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**МЕЖДУНАРОДНАЯ НАУЧНО-ТЕХНИЧЕСКАЯ
КОНФЕРЕНЦИЯ
МЕТАЛЛИЧЕСКИЕ КОНСТРУКЦИИ**

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COMPUTER-AIDED OPTIMAL DESIGN OF STRUCTURES
MADE OF HIGHER STRENGTH STEELS

1. Introduction

The most natural and rigorous way of attacking structural weight minimization is to make use of mathematical programming methods. This approach will be briefly reviewed in this paper, with emphasis on the practically important property of preserving the feasibility of the design. In the field of structural optimization the computer becomes central as a tool for searching and sorting through the similar design concepts and reporting the element details for the most economical design.

A general nonlinear programming /NLP/ problem can be stated as one of choosing $\underline{x} = /x_1, x_2, \dots, x_N/$ such that minimize or maximize

$$F = f /x_1, x_2, \dots, x_N/ \quad /1/$$

subject to constraints

$$\begin{aligned} x_i &\leq x_i \leq x_i \quad U \quad i = 1, 2, \dots, M & /2/ \\ x_i &\geq 0 & /3/ \end{aligned}$$

F is the objective function, expressed in nonlinear terms of the variables; Eqs. /2/ are inequality constraints. The explicit variables $x_i, i = 1, 2, \dots, N$, usually represent physical parameters of the structure to be designed such as dimensions, spacings, bar sizes, plate thicknesses, etc. The implicit variables $x_i, i = N + 1, \dots, M$ are dependent functions of explicit independent variables. The explicit constraints specify, for example, the mechanical behaviour of the structure under load, the known properties of the materials used, fabrication requirements, geometric and layout requirements, etc. The non-negativity condition upon each explicit variable is necessary to ensure that values are obtained for all the problem variables which are real and feasible in an engineering sense.

In our research work we dealt with various programming methods, which can be efficiently used for the solution of nonlinear optimization problems: the combinatorial discrete programming method: backtrack [1,2], the complex method of Box [3,4] and the "Direct Search -

Feasible Direction" /DSEFD/ algorithm of Pappas [5].

In this paper a kind of direct search method is treated; Rosenbrock's "Hill" procedure [6]. At first the method is described, secondly the results of a numerical example are compared to the results of the back-track, complex and DSEFD methods.

2. The Hill algorithm

The procedure is based on the "automatic" method proposed by Rosenbrock [6] according to equations /1,2,3/. The coordinate system is rotated in each stage of minimization in such a manner that the first axis is oriented towards the locally estimated direction of the valley and all the other axes are made mutually orthogonal and normal to the first one. No derivatives are required. The algorithm proceeds as follows:

/i/ Before starting the minimization process, define a set of "initial" step lengths, S_i $i = 1, 2, \dots, N$ to be taken along the search directions $M_{i,j}$ $j = 1, 2, \dots, N$, a starting point that satisfies the constraints and does not lie in the boundary zones, and evaluate the objective function. The boundary zones are defined as follows:

$$\text{Lower Zone: } x_i \leq x_i \leq x_i + \frac{U_i - x_i}{x_i} \cdot 10^{-4} \quad /4/$$

$$\text{Upper Zone: } x_i \geq x_i \geq x_i - \frac{U_i - x_i}{x_i} \cdot 10^{-4} \quad /5/$$

/ii/ Define by F_0 the current best objective function value for a point where the constraints are satisfied.

/iii/ The first variable x_1 is stepped a distance S_1 parallel to the axis and the function is evaluated. If the current point objective function value, F , is worse /larger or less/ than F_0 or if the constraints are violated, the trial point is a failure, and S_1 is decreased by a factor β , $0 < \beta \leq 1,0$, and the direction of movement is reversed. If the move is termed a success, S_1 is increased by a factor α , $\alpha \geq 1,0$, the new point is retained, and a success is recorded.

/iv/ Continue the search sequentially stepping the variables, x_i , a distance S_i parallel to the axis. The same-acceleration or deceleration and reversal procedure is followed for all variables until at least one step has been successful and one step has failed in each of the N directions.

/v/ Compute the new set of directions $M_{i,j}$ /k/ rotating the axes.
/vi/ Search is made in each of the x directions using the new coordinate axes

$$\text{new } x_i /k/ = \text{old } x_i /k/ + S_j /k/ M_{i,j} /k/$$

/vii/ If the current point lies within a boundary zone, the objective function is modified as follows:

$$F / \text{new} / = F / \text{old} / - F / \text{old} / \cdot F_0 / 3 h - 4 h^2 + 2 h^3 /$$

where

$$h = \frac{\text{distance into boundary zone}}{\text{width of boundary zone}}$$

according to /4,5/.

/viii/ The search procedure to find the continuous values of the variables is terminated when the convergence criterion is satisfied.

/ix/ The procedure was modified by a secondary search, to find the discrete values of the variables [4].

3. Minimum weight design of welded plane frames

3.1. Design data

We illustrate the application of the Hill, complex, backtrack and DSEFD methods by the numerical example of a plane frame, made of welded I-section bars as shown in Fig. 1.

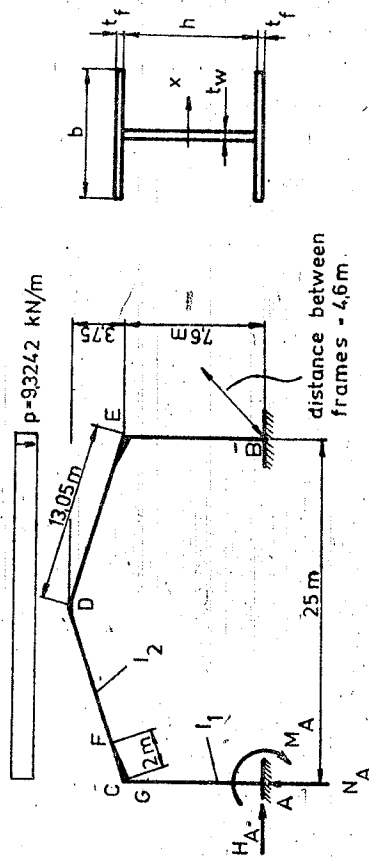


Fig. 1. Geometrical and loading conditions of the plane frame

We consider the sizes of columns and rafters to be different. So the number of unknown variables is eight. Loading conditions:

The load applied to the frame is also shown in Fig. 1. The intensity of the uniformly distributed normal load is $p = 1,1 \cdot 0,57 + 1,4 \cdot 1,0 / 4,6 = 9,3242$ [kN/m²] /9/
Steel Fe 360 or Fe 510 is used.

3.2. Design constraints

a./ Stress constraints

Maximum elastic stresses in the columns and rafters due to bending and compression must be lower than the admissible stress.

We introduce the ratio of moments of inertia

$$k = \frac{I_2}{I_1} \quad /10/$$

For the columns, the maximum stress occurs at the point A. By using the formulae of Glushkov [7] we get

$$M_1 = |M_A| = \frac{3266,8895 k + 989,5919}{1,0175 k^2 + 13,8624 k + 0,73031} \quad [kNm] \quad /11/$$

$$N_1 = |N_A| = \frac{2 ps^2}{L} = 116,6 \quad [kN] \quad /12/$$

For the rafters the maximum stress occurs at the point F, at the end of haunching

$$M_2 = |M_F| = 8,174696 H_A - |M_A| = 206,2564 \quad /13/$$

$$N_2 = |N_F| = 28,373 + 0,957826 H_A \quad /14/$$

where

$$M_C = \frac{4963,8363 k + 88,5588}{1,0175 k^2 + 13,8624 k + 0,73031} \quad /15/$$

$$H_A = (|M_A| + M_C) / 7,6 \quad /16/$$

The maximum stresses in the columns and rafters are as follows

$$\sigma_{Ni} + \sigma_{Ni} = M_i / W_i + N_i / A_i \leq R_{Ni} \quad /17/$$

Considering the I-section sizes as shown in Fig. 1. the cross-section areas are

$$A_i = h_i t_{wi} + 2 b_i t_{fi} \quad /18/$$

The section moduli may be expressed as

$$W_i = h_i (b_i t_{fi} + h_i t_{wi} / 6) \quad /19/$$

The moments of inertia are

$$I_i \approx W_i h_i / 2 \quad i = 1, 2 \quad /20/$$

The subscripts 1 and 2 are valid for columns and rafters, respectively. The limit stress for steel Fe 360 can be taken as $R_u = 200$ MPa and for steel Fe 510 $R_u = 280$ MPa.

b./ Local web buckling constraints

In the case of bending and compression the buckling constraint may be expressed as $t_{wi} / h_i \geq C_1$. /21/

According to Frieze [8], for a web made of steel Fe 360 subjected to bending it is $C_1 = 1/145$, for steel Fe 510 it is

$$C_1 = \frac{1}{145} \sqrt{R_{adm510} / R_{adm360}}$$

For a web subjected to bending σ_M ; compression σ_N and shear τ the following approximate interactive formula may be used:

$$\frac{1}{C_1} = 145 \sqrt{\frac{4(1 + \sigma_N / \sigma_M)^2 + 3(\tau / \sigma_M)^2}{1 + 173(\sigma_N / \sigma_M)^2 + 20(\tau / \sigma_M)^2}} \quad /22/$$

Thus, in our case, neglecting the effect of shear, for steel Fe 360 we can write

$$\frac{h_i}{t_{wi}} \leq 145 \sqrt{\frac{4(1 + \sigma_{Ni} / \sigma_{Mi})^2}{1 + 173(\sigma_{Ni} / \sigma_{Mi})^2}} \quad ; \quad i = 1, 2 \quad /23/$$

and for steel Fe 510

$$\frac{h_i}{t_{wi}} \leq 145 \sqrt{\frac{R_{adm360}}{R_{adm510}} \cdot \frac{4(1 + \sigma_{Ni} / \sigma_{Mi})^2}{1 + 173(\sigma_{Ni} / \sigma_{Mi})^2}} \quad ; \quad i = 1, 2 \quad /24/$$

c./ Local flange buckling constraints

In the case of a compressed flange the buckling constraint can be written as $t_{fi} / b_i \geq C_2$. /25/

For a flange made of steel Fe 360 it is $C_2 = \frac{1}{30}$, for steel Fe 510 it is

$$C_2 = \frac{1}{30} \sqrt{\frac{R_{adm510}}{R_{adm360}}} \quad /26/$$

Thus, we use the following constraints:

$$b_i / t_{fi} \leq 30 \quad ; \quad i = 1, 2 \quad /27/$$

for steel Fe 360, and

$$b_i / t_{fi} \leq 30 \sqrt{\frac{R_{adm360}}{R_{adm510}}} \quad ; \quad i = 1, 2 \quad /28/$$

for steel Fe 510.

3.3. Objective function

When the weight of the structure is selected as the objective to be minimized in $1/L$ this quantity is computed as a sum of member weights.

$$F/x_1 = \sum_{i=1}^p Q_i L_i \quad /29/$$

where A_i and L_i are the area and length of the i th member, respectively and ρ is the material density.

Since ρ is the same for each member, the volume of the frame can be taken as objective function.

$$V/x_1 = 2 \sum_{i=1}^2 L_i A_i \quad /30/$$

$$V/x_1 = 2 / 7,6 \cdot A_1 + 13,05 \cdot A_2 / \cdot 10^5 \text{ [mm}^3] \quad /31/$$

Thus, $/31/$ is a linear function of the unknown sizes.

3.4. The COMMODORE VC-64 microcomputer system as a tool of structural optimization

The computer programs were written in FORTRAN IV and BASIC and run on computer CDC 3300, ODRA 1304 and COMMODORE VC-64, respectively. We elaborated the optimal design of the following structures with the three computers:

- /a/ simply supported I - and box cross-sectional and tubular girders,
- /b/ sandwich beams with thin metal faces and with outer layers of box cross-section,
- /c/ welded portal frames with various columns and rafters,
- /d/ main girders of welded box cross-section of overhead travelling cranes using higher strength steels,
- /e/ cellular plates.

The volume of the CPU at CDC 3300 is 256 K words, at ODRA 1304 is 23 K words and at VIC 64: 64 K bytes. We found in our works that for problems which need not too much memory, the VIC 64 microcomputer system is excellent. The system consists of a VIC-64 microcomputer, a VIC-1541 single drive floppy disk and a GP-100 VC graphic printer. This disk drive allows us to store up on a single mini-floppy diskette for a maximum of over 174 K bytes worth of information storage. It does its own processing without taking any memory away from VIC 64. The VIC 64 is slower than ODRA 1304 and much slower than CDC 3300, but the CPU time can be reduced by using the "Astro-compiler". It converts the BASIC program into machine codes. In our programs the CPU time in this system was reduced to third-fourth part using the compiler. The possibility of interactive design is very precious using this system and the testing time of a new program is much more shorter than that of the other

two computers.

3.5. Results of computations by the four programming methods

Table 1. contains the discrete values, the bounds and the step sizes the eight explicit variables using steel Fe 360.

Table 1.

Final values of discrete explicit variables of the numerical example using steel Fe 360.

method	back-track	complex	DSFD	Rosenbrock	bounds on explicit variables	step size
$x_1 = h_1/\text{mm}$	580	580	660	560	840 520	20
$x_2 = t_{w1}$	5	5	6	5	13 4	1
$x_3 = b_1$	240	260	260	240	420 100	20
$x_4 = t_{f1}$	11	12	9	12	20 4	1
$x_5 = h_2$	500	540	580	560	740 420	20
$x_6 = t_{w2}$	5	6	6	5	12 4	1
$x_7 = b_2$	100	160	100	200	420 100	20
$x_8 = t_{f2}$	16	9	12	7	20 4	1
$F/x_1 = V \cdot 10^{-8} / \text{mm}^3$	2,73	2,98	2,85	2,76		
number of iteration		148	149	15		
number of tested combination	1,95					

Using steel Fe 510 the following results obtained by the Rosenbrock method

Table 2.

Final values of discrete explicit variables using steel Fe 510.

h_1/mm	t_{w1}	b_1	t_{f1}	h_2	t_{w2}	b_2	t_{f2}	$F/x_1 / \cdot 10^{-8} / \text{mm}^3$
Fe 360 580	5	260	12	540	6	160	9	2,98
Fe 510 580	6	220	11	460	6	160	7	2,64

It is shown that using higher strength steel the volume and so the weight of the frame decreased by 11,3 %. The deflection of the frame increased by 20 %.

3.6. Comparison of programming methods

The backtrack method is the slower, depending on the step sizes and the number of unknown variables. The CPU time of complex and DSFD is fifth-tenth part of the backtrack's one. The quickest is the Rosenbrock method. The advantages of the backtrack method become visible at the final values of the merit functions. It is less by 3-4 % than the others. At the DSFD method the starting point may violate the constraints it can find the solution as well. Disadvantage of this method is that it easily gives local optima, so for the feasibility it is necessary to start from another point too.

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RECHNERGESTÜTZTE OPTIMIERUNG VON KONSTRUKTIONEN AUS STÄHLEN HÖHERER FESTIGKEIT

Zusammenfassung

Der Artikel macht die Weiterentwicklung des Hill-Algorithmus nach Rosenbrock bekannt. Er stellt die Verwendungsmöglichkeiten des Mikrocomputersystems COMMODORE 64 bei der Konstruktionsoptimierung vor. Es wird die elastische optimale Bemessung eines einschiffigen Hallenrahmens auf Gewichtsminimum durchgeführt. An diesem Beispiel wird zwischen den Ergebnissen des Hill-Algorithmus, des backtrack-, des Komplex- und des DSFD-Programmes ein Vergleich gezogen. Es werden die Vor- und Nachteile der einzelnen Algorithmen bekanntgemacht. Es wird die Verwendung der Stähle mit erhöhter Fließgrenze und das Mass der Gewichtsabnahme untersucht.

ОПТИМАЛЬНОЕ ПРОЕКТИРОВАНИЕ КОНСТРУКЦИЙ ИЗ СТАЛЕЙ ПОВЫШЕННОЙ ПРОЧНОСТИ

Содержание

В докладе дано развитие алгоритма "HILL" по Розенброку. В докладе показано возможность применения системы микроэлектронных вычислительной машины COMMODORE 64 при оптимизации конструкции. Запроектировано однопролетную стальную раму оптимизируя ее конструкцию иная в виду весовой минимум. На этом примере сравниваются алгоритмы "HILL", "Бектрек", "Комплекс" и "ДСФД". Излагаются преимущества и невыгоды отдельных алгоритмов. Рассматривается применение статей с повышенным пределом текучести и мера снижения веса.

ОПТИМАЛНЕ ПРОЕКЦІВАНІЕ КОНСТРУКЦІЙ ЗЕ СТАЛІ О ПОДВІЙШЕНОЇ ВИТРІМЛІВОСТІ

Streszczenie

W referacie przedstawiono rozwinięcie algorytmu "Hill" wg. Rosenbrocka. Wykazano możliwość zastosowania systemu mikrokomputerowego COMMODORE VC-64 jako narzędzia optymalizacji konstrukcji. Zaprojektowano jednonawową ramę stalową optymalizując jej konstrukcję ze względu na minimum wagi. Na przykładzie tym przeprowadzono porównanie algorytmów "Hill", "backtrack", "kompleks" i "DSFD".