



**Hungarian Delegation**

**Cost comparison of optimized welded square cellular plates supported at four corners with two different kinds of stiffeners**

**József Farkas<sup>1</sup>, Károly Jármai<sup>2</sup>**

<sup>1,2</sup>University of Miskolc, H-3515 Miskolc, Hungary,

<sup>1</sup>Professor emeritus, Dr.sci.techn. altfar@uni-miskolc.hu

<sup>2</sup>Professor, Dr.sci.techn. altjar@uni-miskolc.hu,

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**Abstract:** Cellular plates are constructed from two face plates and a stiffener grid welded between them. It is shown that the square cellular plates can be calculated as isotropic ones. Therefore, the classic formulae for maximum bending moment and deflection, valid for isotropic plates, can be used. The stiffeners can be made from halved rolled I-profiles, or from welded T-sections. These two kinds of cellular plates are optimized, and their minimum volumes and costs are compared to each other. The comparison shows that the cellular plate with welded T-stiffeners is more economic.

**Keywords:** cellular plates, welded stiffened plates, fabrication cost calculation, economy of welded structures, structural optimization

## 1. Introduction

Cellular plates consist of two face plates and a grid of stiffeners welded between them. The cells produce a large torsional stiffness; thus, the cellular plates can be calculated as isotropic ones.

In their previous studies, the authors have designed cellular plates with halved rolled I-stiffeners. In the present study, these rolled stiffeners are replaced by welded T-stiffeners. The comparison of the cellular plates with the two different kinds of stiffeners shows that using welded T-stiffeners significant savings in mass and cost can be achieved.

The formulae for the two kinds of stiffeners are nearly the same; thus, the formulae for halved rolled I-stiffeners are described and then the differences for welded T-stiffeners are given.

## 2. Bending and torsional stiffness of a cellular plate

The Huber's equation for orthotropic plates in the case of a uniform transverse load

$$B_x w'''' + 2Hw'''' + B_y w'''' = p \quad (1)$$

where

$$H = B_{xy} + B_{yx} + \frac{\nu}{2}(B_x + B_y) \quad (2)$$

is the torsional stiffness of an orthotropic plate.

The corresponding bending and torsional stiffnesses are defined as

$$B_x = \frac{E_1 I_y}{a_y}; B_y = \frac{E_1 I_x}{a_x}; E_1 = \frac{E}{1 - \nu^2} \quad (3)$$

for cellular plates

$$B_{xy} = \frac{GI_y}{a_y}; B_{yx} = \frac{GI_x}{a_x}; G = \frac{E}{2(1 + \nu)} \quad (4)$$

$$H = B_{xy} + B_{yx} + \frac{\nu}{2}(B_x + B_y) = \frac{E_1}{2} \left( \frac{I_y}{a_y} + \frac{I_x}{a_x} \right) \quad (5)$$

for plates of quadratic symmetry

$$H = B_x = B_y \quad (6)$$

Thus, the torsional stiffness of a cellular plate of quadratic symmetry equals to its bending stiffness.

## 3. Bending moments and deflections

Lee et al. (1971) have solved the differential equation for rectangular orthotropic plates (Eq.1) supported at four corners by using a polynomial function.

Formulae have given for bending moments and deflections as a function of bending and torsional stiffnesses. In the case of a square cellular plate, the bending stiffnesses are equal to the torsional stiffness ( $B_x = B_y = H$ ) and the maximum bending moment is

$$M_{max} = 0.15pL^2 \quad (7)$$

and the maximum deflection is expressed by

$$w_{max} = 0.025p_0L^4/B_x \quad (8)$$

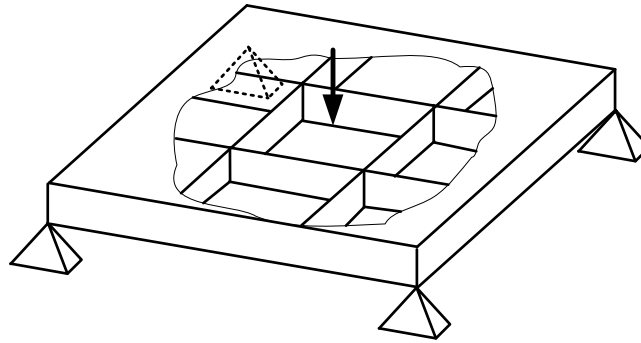


Figure 1. A cellular plate supported at four corners

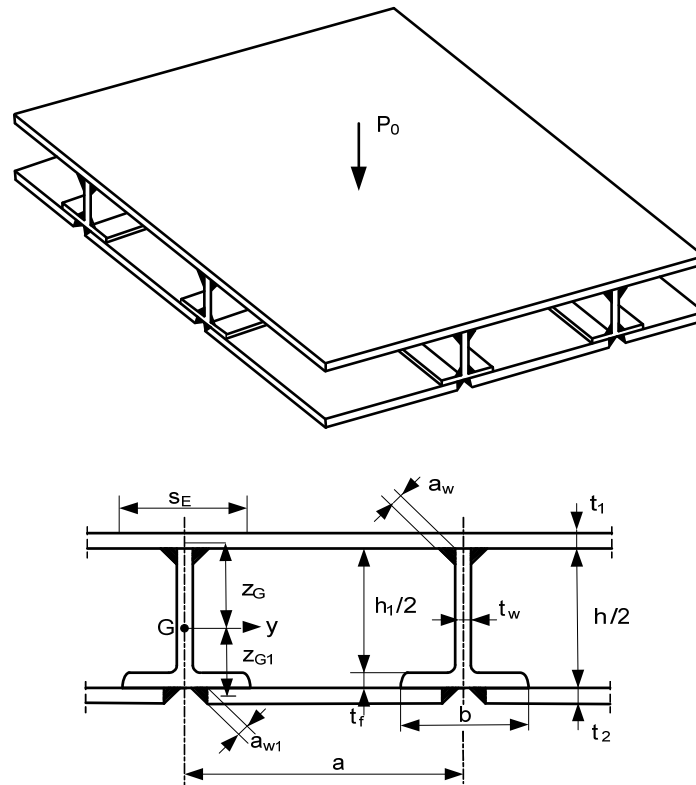


Figure 2. Cellular plate and dimensions of halved rolled I-section stiffener

where  $L$  is the plate edge length,  $p_0$  is the factored intensity of the uniformly distributed normal load and  $p$  is the load intensity including the self mass of the plate.

Results for square isotropic plates according to Timoshenko & Woinowsky-Krieger (1959)

for  $v = 0.3$

$$M_{max} = 0.1404pL^2 \quad (9)$$

and

$$w_{max} = 0.0249p_0L^4/B_x \quad (10)$$

It can be seen that the constants are nearly the same.

#### 4. Cellular plate with halved rolled I-section stiffeners

##### 4.1 Geometric characteristics (Fig. 2)

The upper face plate parts can locally buckle from the compression stresses caused by bending. This local buckling is avoided by using effective plate widths according to Eurocode 3 (2007)

$$s_e = \rho s, s = \frac{a}{n} \quad (11)$$

$$\rho = \frac{\lambda_p - 0.22}{\lambda_p^2}, \lambda_p = \frac{s}{56.8\epsilon t_1}, \epsilon = \sqrt{\frac{235}{f_y}} \quad (12)$$

$n$  is the number of spacing,  $f_y$  is the yield stress.

Cross-sectional area of a halved rolled I-section stiffener

$$A_s = \frac{h_1 t_w}{2} + b t_f, \quad h_I = h - 2t_f \quad (13)$$

Cross-sectional area of a stiffener with upper and bottom base plate parts

$$A = s_e t_1 + a t_2 + A_s, \quad a = \frac{L}{n+1} \quad (14)$$

Distances of the gravity center

$$z_G = \frac{1}{A} \left[ a t_2 \left( \frac{h}{2} + \frac{t_1}{2} + \frac{t_2}{2} \right) + b t_f \left( \frac{h_1 + t_1 + t_f}{2} \right) + \frac{h_1 t_w}{2} \left( \frac{h_1}{4} + \frac{t_1}{2} \right) \right] \quad (15)$$

$$z_{G1} = \frac{h + t_1 + t_2}{2} - z_G \quad (16)$$

Moment of inertia

$$I_y = s_e t_1 z_G^2 + a t_2 z_{G1}^2 + b t_f \left( \frac{h_1 + t_1 + t_f}{2} - z_G \right)^2 + I_{y1} \quad (17)$$

$$I_{y1} = \frac{h_1^3 t_w}{96} + \frac{h_1 t_w}{2} \left( \frac{h_1}{4} + \frac{t_1}{2} - z_G \right)^2 \quad (18)$$

Bending stiffness

$$B_x = \frac{E_1 I_y}{a}, E_1 = \frac{E}{1 - \nu^2} \quad (19)$$

Structural volumes corresponding to each fabrication phase are as follows:

$$V_1 = L^2 t_1, V_2 = V_2 + (n + 2) A_s L, V_3 = V_2 + (n + 2) A_s L \quad (20)$$

$$V_4 = V_3 + L^2 t_2 \quad (21)$$

Load intensity including the self mass

$$p = p_0 + \frac{\rho_0 V_4}{L^2} \quad (22)$$

## 4.2 Design constraints

Stress constraint including normal stress due to local bending of an upper base plate part with built-in edges according to Timoshenko & Woinowsky-Krieger (1959)

$$\sigma_p = 0.0513 \frac{p_0 a^2}{t_1^2 / 6} = 0.3078 \frac{p_0 a^2}{t_1^2} \quad (23)$$

$$\sigma_2 = \frac{0.15 p L^2 z_G}{I_y} + \sigma_p \leq \frac{f_y}{1.1} \quad (24)$$

Constraint on stress in the lower face plate

$$\sigma_1 = \frac{0.15 p L^2 z_{G1}}{I_y} \leq \frac{f_y}{1.1} \quad (25)$$

Deflection constraint

$$w_{max} = \frac{0.025 p_0 L^4}{B_x} \leq w_{allow} = \frac{L}{300} \quad (26)$$

Shear stress constraint at the corners

$$\tau = \frac{p L^2}{4 h_1 t_w} \leq \frac{f_y}{1.1 \sqrt{3}} \quad (27)$$

### 4.3 Fabrication constraints

Thickness limitation:  $t_{min} = 4$  mm.

Limitation of the distance between stiffener flanges to allow the welding of the stiffener web to the upper base plate:

$$a - b \geq 300 \text{ mm.} \quad (28)$$

### 4.4 Structural characteristics to be changed (variables)

- number of stiffeners in one direction (square symmetry)  $n$ ,
- thicknesses of the upper and bottom base plates  $t_1$  and  $t_2$ ,
- height of the rolled I-section stiffener  $h$ .

Dimensions of UB profiles are given in Table 1. (ArcelorMittal Profile Catalogue 2013)

Table 1 Selected UB profiles according to the ArcelorMittal catalogue

UB Profile	$h$ Mm	$b$ mm	$t_w$ mm	$t_f$ mm	$A_s$ mm <sup>2</sup>	$I_y \times 10^{-4}$ mm <sup>4</sup>
152x89x16	152.4	88.7	4.5	7.7	2032	834
168x102x19	177.8	101.2	4.8	7.9	2426	1356
203x133x25	203.2	133.2	5.7	7.8	3187	2340
254x102x25	257.2	101.9	6.0	8.4	3204	3415
305x102x28	308.7	101.8	6.0	8.8	3588	5366
356x127x39	353.4	126.0	6.6	10.7	4977	10172
406x140x46	403.2	142.2	6.8	11.2	5864	15685
457x152x60	454.6	152.9	8.1	13.3	7623	25500
533x210x92	533.1	209.3	10.1	15.6	11740	55230
610x229x113	607.6	228.2	11.1	17.3	14390	87320
686x254x140	683.5	253.7	12.4	19.0	17840	136300
762x267x173	762.2	266.7	14.3	21.6	22040	205300
838x292x194	840.7	292.4	14.7	21.7	24680	279200
914x305x224	910.4	304.1	15.9	23.9	28560	376400
1016x305x349	1008.1	302	21.1	40.0	44420	722300
1016x305x393	1016.0	303	24.4	43.9	50020	807700

### 4.5 Numerical data

Plate edge length:  $L = 18$  m, factored load intensity  $p_0 = 150 \text{ kg/m}^2 = 0.0015 \text{ N/mm}^2$ , yield stress of steel  $f_y = 355 \text{ MPa}$ , elastic modulus  $E = 2.1 \times 10^5 \text{ MPa}$ , Poisson ratio  $\nu = 0.3$ , steel density  $\rho = 7.85 \times 10^{-6} \text{ kg/mm}^3$ ,  $\rho_0 = 7.85 \times 10^{-5} \text{ N/mm}^3$ .

### 4.6 Cost function

The cost function is formulated according to the fabrication sequence.

(a) Welding of the upper base plate (18x18 m) from 36 pieces of size 6 m x 1.5 m using single or double bevel welds with complete joint penetration (GMAW-C gas metal arc welding with CO<sub>2</sub>):

$$K_{w1} = k_w \left[ \Theta \sqrt{36 \rho V_1} + 1.3 C_1 t_1^{n1} 13L \right] \quad (29)$$

welding cost factor  $k_w = 1$  \$/kg, factor for the complexity of assembly  $\Theta = 3$ ,

$$\text{for } t_1 < 15 \text{ mm } C_1 = 0.1939 \text{ and } n1 = 2 \quad (30a)$$

$$\text{for } t_1 > 15 \text{ mm } C_1 = 0.1496 \text{ and } n1 = 1.9029. \quad (30b)$$

(b) Welding of  $n+2$  continuous stiffeners to the upper base plate by double fillet welds (GMAW-C)

$$K_{w2} = k_w \left[ \Theta \sqrt{(n+3) \rho V_2} + 1.3 \times 0.3394 \times 10^{-3} a_w^2 2(n+2)L \right] \quad (31)$$

$a_w = 0.4t_w$ , but  $a_{wmin} = 4$  mm.

(c) Welding of  $n+2$  intermittent stiffeners to the upper base plate and to the continuous stiffeners (webs with fillet welds, flanges with butt welds GMAW-C)

$$K_{w3} = k_w \left[ \Theta \sqrt{(n^2 + 3n + 3) \rho V_3} + T_1 + T_2 \right] \quad (32)$$

$$T_1 = 1.3 \times 0.3394 \times 10^{-3} a_w^2 (h_1 + b) 2(n+1)(n+2) \quad (33)$$

$$T_2 = 1.3 C_1 t_f^{n1} 2b(n+1)(n+2) \quad (34)$$

(d) Welding of the bottom plate parts to the flanges of stiffeners by fillet welds (GMAW-C)

$$K_{w4} = k_w \left[ \Theta \sqrt{(n^2 + 2n + 2) \rho V_4} + 1.3 \times 0.3394 \times 10^{-3} a_{w1}^2 4L(n+1) \right] \quad (35)$$

$a_{w1} = 0.4t_2$ , but  $a_{w1min} = 3$  mm.

Cost of material

$$K_M = k_M \rho V_4, \quad k_M = 1 \text{ $/kg}, \quad (36)$$

Cost of painting

$$K_P = k_P \Theta_P S_P, \quad \Theta_P = 3, \quad k_P = 14.4 \times 10^{-6} \text{ $/mm}^2 \quad (37)$$

surface to be painted

$$S_P = 3L^2 + 2L(h_1 + b)(n+2) \quad (38)$$

Total cost

$$K = K_M + K_{w1} + K_{w2} + K_{w3} + K_{w4} + K_P \quad (39)$$

## 4.7 Optimization and results

A systematic search for the optima is performed using a MathCAD algorithm. The results are given in Table 2.

Table 2. Results of a systematic search. Halved rolled I-stiffeners. The optima are marked by bold letters. Allowed normal stress ( $\sigma_2$ ) 322 MPa, allowed deflection  $w_{\max} = 60$  mm. Dimensions in mm

$h$	$n$	$t_1$	$t_2$	$\sigma_2$ MPa	$w_{\max}$ mm	$V \times 10^{-9} \text{ mm}^3$	$K \times 10^{-5} \$$
683.5	4	11	4	286	38	6.766	1.273
	5	9	4	315	40	6.435	1.258
	6	8	4	317	41	6.429	1.286
<b>607.6</b>	3	13	4	311	49	6.791	1.253
	<b>4</b>	<b>11</b>	<b>4</b>	316	50	6.400	<b>1.224</b>
	5	10	4	302	50	6.332	1.240
	6	9	4	302	51	6.265	1.258
	<b>7</b>	<b>8</b>	<b>4</b>	313	52	<b>6.197</b>	1.277
533.1	3	14	5	312	55	7.200	1.292
	4	12	5	314	57	6.761	1.255
	5	11	5	298	57	6.646	1.263
	6	10	5	296	58	6.530	1.272

## 5. Cellular plate with welded T-stiffeners

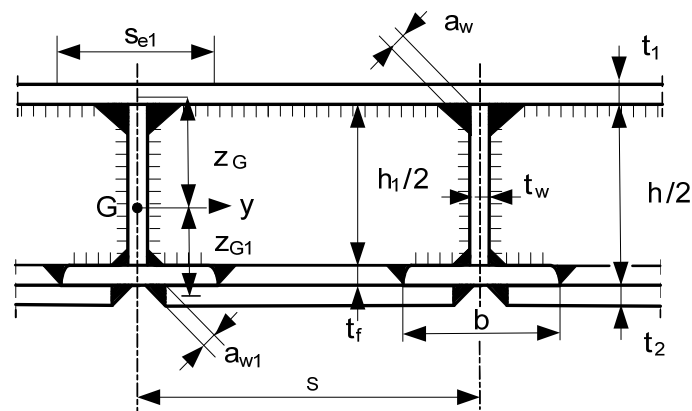


Figure 3. Cellular plate with welded T-stiffeners

### 5.1 Geometric characteristics (Fig.3)

The formulae of the characteristics of halved rolled I-section stiffeners are valid also for welded T-stiffeners with two differences:  $b = 30 \text{ mm}$  and  $t_f = t_2$ .

## 5.2 Buckling constraint for the stiffener web

The web thickness should be taken to fulfil the following buckling constraint: according to Eurocode 3-1-5 for plates with varying longitudinal stress



$$\frac{0.15pL^2z_G}{I_y} \leq C_x f_y \quad (40)$$

where

$$C_x = 1 \text{ when } \bar{\lambda}_p \leq 0.673 \quad (41a)$$

$$C_x = \frac{\bar{\lambda}_p - 0.055(3 + \psi)}{\bar{\lambda}_p^2} \text{ when } \bar{\lambda}_p \geq 0.673 \quad (41b)$$

$$\bar{\lambda}_p = \frac{h_1 / 2}{t_w} \frac{1}{28.4 \varepsilon \sqrt{k_\sigma}} \quad (42)$$

for  $-1 \leq \psi < 0$

$$k_\sigma = 7.81 - 6.29\psi + 9.78\psi^2 \quad (43)$$

$$\psi = -\frac{h_1 / 2 - z_G}{z_G} \quad (44)$$

### 5.3 Cost function

In the cost function, the following changes are considered. Instead of Eqs (31, 32) the following formulae are used

$$K_{w2} = k_w \left[ \Theta \sqrt{(2n+5)\rho V_2} + 1.3 \times 0.3394 \times 10^{-3} a_w^2 4(n+2)L \right] \quad (45)$$

$$K_{w3} = k_w \left[ \Theta \sqrt{(2n^2 + 6n + 5)\rho V_3} + T_1 + T_2 \right] \quad (46)$$

### 5.4 Optimization and results

The results of a systematic search are given in Table 3.

Table 3. Results of a systematic search. Welded T-stiffeners. The optima are marked by bold letters. Allowed normal stress ( $\sigma_2$ ) 322 MPa, allowed deflection  $w_{\max} = 60$  mm. Dimensions in mm

$h$	$n$	$t_1$	$t_2$	$t_w$	$\sigma_1$ MPa	$w$ mm	web buckling MPa	$V \times 10^{-9} \text{ mm}^3$	$K \times 10^{-5} \$$
1400	4	7	4	6	309	12	187<201	4.492	1.075
<b>1300</b>	<b>5</b>	<b>6</b>	<b>4</b>	<b>6</b>	320	16	204<215	<b>4.247</b>	1.074
1200	3	9	4	6	321	16	206<232	4.877	1.073
<b>1100</b>	<b>4</b>	<b>8</b>	<b>4</b>	<b>6</b>	309	20	215<251	4.622	<b>1.068</b>
1000	5	7	4	7	317	24	232<310	4.469	1.070
900	4	9	4	5	311	26	237<258	4.720	1.070
800	5	8	4	7	322	37	258<375	4.622	1.072

## **6. Comparison of the two optimized cellular plates with different stiffeners**

According to the results summarized in Tables 2 and 3 it can be concluded that the cellular plate with welded T-stiffeners is more economic, than that with halved rolled I-section stiffeners, since the mass is  $(6.197-4.247)/6.197 \times 100 = 31\%$  smaller and the cost is  $(1.224-1.068)/1.224 \times 100 = 12\%$  smaller.

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