

METHODS FOR SIMULATING HYDRODYNAMIC EFFECTS IN TRIBOTECHNICAL PROCESSES

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Abstract: In this paper variational and analytical methods for the solution of multilinked contact mechanical problems of deformable solids and fluid have been considered. Numerical methods have been found cumbersome and inaccurate. In this connection a new accurate and quick variational–analytical method for the lubricant flow analysis has been developed.

Keywords: Hydraulic pressure, Hydrodynamics, Interference–fit joint, Mode of deformation, Viscous fluid flow

1. Introduction

Nowadays interference–fit joints are widely used for transmitting axial forces and torques varied by direction and magnitude. Their use with continuously increasing loads requires increasing their load capacity and strength. Thus obtaining a joint with the maximum strength and required operational effectiveness is a very important problem for modern machinery.

One of the most efficient technological methods for joints load capacity increase is female parts elastoplastic deformation (ED). It is implemented under conditions of dry, liquid and boundary friction types. At the same time, one of the most efficient ways to assemble an interference–fit joint is using the hydraulic assembly method, which is carried out in the liquid friction mode.

Performing the processes in the liquid friction mode provides the following advantages: press force reduction with the same productivity, mating parts safety; there is no micro asperities smashing, hence increase the actual contact area and mating

surfaces friction ratio in the designed joint. Changing the press speed, geometrical and material properties of the parts, rheological properties of the lubricant, using various lubricant supply schemes and pressure gain methods allow controlling the resulting shapes of the parts and their residual mode of deformation. In this connection hydrodynamic effects simulation allowing for the mentioned factors will help determine the best technological modes for the process and hence control the properties of the resulting joint.

The feature of the ED process in the liquid friction mode is that the surfaces of the parts are being deformed under the influence of pressure in a thin and long lubricating layer with non-constant height. At each time step of the whole process the analysis is done in several phases: a) the lubricant pressure values distribution is calculated first; b) the lubricant pressure forces are then applied onto the surfaces of the parts; c) the elastoplastic mechanical problem of deformable solids is being solved using the finite element method (FEM). As all numerical methods, FEM allows to solve complex differential tasks by just solving a linear equations set. As a result nodal displacements and the mode of deformation parameters' values are calculated.

2. The investigated model

Pressure distribution in a thin and long lubricating layer has been analyzed by means of analytical and variational methods [1, 2, 3, 4]. Experimental methods are expensive and unprofitable. Analytical methods are applicable just for simple shapes of the parts and the clearance. But in a real process the parts and the clearance shapes are changing unpredictably. Using FEM [5] and other numerical methods for the fluid mechanical problem in the present task provides rather inaccurate results because the height of the analyzed fluid layer is very low comparing to its length. For an ED process the typical height-to-length ratio value interval is $7,5 \cdot 10^{-5} \leq h/l \leq 7,5 \cdot 10^{-3}$. Analyzing fluid flow between two parts by means of the finite difference method (FDM) [6] allowed determining the influence of design of the joint and assembly technology on the contact pressure distribution along the joint, mode of deformation and find risk areas with their maximum stresses.

The analysis of the hydro-press joint assembly method (*Fig.1*), that involves the same physical processes as in ED process in the liquid friction mode, has showed that the parts are being separated by a thin lubricating layer maintained by an external pressure source. In the mating zone the parts are being affected by contact pressures occurring there due to the difference in the mounting dimensions of the parts and hydrodynamic pressure in the lubricating layer of non-constant height. From the outer side the parts are affected by technological rigging providing a qualitative assembly process [3].

When analyzing the contact of parts and fluid, multilinked contact conditions must be taken into account. The presence of metallic layers, applied onto the joint's elements, is considered when composing the resolving linear equations set, which in the FEM formulation [4] is as follows:

$$\begin{aligned} [K_i]\{\delta_i\} + \{F_i\}_p + \{F_i\}_b + \{F_i\}_{\varepsilon_0} + \{F_i\}_{\sigma_0} - \{R_i\} &= 0, \\ [H]\{\delta^*\} - \{\Phi\} &= 0, \end{aligned} \quad (1)$$

where K and H – the coefficient matrixes of the equations set,

$$i \text{ is an index, } i = \begin{cases} 1, & \text{for the female part,} \\ 2, & \text{for the male part,} \\ 3, & \text{metallic layer on the female part,} \\ 4, & \text{metallic layer on the male part,} \\ 5, & \text{for the rigging, affecting the female part,} \\ 6, & \text{for the rigging, affecting the male part,} \end{cases}$$

$\{\delta_i\} = \begin{Bmatrix} x_{ri} \\ x_{zi} \end{Bmatrix}$ is nodal displacements in the finite–element grid,

$\{F_i\}_p, \{F_i\}_b, \{F_i\}_{\varepsilon_0}, \{F_i\}_{\sigma_0}$ is nodal forces due to distributed (with the index p) and boundary (with the index b) loads, initial strains (with the index ε_0) and initial stresses (with the index σ_0).

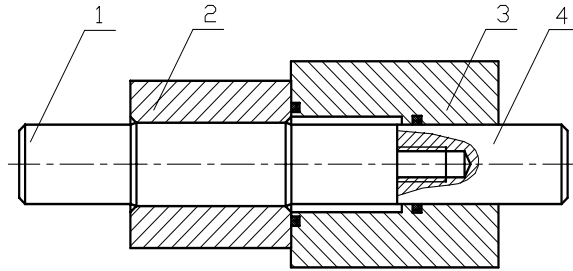


Fig. 1. An interference–fit joint's parts and rigging for hydro–press assembly
1–shaft; 2–bush; 3–work tool; 4–false shaft

The elements in the second equation of the system (1) are

$$\{\delta_k^*\} = \begin{Bmatrix} v_{rk} \\ v_{zk} \\ p_k \end{Bmatrix} \text{ is the velocities and the pressure vector,}$$

$\{\Phi\}$ is the vector of mass forces.

The influence of resistance forces and inertias, according to the D'Alembert's principle, is replaced by their static equivalent. Forces affecting the parts surfaces in their interfacing zone, occurred there due to the contact pressure, are equal by magnitude and opposite by direction

$$\{F_1\} = -\{F_2\}. \quad (2)$$

Under the forces influence the mating surfaces are moving (*Fig. 2*). Their displacements depend on the interference value

$$\frac{1}{2}N_i + h_i - \Delta_i = \pm x_{1ri} \mp x_{2ri} + \sum x_{\lambda i} + \zeta_i \quad (3)$$

where N_i is the interference of the analyzed joint, h_i is the lubricating layer height, Δ_i is the gap size, only for joints assembled by auto-fastening and auto-fretage, x_{1ri} and x_{2ri} are the nodal radial displacements of the female part, the first case ($x_{1ri} > 0$ and $x_{2ri} < 0$) conforms to assembling with an interference, the second one ($x_{1ri} < 0$ and $x_{2ri} > 0$) conforms to assembling with a gap, $\sum x_{\lambda i}$ is the total contact juncture ductility, ζ_i is the allowed displacements computing error.

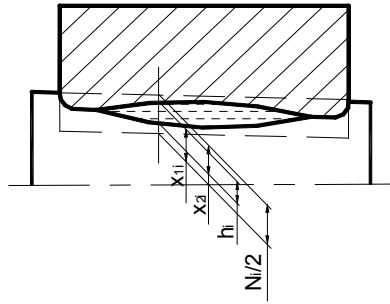


Fig. 2. Displacements consistency conditions in HPJ (assembling with autofretage)

The pressure distribution in the lubricating layer is calculated after the solution of the fluid mechanical problem for known geometry of the clearance geometry.

Forces in the contact zone are determined as

$$\{F_i(p_k, p_M)\}_K = \begin{cases} \int_S [N]^T \{p_K\} dS, & \text{the radial force for dry and mixed friction zone,} \\ \int_S [N]^T \{p\}_K f(p) dS \text{ or } \int_S [B]^T \{t\}_C p dS, & \begin{cases} \text{axial force for dry and} \\ \text{mixed friction zone,} \end{cases} \\ \int_S [N]^T \{p\}_L dS, & \text{radial force for liquid friction zone,} \\ \int_S [B]^T \{t\}_L dS, & \text{axial force for liquid friction zone.} \end{cases} \quad (4)$$

The forces, clearance dimensions and fluid flow speed values are computed using an iterative method for solving the equations set (1). Contact pressure, the mode of deformation of the parts in an assembled joint are determined allowing for the parts

loading history when assembling. The lubricating layer removed in the end is not considered.

Explorations [1, 2, 3] showed that in a hydro–press assembly process the maximum pressure gradient value is located at the butt of the joint, because there the clearance shape resembles a parabola. At all other locations the clearance shape resembles a wedge with a negligible obliquity. The pressure gradient value depends on the insertion length.

The change in the rigging hollow length with the invariable chamfer, hollow height, assembly speed and productivity showed that the pressure changes only in the inlet, while in the outlet it mostly remains constant. If the hollow length is negligible, the inlet pressure value is 192 MPa. This value is obtained for a joint with a female part containing a chamfer of 2.5 mm size and 45° angle. The chamfer reduction will result in the pressure increase. If the hollow length equals to 5 mm, the pressure decreases to 180 MPa, or to 176 MPa if the hollow length value is 7.5. The hollow length increase up to 10–15 mm results in the pressure decrease down to 175.6–174.8 MPa

The mode of deformation computation showed that the presence of a 45° angle chamfer on the female part results in the pressure increase both in the rigging hollow and in the parts mating zone. That conditions a significant change in the mode of deformation of the part, in comparison with longitudinal pressing or hydrostatic lubricant distributing (Fig. 3).

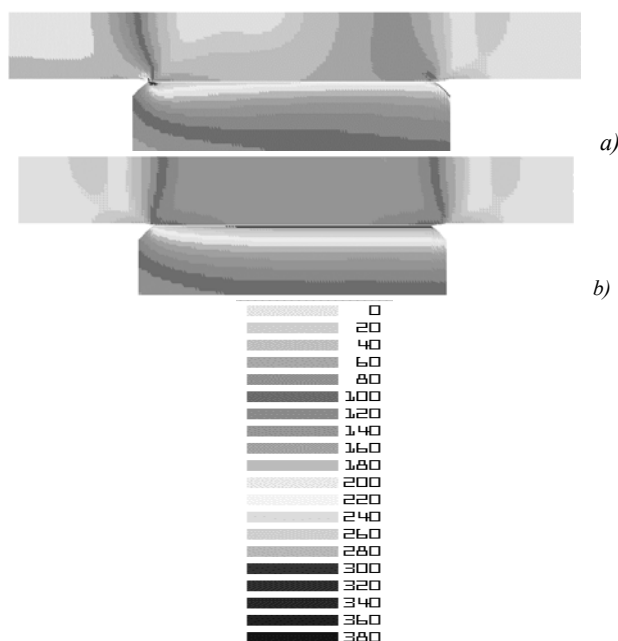


Fig.3. Stress intensity field in the parts of a joint assembled using
a) press and b) hydro–press methods, a) friction coefficient $f_0=0.23$; $l_{ins}=50$ mm,
b) lubricant pressure is hydrostatic, $P_M=126$ MPa, insertion length $l_{ins}=50$ mm,
hollow length $l_{Hol}=0.0$ mm

The performed extensive studies allowed determining the influence of chamfers in design of the female part on the assembly process parameters: lubricant pressure and press force (Fig. 4).

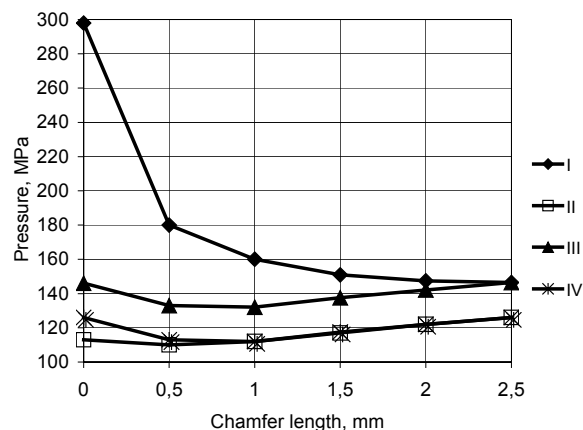


Fig. 4. The influence of chamfers in design of the female part on the lubricant pressure, (hollow length $l=5$ mm, pump productivity $Q=0,37 \cdot 10^{-5} \text{ m}^3/\text{s}$, assembly speed $v=4,0 \cdot 10^{-3} \text{ m/s}$, viscosity $\mu = \text{Pa} \cdot \text{s}$)

I – both inlet chamfers are being changed, II – both outlet chamfers are being changed, III – one inlet chamfer is being changed, IV – one outlet chamfer is being changed

The diagrams analysis shows that using chamfers in the female part structure allows controlling the pressure and hence the mode of deformation of the parts and the load carrying capacity of the joint.

Developed mathematical model of an assembly process helped determine the influence of the technological rigging pressing force on the assembly parameters and the mode of deformation of the parts.

Hydro–press joints (HPJ) load–carrying capacity analysis showed that when assembling at high contact and fluid pressure values that exceed the yield point, the residual interference and contact pressure change occurs. The HPJ load carrying capacity is then lower than if it was assembled using the thermal method.

3. The applied method

Variational methods application to the concerned problem showed that multilinked contact problems solution involves consuming a long–time and high system requirements. And so, taking into account the complication of FEM and FDM application to fluid mechanics two–directional problem [7], a variational–analytical solution method has been developed.

The fluid flow in a thin lubricant layer between the mating surfaces is performed with low assembling (deforming) speed values. Hence the fluid flow can be considered to occur at low Reynolds numbers. For the present task we can use the steady

Newtonian viscous fluid flow model governed by the Navier–Stokes differential equations set [8].

The used assumptions are: fluid flows mostly in the axial direction, the component v_r value is much lower than v_z ; fluid sticks to the walls, v_z is changing much slower than v_r .

The following set of approximating equations can be obtained from the initial equations

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) &= \frac{1}{\mu} \frac{\partial p}{\partial z}, \\ \frac{\partial p}{\partial r} &= 0, \\ \frac{1}{r} \frac{\partial (r v_r)}{\partial r} + \frac{\partial v_z}{\partial z} &= 0. \end{aligned} \quad (5)$$

To investigate the fluid flow in the layer with non-constant height we split it into a number of simple-shaped regions (*Fig. 5*).

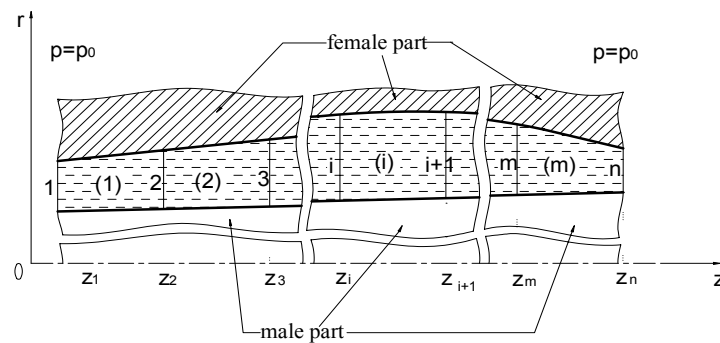


Fig. 5. Lubricating layer split into simple-shaped regions

Such a shape of a region can be easily and accurately enough approximated by a simple function. As a result the lubricating layer is represented by a sequence of m simple regions confined by $n=m+1$ sections ($m, n \in N$). In *Fig. 5* a number in round brackets defines a region, and a number without brackets defines a section.

The insertion direction is supposed to be opposite to the z -coordinate axis direction. Then the following boundary conditions conform to each region i (here and further $i = \overline{1, m}$).

$$\begin{aligned} \text{if } r = a_i(z), \ z_i \leq z \leq z_{i+1}, \text{ then } v_z &= -U, \ v_r = 0, \\ \text{if } r = b_i(z), \ z_i \leq z \leq z_{i+1}, \text{ then } v_z &= 0, \ v_r = 0, \end{aligned} \quad (6)$$

where $a_i = a_i(z)$, $b_i = b_i(z)$ are the male part outer radius and the female part inner radius correspondingly, z_i , z_{i+1} are the axial coordinates of the left and the right boundaries of the region, correspondingly, and U is the pressing speed.

A sequence of math transformations of the equations in the set (5) for region i results in a number of equations, including one for fluid flow (7) through any section within region i

$$Q_i(z) = Q_{Ui}(z) + \frac{\partial p_i}{\partial z} Q_{Pi}(z), \quad (7)$$

where

$$Q_{Ui}(z) = \frac{U\pi}{2 \ln \frac{a_i}{b_i}} \left(b_i^2 - a_i^2 + 2a_i^2 \ln \frac{a_i}{b_i} \right), \quad (8)$$

$$Q_{Pi}(z) = \frac{\pi}{8\mu_i \ln \frac{a_i}{b_i}} \left(\ln \frac{a_i}{b_i} (a_i^4 - b_i^4) - (a_i^2 - b_i^2) \right), \quad (9)$$

an equation determining the pressure drop through element i (10), and another one for pressure distribution along the lubricating layer (11)

$$\Delta p_i = \int_{z_i}^{z_{i+1}} \frac{\partial p_i}{\partial z} dz = \int_{z_i}^{z_{i+1}} \frac{Q_i(z) - Q_{Ui}(z)}{Q_{Pi}(z)} dz, \quad (10)$$

$$\forall z_p \in [z_i; z_{i+1}], \quad p(z) = p(z_1) + \sum_{k=1}^{i-1} \Delta p_k + \int_{z_i}^z \frac{Q_i(z) - Q_{Ui}(z)}{Q_{Pi}(z)} dz. \quad (11)$$

For region i of an incompressible fluid layer can be formulated

$$\Delta V_i = \pi U [a_i^2(z_i) - a_i^2(z_{i+1})] \quad \text{or} \quad \Delta V_i = Q_i(z_i) - Q_i(z_{i+1}), \quad (12)$$

where ΔV_i – volume change for region i .

Using the fluid incompressibility assumption and equation (12) when analyzing a sequence of elements, we get an equation for fluid flow $Q_i(z_q)$ ($z_i \leq z_q \leq z_{i+1}$) through any section of the lubricating layer (13).

If the lubricant or some quantity of it is distributed via one or a number of oil-distributing grooves, it is needed to subtract from the right part of equation (13) the total output of the pumps providing lubricant distribution in the interval $z_1 \leq z \leq z_q$

$$Q_i(z_q) = Q_1(z_1) - \pi U \left(\sum_{j=1}^{i-1} [a_j^2(z_j) - a_j^2(z_{j+1})] + a_j^2(z_i) - a_i^2(z_q) \right). \quad (13)$$

In that way the lubricant flow distribution along the lubricating layer is calculated relatively to its smallest z coordinate that equals to z_l . In the case of one-directional fluid flow from the contact zone (lubricant is distributed from butt of part) it is very easy to determine the fluid flow through this section,

$$Q_1(z_1) = k_1 Q_T + k_2 \left(Q_K - \sum_{k=1}^m \Delta V_k \right), \quad (14)$$

where Q_T is the quantity of lubricant, distributed from the butt of part, Q_K is the total pumps productivity, providing lubricant distribution via oil-distributing grooves, and

$$k_1 = \begin{cases} 1, & \text{if fluid flows codirectionally with axis } z, \\ -1, & \text{in opposite case.} \end{cases}$$

In most cases (for instance when high-viscosity lubricant is preliminary applied onto the conical mounting surface of the female part or when distributing lubricant via one or a number of oil-distributing grooves) a two-directional fluid flow from the contact zone occurs. For such flow the presence of a section corresponding with the only pressure value maximum point is typical. The maximum location depends on the $Q(z)$ values distribution. Using once again the fluid incompressibility assumption, write the equation for the total lubricant quantity Q flowing out from the both sides

$$Q - Q_m(z_n) - Q_1(z_1) = Q_K - \sum_{k=1}^m \Delta V_k. \quad (15)$$

Then the fluid flow through the section $z=z_l$ equals to

$$Q_1(z_1) = k \cdot Q, \quad k \in R, \quad 0 \leq k \leq 1. \quad (16)$$

In addition due to the constancy of external pressure on the lubricating layer the following equality is correct

$$\sum_{i=1}^m \Delta p_i = 0. \quad (17)$$

The coefficient value k providing the verity of condition (17), is determined by an iterational approaching method using equations (8)–(10), (13), (15) and (16) for recomputing pressure drops through the layer regions.

4. Conclusion

It is experimentally proved that the dynamic viscosity coefficient value used in equations (5), (9), is highly dependant on the fluid pressure and temperature. To recalculate the viscosity coefficient value in section j ($j = \overline{1, n}$) and the pressure distribution along the layer dependant on the viscosity an iterational method is used.

Approaching continues until two neighbor iterations' values divergence does not exceed 1 %. Dynamic viscosity coefficient values within each region are approximated linearly depending on the viscosity values in the region's boundary sections.

The suggested math model of a fluid flow in a thin and long circular gap between the parts of an assembled joint is implemented as a program application. Extensive studies showed the high accuracy and speed of the algorithm, that are high advantages comparing to numerical methods.

The suggested model adequacy is confirmed experimentally and in comparison with the results of a computation performed using an analytical method for a simple-shaped gap, considered in [8]. The divergence of the results was less than 10^{-4} %.

It is ascertained that the results accuracy obtained by the suggested method depends on: compliance of the process conditions with the assumptions used for the derivation of differential equations set (5), the accuracy of the lubricating layer shape approximation by the functions $a_i=a_i(z)$, $b_i=b_i(z)$.

Depending on the functions a_i and b_i types, the application of a numerical integrating method might be needed when using equations (10) and (11). The chosen integrating method also affects the computing accuracy.

The suggested analytical method allows to: a) productively and very accurately analyze the hydrodynamic effects occurring in the lubricating layer, and hence choose the best parameters and modes for hydro–press joints assembling and female parts ED processes using parts with any shapes and dimensions, b) analyze high–strength joints with guaranteed interferences at any life cycle phase, except for disposal, c) determine the load–carrying capacity of the joint or deformed part allowing for a technological factor influence and calculate the parameters for technological processes with various contact events occurring in joints (parts) assembled (deformed) using various methods.

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