

FUZZY–RULE BASED DIFFUSION IN THERMAL IMAGE PROCESSING

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Abstract: In this paper a non–linear parabolic PDE–based diffusion model is applied to enhance thermal images. The proposed smoothing technique is a non–linear time–variant system, in which the diffusivity field depends on the local characteristics of the image. The non–linear diffusion is controlled by human knowledge based linguistic rules. Some examples of intensity dependent diffusion, of edge preserving filters and their aggregates are presented. The image details in the interested regions are preserved while in other regions the variations of pixel intensities are attenuated. With the proposed fuzzy–technique a strongly non–linear diffusivity function can be defined in a user–friendly way.

Keywords: Non–linear diffusion, Fuzzy–rule based diffusivity, Thermal image processing

1. Introduction

Partial differential equation (PDE) based diffusion models are widely used to image enhancement [1]. Diffusion can precede the higher level image processing techniques such as edge detection, image segmentation, classification, object identification, etc. Diffusion is equivalent to smoothing of image data. Smoothing allows an exchange of information between neighboring parts of an image, thus extending the effect of local data to a wider area. The main problem of filtering is the dilemma of concurring image properties, such as sharpness and smoothness, respectively. Removing noises in an image, the fine details are usually also filtered out. Enhancing the edges and fine structures, on the other side, the noise will also be amplified. To solve the problem, many approaches have already been developed. For example isotropic or anisotropic

diffusions with adaptive coefficients are widespread used [2, 3, 4]. Traditionally, the non-linear diffusion filtering is performed on the nearest neighborhood pixels. Taking into account an extended neighborhood was proposed in [5]. A structural adaptive smoothing has been suggested in [6].

This work focuses on the pre-processing of thermal images. Infrared (IR) images can be degraded not only by the error of imaging processes but also by the non-uniform properties of the surface where data are collected. At the pixel level, the noise in thermal image is additive, of Gaussian nature and of high frequency with respect to the useful signals [7]. In a thermal image edges are originally blurred and consequently they are hard to detect with the classical edge-preserving algorithms. In this work the diffusivity will be defined in an implicit way, in form of fuzzy IF-THEN rules, i.e. the PDE-based non-linear diffusion algorithm is extended with a fuzzy-diffusion coefficient. The non-linear diffusion is controlled by the human knowledge based linguistic rules. The proposed technique belongs to the fuzzy extensions methods [8, 9, 10]. The smoothing properties of the diffusion correspond to the user demands: removing noises or smoothing only the predefined intensity region(s), without blurring and moving edges and so on. The proposed fuzzy-technique results a strongly non-linear diffusivity function in a user-friendly way.

2. Diffusion models

The oldest and most investigated problem in the grayscale image processing is the parabolic linear heat equation,

$$\frac{\partial I(t, x)}{\partial t} = c \Delta I(t, x), \quad I(0, x) = I_0, \quad (1)$$

where the processed image is $I(t, x)$, the constant c is the diffusion coefficient, the initial condition is the original image I_0 and usually adiabatic boundary conditions are assumed. One time slice of the solution to the uniform diffusion Eq. (1) is equivalent to the convolution of an image I with the Gaussian kernel having the global parameter σ constant for the whole image at a time instant. This smoothing is homogeneous: it does not depend on the image, and it is the same in all directions.

The non-linear diffusion models are generalizations of the heat Eq. (1). The non-linear diffusion equation for image I can be given by the formulation

$$\frac{\partial I(t, x)}{\partial t} = \nabla \cdot [c(I(t, x)) \nabla I(t, x)], \quad (2)$$

where the diffusivity $c(I(t, x))$ is non-uniform. The crucial point is how to define a diffusivity function so that it reflects both knowledge on statistics of noise degradation and edge strength in the image. Perona and Malik (PM) proposed initially a variable, adaptive, diffusion coefficient to reduce the smoothing effect near edges in [11]. The

gradient dependent diffusivity $c(|\nabla I|^2)$ of the PM model is a monotonically decreasing function: $c(0)=1$, and

$$\lim_{|\nabla I|^2 \rightarrow \infty} c(|\nabla I|^2) = 0.$$

With this choice inside the regions where the magnitude of the gradient of I is weak, (3) acts like a heat equation in isotropic smoothing. Near the boundaries of the regions where the magnitude of the gradient is large, the regularization is stopped and edges are preserved. If the initial data is very noisy then the PM model cannot distinguish between “true” and “false” edges created by the noise. This problem can be removed by smoothing or pre-smoothing I before its derivatives are computed. The PM model has been regularized by Catté et al [5] with changing the diffusivity function to

$$c(|\nabla I_\sigma|^2) = c(|\nabla (G_\sigma I)|^2) \quad (3)$$

where G_σ denotes the two-dimensional Gaussian kernel. The isotropic diffusion reduces diffusivity in all directions in the presence of discontinuities. Therefore data with lots of discontinuities is not properly smoothed anymore. Weickert [4] proposed an anisotropic extension with a diffusion tensor, which had the ability to smooth along discontinuities but not across them. The stopping criterion for the smoothing process at the optimum stage has been designed in [12], by introducing the so-called relative SNR. The iteration process can be terminated, if the change in relative SNR decreases below a predefined limit.

In the previous work of the authors [13] the diffusivity has been considered to be a hysteresis function of the pixel intensity. The hysteresis is approximated by the Preisach hysteresis model [14] with Gaussian distribution. With this diffusivity function not edges but hot or cold regions can be outlined in a thermal image. The drawback of this method is the intensive computation requirement. Fuzzy-logic is an efficient way to determine a flexible diffusivity function for smoothing thermal images so that the result is more suitable than the original image for specific applications.

3. The semi-implicit, finite-difference scheme

Discretising the spatial domain, finite difference schemes are widely used in image processing. This is due to the digital structure of an image as a set of uniformly distributed pixels. The image can be associated with a uniform grid i.e. the grid spacing in the x and y directions are equal, $\Delta x = \Delta y = h$. The temporal domain can be discretized with an explicit, implicit or a semi-implicit scheme. Explicit scheme is easy to implement, but it requires small time steps for stability reasons. Hence, much iteration is needed to reach some interesting diffusion time, and the entire procedure is

relatively inefficient. Typical example for this diffusion algorithm is the Gaussian filtering that can be written in a matrix form

$$\mathbf{I}^{k+1} = \mathbf{A}\mathbf{I}^k. \quad (4)$$

The superscript refers to the time step. The kernel of the coefficient matrix $\mathbf{A}$ has the form of

$$\mathbf{A} = \frac{\Delta t}{h^2} \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad (5)$$

where h is the uniform mesh size and Δt is the temporal grid spacing.

Discretizing with a semi-implicit scheme and assuming intensity dependent diffusivity yields a system of equations that can be written in the matrix form,

$$\mathbf{A}^{k+1,l+1} \mathbf{I}^{k+1,l+1} = \mathbf{B}^{k+1,l+1} \mathbf{I}^k. \quad (6)$$

The first superscripts refer to the time steps, the second superscripts are the iteration cycle numbers. The kernels of the coefficient matrixes of (6) have the next forms

$$\mathbf{A}^{k+1,l+1} = \begin{bmatrix} 0 & -rc_{i,j-1/2}^{k+1/2,l+1} & 0 \\ -rc_{i-1/2,j}^{k+1/2,l+1} & (1+p) & -rc_{i+1/2,j}^{k+1/2,l+1} \\ 0 & -rc_{i,j+1/2}^{k+1/2,l+1} & 0 \end{bmatrix}, \quad (7)$$

$$\mathbf{B}^{k+1,l+1} = \begin{bmatrix} 0 & -rc_{i,j-1/2}^{k+1/2,l+1} & 0 \\ -rc_{i-1/2,j}^{k+1/2,l+1} & (1-p) & -rc_{i+1/2,j}^{k+1/2,l+1} \\ 0 & -rc_{i,j+1/2}^{k+1/2,l+1} & 0 \end{bmatrix}, \quad (8)$$

where $r = \Delta t / 2h^2$ and $p = \frac{\Delta t}{2h^2} \left(c_{i-1/2,j}^{k+1/2,l+1} + c_{i+1/2,j}^{k+1/2,l+1} + c_{i,j-1/2}^{k+1/2,l+1} + c_{i,j+1/2}^{k+1/2,l+1} \right)$,

the subscripts denote the spatial coordinates, $i, j = 1, \dots, n-1$, the mesh size is $n+1$ in both directions. This discretisation scheme requires diffusivities at the half time-steps and at the half mesh points i.e. the diffusivity has been determined with the so called time-centered method. The time step can be increased by this semi-implicit scheme but the computational cost increases as well. The upper limit to the time step can be evaluated from the following inequality $\Delta t \leq h^2 / c_{\max}$. A multigrid solver can reduce the solution time. Multigrid methods are more efficient in solving anisotropic diffusion problems than other iterative methods [2].

4. The fuzzy-rule based diffusivity

Fuzzy logic is a powerful tool to solve many image processing problems because of its ability to deal with ambiguous data. Some popular models are: the use of fuzzy membership function, fuzzy clustering, fuzzy-rule based systems, fuzzy entropy (measure of fuzziness), etc. [8]. Fuzzy rules serve to describe the quantitative relationship between variables in linguistic terms. In the proposed model, the rule-base is constructed so that it represents expert's conceptions about how to control the diffusivity. The fuzzy system is based on an I/O function that maps each very low-resolution quantization interval of the input domain into a very low-resolution quantization interval of the output domain. The mapping relationship can be very easily expressed using the IF-THEN formalism. The overlapping of these fuzzy domains and their linear membership functions will eventually allow achieving a rather high-resolution non-linear I/O function between crisp input and output variables.

The general structure of the proposed fuzzy-rule based image processing system is shown in Fig. 1. The inference mechanism emulates the expert's decision making. The fuzzification interface converts crisp input data into information that the inference mechanism can easily use to activate and apply IF-THEN rules. The defuzzification interface converts the conclusions of the inference mechanism into the actual crisp output. This fuzzy-diffusion process is a closed loop system.

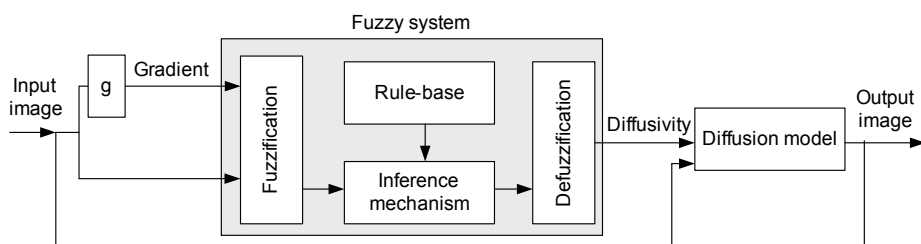


Fig. 1. The general structure of the fuzzy-rule based image smoothing

Membership functions are often determined heuristically and subjectively, however for example in [10] can be found some suggestions how to tune the membership functions with training data. Fuzzy rules can either be extracted from expert's knowledge or learned from numerical data. Linguistic rules and linguistic values are not precise representations of the underlying quantities that they describe, but at a level of abstraction that humans are often comfortable with in terms of specifying how to adjust an image.

In all presented examples the Mamdani max-min inference mechanism has been applied and the Center of Gravity (CoG) defuzzification method is approximated by the Center of Maximum (CoM) method. If the rule-base has more input variables, and conjunction has been supposed between the inputs and the activity index of the rule is the minimum of the actual membership values of the fuzzy inputs. For more detailed description of these methods can be found in the corresponding literature, for example

in [8] or [15]. Examples presented in this paper are very simple rule based systems serving demonstrative purposes.

5. The diffusivity is a function of the intensity

The first example is a single-input single-output fuzzy-rule based system. The input variable is the pixel intensity related to the temperature, the output variable is the diffusivity. Both linguistic variables have three values or terms: “low”, “medium” and “high”. The universe of the intensity and the diffusivity are normalized in $[0,1]$. The linguistic terms are mapped to the base scales with membership functions of triangular shapes. Triangular type membership functions are most commonly used in fuzzy logic applications because of its computational efficiency and simplicity. If the details of the hot regions have to be retained with degrading the other temperature regions, the membership functions presented in *Fig. 2* can be applied. The system consists of three rules, see *Table I*.

Table I

Rule base I

Rule	1.	2.	3.
IF intensity is	“low”	“medium”	“high”
THEN diffusivity is	“high”	“medium”	“low”

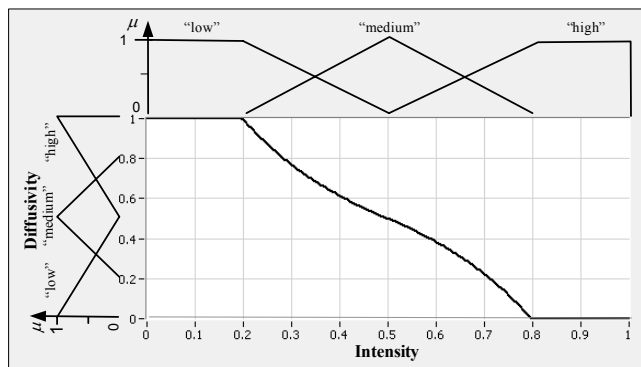


Fig. 2. The I/O characteristic of the rule base I

Rules listed above are “linguistic rules” since they are formed solely from linguistic variables and values. The I/O characteristic of this fuzzy system is shown in *Fig. 2*.

Although this system is very simple, the shape of the diffusivity function is non-linear. The diffusivity function decreases continuously in the intensity regions of overlapping membership functions. Where there is no overlapping, the diffusivity is

constant. The less the overlapping the more plateaus are in the diffusivity function. Overlapping of the output terms is marginal in CoM defuzzification.

An IR image of a turbulent steam flow can be seen in *Fig. 3a*. The hot steam jet has been mixed with the ambient, cooler air. The emission of the condensed water droplets has been imaged. Low temperatures are represented by dark shades and high temperatures by bright shades. The temperature provided by the camera did not represent the true gas or background temperature, since the camera used a fictitious total emissivity value, while the actual emissivity varied spectrally and spatially. The indicated fictitious detector temperature has been only slightly higher than the background temperature. This suggests that the steam transmissivity is quite high. The spatial variations of the background cannot be eliminated with averaging of the sampled images, because the summation approximates the Reynolds-averaged flow pattern. The results of the Gaussian isotropic diffusion are shown in *Fig. 3b*. Image resulted by the fuzzy rule-based diffusion is depicted in *Fig. 4a*. The Gaussian smoothing generates a blurred image. The fuzzy-diffusion preserves the region of the highest intensities while attenuates the diversity of regions with lower intensities. The corresponding diffusivity field can be seen in *Fig. 4b*.

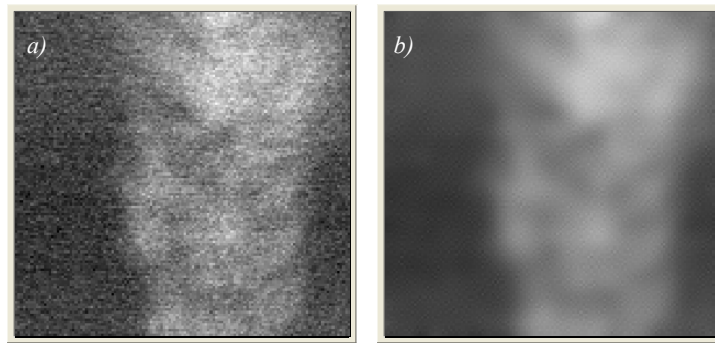


Fig. 3. a) Thermogram of a turbulent steam flow, b) result of the Gaussian diffusion

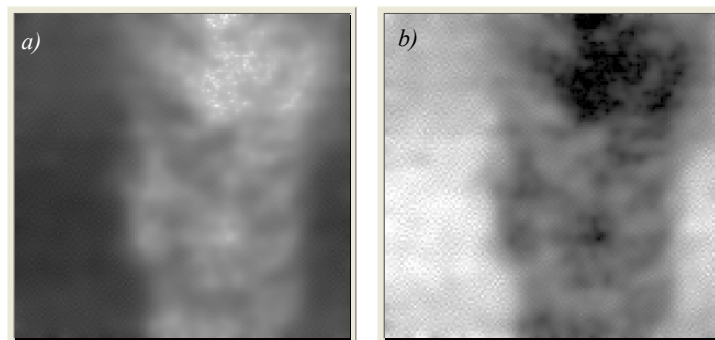


Fig. 4. a) Result of the fuzzy diffusion, b) diffusivity field, $k=3$

Analyzing the histograms of the processed images, the main differences appear at the high-, and low-intensity regions, see *Fig. 5*. The convolution with the Gaussian kernel has suppressed the intensity oscillations in the whole domain, the highest and the lowest intensities are fully damped. The lowest intensities are damped with the fuzzy filter as well, but higher intensities are preserved.

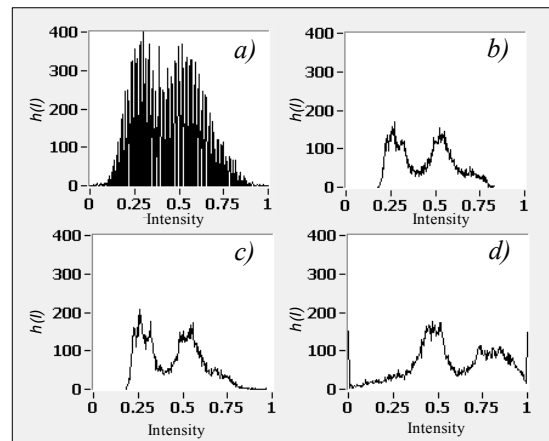


Fig. 5. Histograms of the images, a) input image, b) Gaussian-smoothed, c) fuzzy-diffusion processed, d) diffusivity field, $k=3$

The regions of medial intensities can be retained by changing the *rule base I* to the *rule base II*, see *Table II*. The I/O characteristic of the fuzzy system is shown in *Fig. 6*.

This type of filtering can precede for example a threshold operation. The middle-intensity values in a normalized and linear contrast-stretched image can be a borderline between the hot and cold image regions, and therefore this type of fuzzy-diffusion provides an edge-preserving filter.

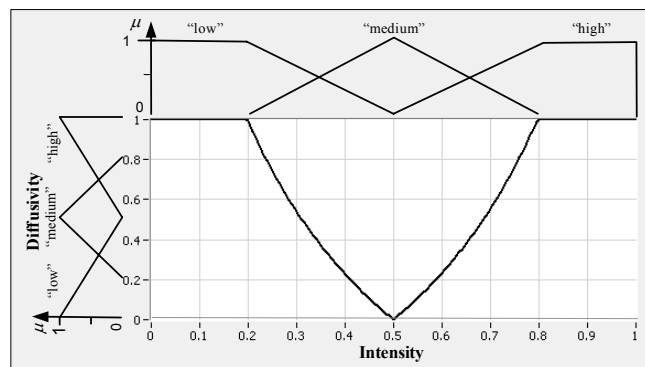


Fig. 6. The I/O characteristic of the *rule base II*

Table II

Rule base II

Rule	1.	2.	3.
IF intensity is	“low”	“medium”	“high”
THEN diffusivity is	“high”	“low”	“high”

Fig. 7a shows the thermogram of an opened valve in a steam pipework. Fig. 7b presents the result of the fuzzy diffusion with rule base II, $k = 2$. The background and the hot regions are smoothed, edges around the valve are preserved. The diffusivity field shows the edges in Fig. 7c. This type of image degradation can precede for example an automated object detection process.

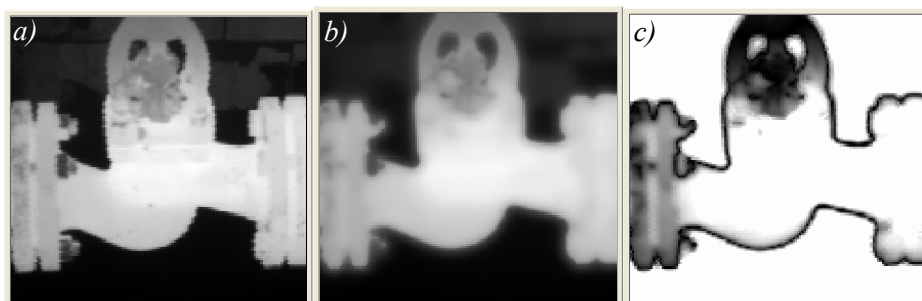


Fig. 7. a) The original image, b) smoothed with rule base II and c) the appropriate diffusivity field, $k=2$

6. The diffusivity is a function of the gradient

An edge preserving system distinguishes flat regional parts of the image from abruptly changing ones, such as edges and set the system parameters for each part independently. If the intensity difference between two adjacent pixels is small they are included probably in the same flat region. However it is difficult to find a definite threshold to distinguish small differences from large differences. In some part of the edges, the intensity difference may be relatively small and in some parts of the flat region the intensity difference may be relatively large because of noises. The definition of the signal characteristics and the rules to describe how to control the system parameters are usually expressed in an ambiguous form. For example a simple edge-preserving filter can be obtained by applying the next fuzzy-rule based system. The input of the system is the gradient measured at the pixel the output remains the diffusivity.

The terms of the input linguistic variable of gradient are “small” “medium” and “large”. Rules are presented in Table III.

Table III

Rule base III

Rule	1.	2.	3.
IF gradient is	“small”	“medium”	“large”
THEN diffusivity is	“high”	“medium”	“low”

The crisp value of the gradient is calculated from the neighboring pixel differences,

$$g_{ij} = \max \left(|I_{ij} - I_{i-k, j-m}| \right), \quad k, m = -1, 1, \quad i = 1, \dots, n-2, \quad j = 1, \dots, n-2. \quad (9)$$

The membership functions of the terms and the I/O characteristic of the system are shown in Fig. 8. The membership functions of the input terms are overlapped, so the transition from low to high diffusivity is continuous.

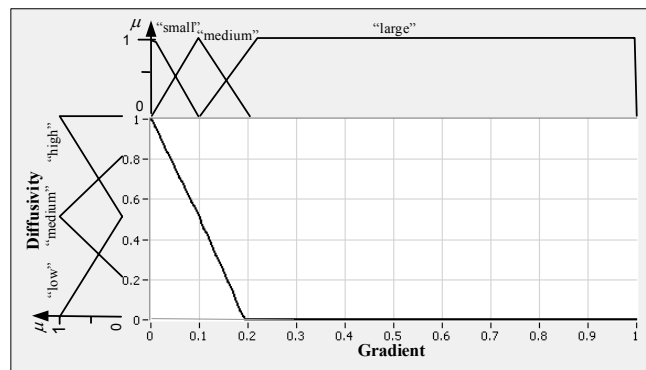


Fig. 8. The I/O characteristic of an edge preserving system with rule base III

The IR-image of a closed steam-valve can be seen in Fig. 9a. The initial diffusivity field is shown in Fig. 9b. Contours of fine details are recognizable. The result of the diffusion process is shown in Fig. 10a, $k = 3$. Only the main edges remained in the corresponding diffusivity field, see Fig. 10b.

Though the fuzzy system has found the edges but not the same certainty. Diffusion passes through edges of higher diffusivity, consequently edges of lower diffusivity could be better retained. The presented edge-preserving filter has to be adapted to the processed images.

7. The diffusivity depends on the intensity and the gradient

The above-mentioned fuzzy systems determine their outputs from one input condition, from the intensity *or* from the gradient. If both conditions have to be considered, a fuzzy system with two input variable can be constructed, with *and*

relationship between the terms. The rule base consists of 9 rules and can be written in a matrix form, see *Table IV*. Consequences are the elements of the matrix. *Fig. 11* shows the characteristic of the constructed system in 3D. The fuzzy decision process with two activated rules is represented in *Fig. 12*.

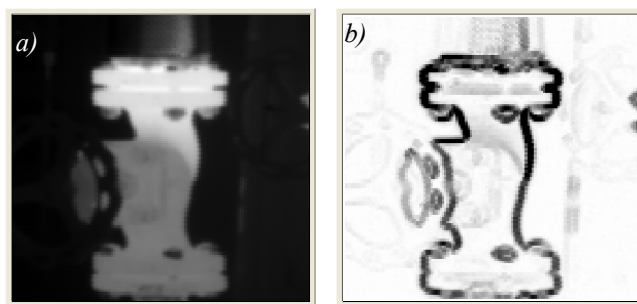


Fig. 9. a) The original image, b) the initial diffusivity field

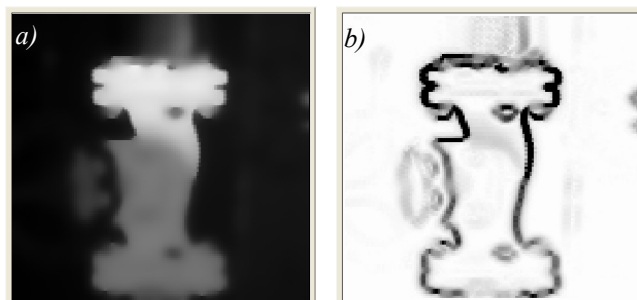


Fig. 10. a) The filtered image and b) the corresponding diffusivity, after two smoothing sweep with the system of *rule base III*

Table IV

Rule base IV

Intensity Gradient	“low”	“medium”	“high”
“small”	“high”	“low”	“medium”
“medium”	“medium”	“low”	“medium”
“large”	“low”	“low”	“low”

When the rule has two antecedents, the activity index can be calculated with some of the fuzzy intersection operators [12]. In the presented examples *and* conjunction is supposed and the activity index is calculated by the *min* operator.

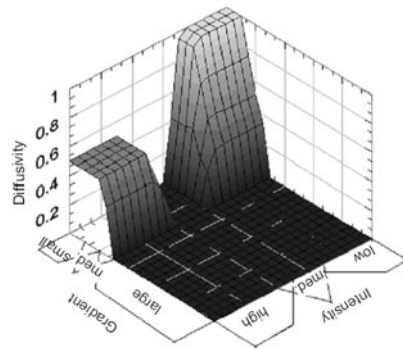


Fig. 11. I/O characteristic of the fuzzy system with two inputs and one output with *rule base IV*

Smoothing the original image in Fig. 9a with the system of *rule base IV*, the processed image and the diffusivity are shown in Fig. 13. The diffusivity field can be applied to detect the state of the valve. The average temperatures of the hot and cold valve regions can be easier evaluated from the smoothed image than from the original.

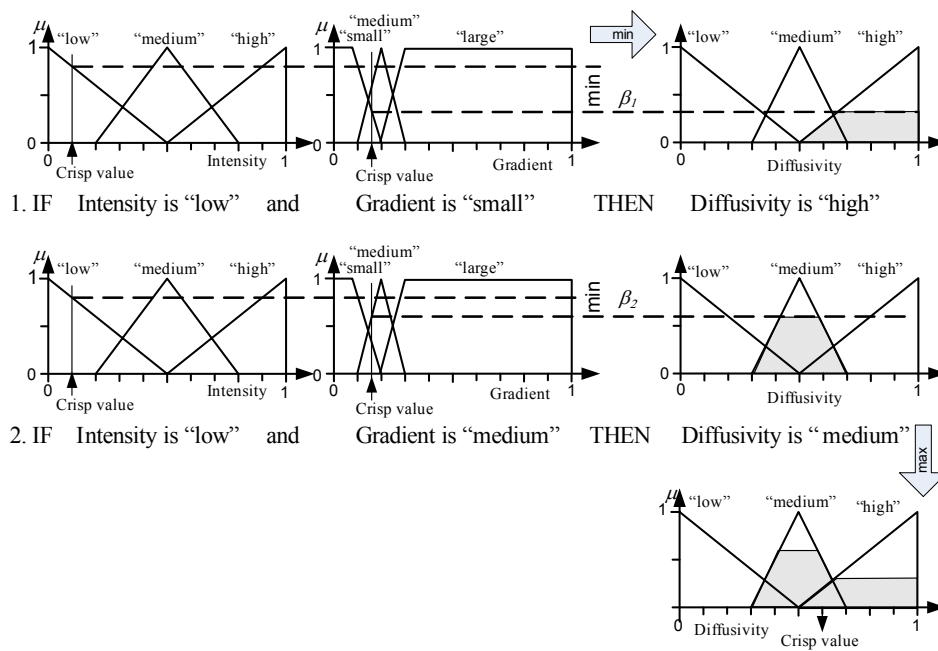


Fig. 12. Graphical representation of fuzzy system operation with two inputs – one output

Working with same linguistic variables and membership functions but modifying the rule base according to the *rule base V* in Table V, the brightest region can be retained and the other areas can be smoothed without blurring edges. The diffusivity function of the system is shown in Fig. 14. Result of the diffusion process and the diffusivity field can be seen in Fig. 15a and Fig. 15b, respectively.

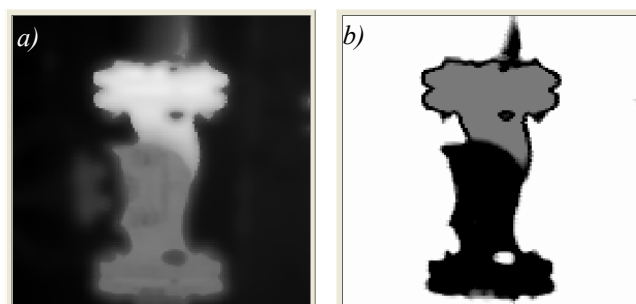


Fig. 13. The image in Fig. 9a is filtered with the rule base IV
a) result of diffusion, b) the diffusivity field, $k=3$

Table V

Rule base V

Intensity \ Gradient	“low”	“medium”	“high”
“small”	“high”	“high”	“low”
“medium”	“medium”	“medium”	“low”
“large”	“low”	“low”	“low”

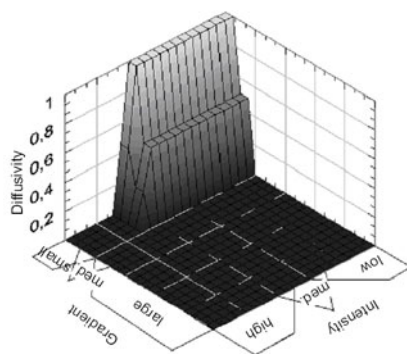


Fig. 14. The I/O characteristic of the fuzzy system of rule base V

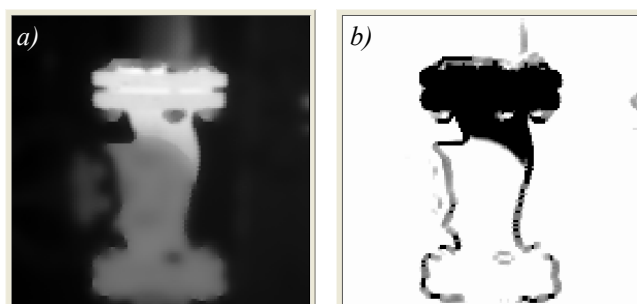


Fig. 15. a) Image in Fig. 9a has been smoothed with the rule base V, b) the diffusivity field, $k=3$

Image details in the regions of highest intensities are retained. Because the low intensity of the edges, the intensities in the other areas have been smoothed only intra-regionally, i.e. temperature variations only in bounded regions have been attenuated, averaged. This type of image enhancing can be useful for better visualization and for automated supervising of equipments.

8. Conclusions

The fuzzy-rule based system is a new technique for the composition of the diffusivity function. The non-linear diffusion can be a very useful pre-processing technique in the enhancement of thermal images. The main advantage of the proposed image processing technique is its ability to handle the human's conceptions in form of linguistic rules. It is easy expanded and formed to the user demands. Development of a system to an actual series of images, for example for computer vision, or to other engineering applications must be an iterative process that is based on the expert knowledge. The smoothing effect can be influenced by the number of the terms, by the shapes and overlaps of the membership functions. Not only the filtered thermal image, but also the diffusivity field can be a basis of segmentation or object detection processes.

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