loss of stability of the complete structure. This train of thought eliminated major problem spheres from the program of theoretical and experimental investigations. In the course of our investigations and analyses it was first the experimental results that indicated and then proved very convincingly that this type of viewpoint simplifies the behaviour of the structure.

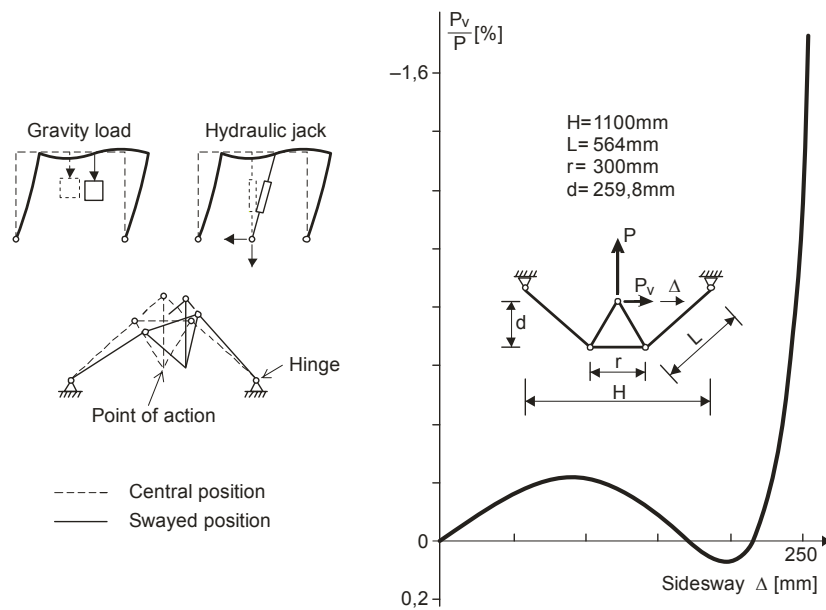


Fig. 19. Gravity load simulator

A full knowledge of the behaviour of the structure also involves the knowledge of the behaviour of structural elements and thus it is not only a demand from the viewpoint of “comfort” that when investigating the supporting structures also the descending section of the load–displacement curve should be considered, but this is also required by the demand of a complete knowledge.

The relationship of the loading characteristics and the model of supporting structure can be seen for a simple model in Fig. 20a. The state indicated in Fig. 20b develops as an effect of gravity load type loading. The state shown in Fig. 20c – taking also into consideration the descending section of the load–displacement curves of individual units – develops as an effect of deformation type loading. The maximum load capacity characterizes the behaviour of the structure sufficiently well so the use of deformation–type loading is not absolutely necessary in this case. However, it is expedient to carry out the standard experiments for structural elements, beams, columns, connections, column bases, etc. first and foremost with deformation–type loading if both the ascending and descending parts of the load–displacement curve are to be considered and if the displacement capability of the supporting structures is to be determined.

4.1.2. Test of frame C–3/2

The load–deflection diagram is shown in Fig. 21. Failure was due to plate buckling in the plastic hinge below the haunch in the column (to develop the earliest) and

lateral buckling around the midspan. Load–deflection diagram proves the formation of the predicted yield mechanism.

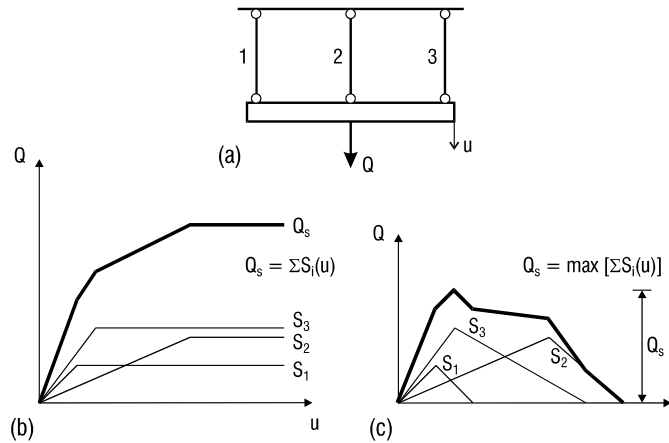


Fig. 20. Effect of behaviour of loading system

4.2. Results of theoretical and experimental investigations for gable-frame structures

4.2.1. Application of computer programs

The load–deflection curve for the experimental frame C–3/2 is shown in Fig. 21. On the side of the horizontal load, the first inelastic hinge develops due to the residual stresses and deformations in the cross section beneath the frame knee and this hinge develops at 52 % of the maximal frame load. At 97 % of the maximal load, Zone (L) describing the effect of plate buckling develops also in this cross section, i.e. in the frame cross section an “unstable” state – a descending characteristic curve – develops.

For comparison Fig. 21 also introduces the characteristic load–displacement curve of the frame structure in the case when the basis of the computations is the traditional plastic hinge.

The results show well that the presence of residual stresses influences in a major way the range of limited plastic deformations, however, mainly because of the cross section geometry of the experimental beam, the maximal load bearing capacities computed with the traditional (elastic–ideally plastic) hinge as well as those obtained by the interactive hinge coincide with the experimental results.

4.2.2. Application of the approximate engineering method

The Approximate Engineering Method is presented for test frame C–3/2. The column base of the test frame was supported by the foundation, but it did not act as a fix end, so plastic load bearing capacity of the column base was determined by experiments. These values were used in the calculations.

The Approximate Engineering Method is suitable for the analyses of the strain hardening and to take into the effect the plate buckling. The most important steps of the analysis of frame C-3/2 given in *Fig. 21* are as follows:

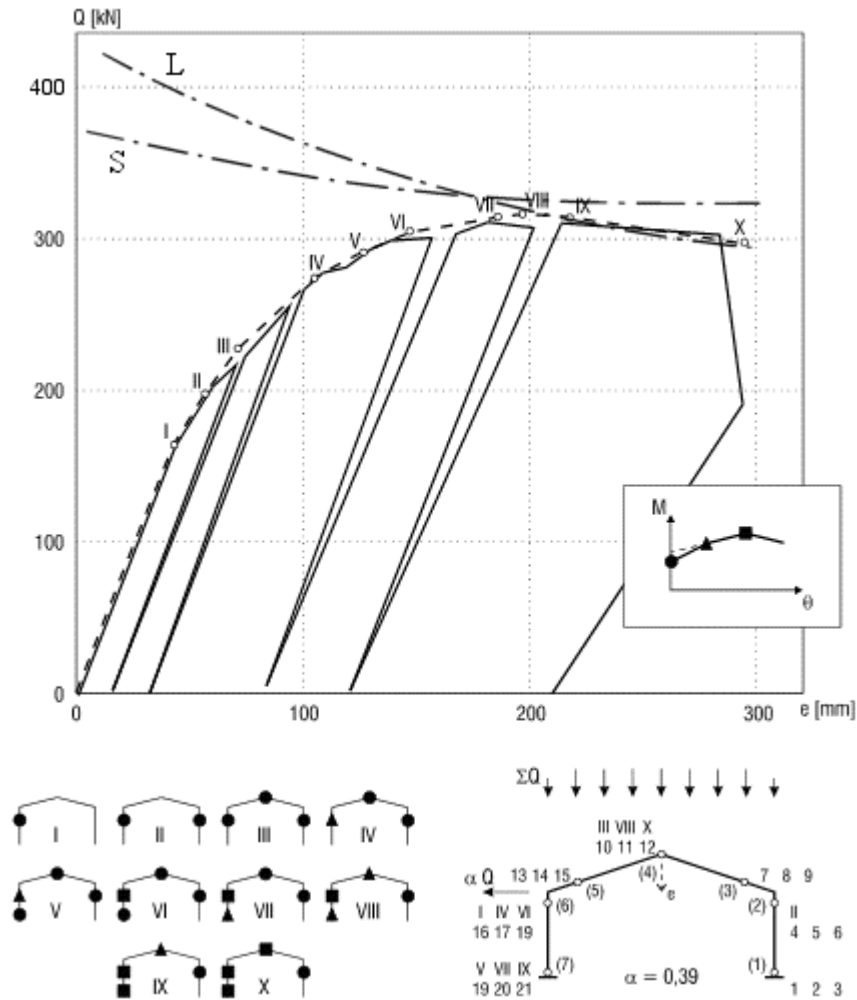


Fig. 21. Experiment (continuous line) and analysed (dashed) result for gable frame structure

- plastic load bearing capacity analysis by a mechanism chosen according to the test results;
- analysis of effects of finite deformations with the help of the chosen mechanism;

- effect of strain-hardening: moment-rotation relationship of inelastic zones can be replaced by straight lines, therefore rigid-hardening model can be applied to determine m_s .
- effect of plate buckling: characteristic curves of interactive hinges can be replaced by straight lines too, therefore the rigid-hardening model can be applied to determine M' and m_L .

Fig. 21 compares the test result and the results of the Approximate Engineering Method taking into consideration the strain-hardening of steel and the effect of the plate buckling. The comparison shows that the Approximate Engineering Method gives a satisfactory result for the maximum loads and the unstable equilibrium state path of the whole structure as well. At the same time the analysis can be done at the “desk of the designer”.

4.3. Analysis of the influence parameters for gable-frame structures

4.3.1. Column bases

Column bases were fixed or hinged. The hinges were not ideal: columns have been supported by large base-plates. *Fig. 22* compares the measured bending moment due to vertical and horizontal load with the calculated ones assuming pinned (dashed line) and fixed (solid line) frames. *Fig. 23* shows the corresponding moment-rotation diagrams for the pinned and fixed frames. Moreover *Fig. 23* also shows the experimental results, which fall between the theoretical bounds. However this curve is identical to the curve, which can be obtained by using an interactive hinge method.

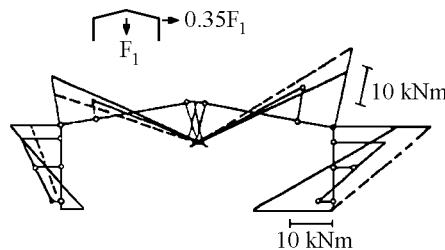


Fig. 22. Measured and calculated bending moments

4.3.2. Lateral supports

Spacing and efficiency of lateral supports proved to be of basic importance. Their effect is illustrated in *Fig. 24*.

The importance of adequate spacing of lateral supports and their efficiency in preventing the rotation of the cross section around the bar axis has to be emphasized as purling and rails connected to tension flanges often cannot be regarded fully effective in the case of thin webs.

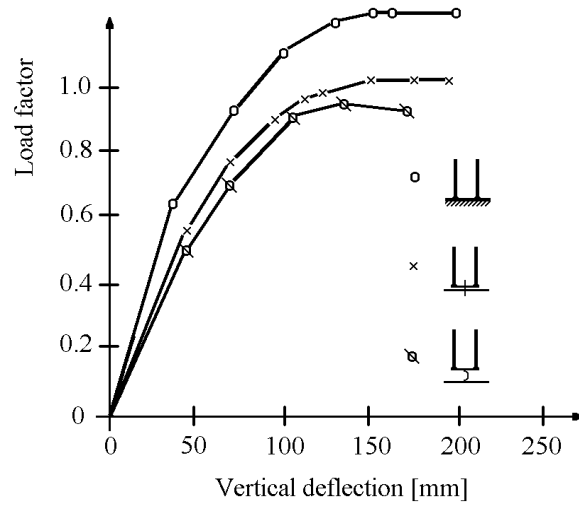


Fig. 23. Effect of column-bases for the behaviour of gable-frame structure

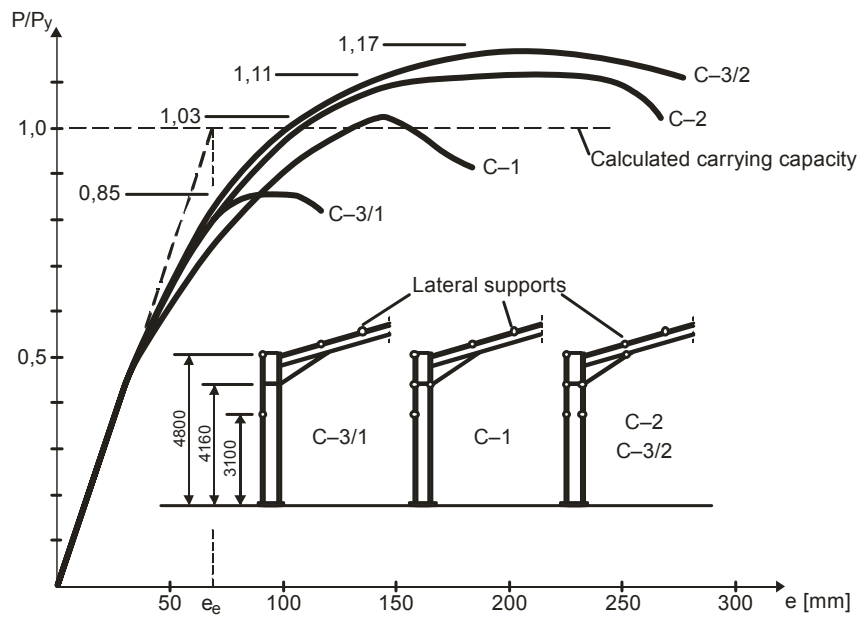


Fig. 24. Effect of lateral supports

Not only the load carrying capacity can thus be substantially reduced (as by elastic lateral buckling in the case of frame *C-3/I* in *Fig. 24*), but the yield plateau in the load deflection diagram can be too short (as in the case of frame *C-1* in *Fig. 24*), rendering the structure sensitive against initial imperfections.

4.4. Test program for multi-storey frame structures

The experimental research project was carried out in the Laboratory of the Department of Structural Engineering, Budapest University of Technology and Economics [21].

An overall view of the testing arrangement is shown in *Fig. 25*. The examined frames are two-storey single-bay frames. Both the columns and the beams are welded I sections. Columns are connected to a rigid steel base element by two bolts through an end plate (layout generally regarded as pinned joint in practice). Beams and columns are connected with flush end plate joints.

In order to avoid lateral-torsional buckling, lateral restraints are applied to the frame at the beam-to-column joint locations and at the mid-spans of the beams, see *Fig. 25*. The frame is loaded by two vertical concentrated loads at the mid-spans of the beams, and two horizontal loads applied at one side of the frame in the levels of the beams (*Fig. 25*). The two vertical loads are increased and decreased proportionally using three hydraulic jacks (one larger to the lower beam and two smaller and identical to the upper) connected into one oil circuit. Because of the slight difference between the pressure surfaces of the larger jack on one hand and the two smaller jacks on the other, the lower beam was loaded by a concentrated load 89 % in magnitude of the load on the upper beam. The vertical loads are applied through so-called gravity load simulators [35], devices, which ensure the verticality of the loads within certain limits of lateral displacements of the points of application of the loads. The horizontal loads are applied using one hydraulic jack through a simply supported vertical beam, which ensures the applied load, to be equally distributed between the two beam levels. The direction of these horizontal loads is reversible.

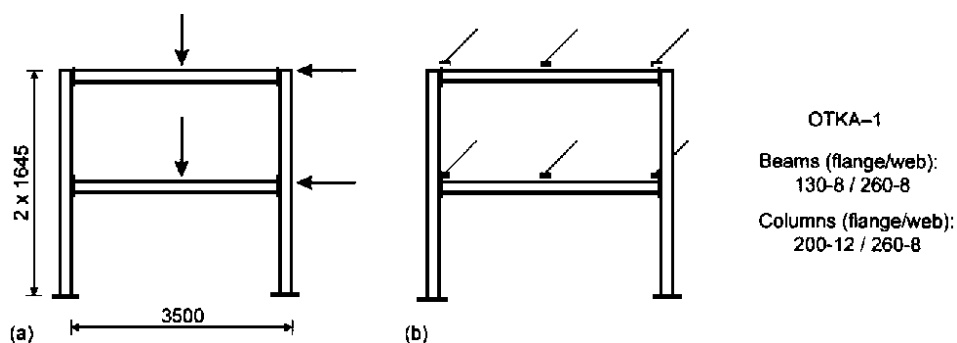


Fig. 25. The tested frame: (a) main geometry and (b) lateral restrains

4.5. Results of theoretical and experimental investigation for multi-storey frame structures

The load–deflection curve for the experimental frame OTKA–1 is shown in Fig. 26. The Approximate Engineering Method is also presented for test frame OTKA–1, shown in Fig. 26. The comparison shows that the Approximate Engineering Method gives satisfactory results for the maximum loads and the descending state path of the whole structure as well. Furthermore the analysis can be done at the “desk of the designer”.

5. Final remarks

Analysis of structural response is becoming a very sophisticated field in the view of ductility. Specifications have to incorporate an increasing number of design rules, sometimes in the form of tables and diagrams often derived directly from experiments, making specifications voluminous, the number of their appendices increasing.

Although we are in possession of all facilities to carry through any complicated calculations, it seems necessary to regard them mainly as a tool for research and to find a good balance between the real needs of a safe and economic everyday design praxis and the way of feeding back the accumulated information gained by research.

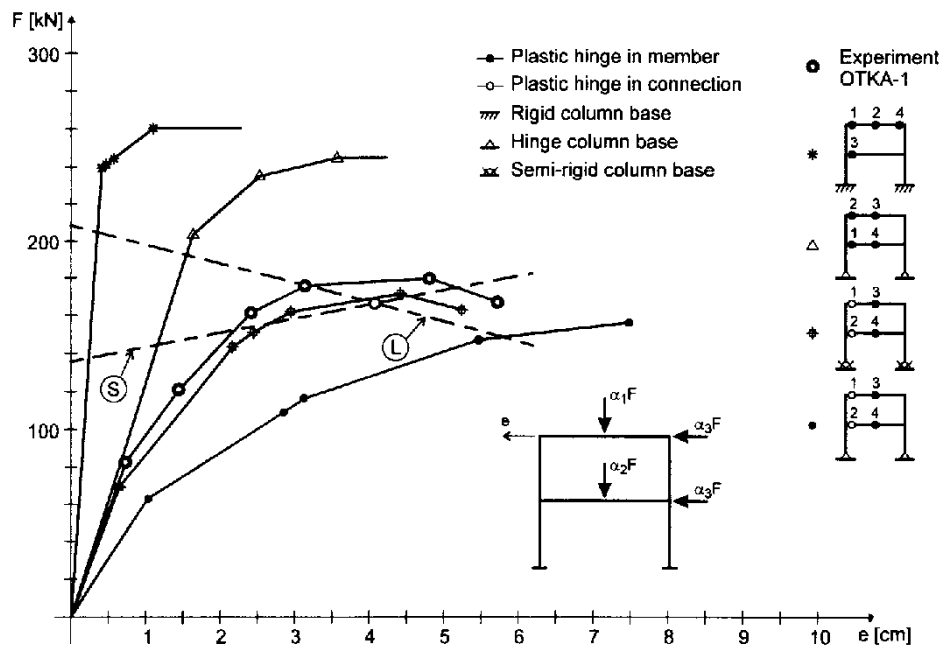


Fig. 26. Load–displacement curve of test OTKA–1

Acknowledgements

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