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# Seismic resistant optimum design of a welded steel frame supporting a pressure vessel

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## ABSTRACT

In the optimum seismic design of a simple steel frame the constraints relate to the stability of columns and beams, to sway limitation due to horizontal seismic forces and to the local buckling of square hollow sections (SHS). The objective function is the structural volume or mass, since the cost is proportional to the mass. In order to ease the fabrication of corner joints the widths of columns and beams should be equal. Thus, the unknowns are the common width and the two different thicknesses. The optimum solution is obtained by a systematic search using a discrete series of SHS.

*Keywords:* welded steel frame, seismic design, structural optimization, tubular structures, sway limitation.

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# 1 INTRODUCTION

Pressure vessels are expensive and dangerous devices, which need safe supports. Their fracture caused by earthquake can be very dangerous. Thus, the design constraints should be very strict.

A simple supporting frame consists of 4 columns and 4 beams (Fig.1). The pressure vessel is fixed at the middle of the beams. This is a simplified model of a frame structure of a chemical plant producing PVC powder. This structure consists of a lot of pressure vessels, connecting by pipelines and equipped by ladders and service walk-ways. Thus it is possible to apply the rules of Eurocode 8 (2003) treating this structure as a building with special sensitive equipment. The horizontal seismic load is calculated according to Eurocode 8 (2003). Since the horizontal forces cause large bending moments in the horizontal plane and the beams should transfer at the frame corners large bending moments, they suitable profile is a welded box section or tubular hollow section. Therefore the columns are constructed with box section as well. The welded corners are assumed to be rigid.

Eurocode 8 prescribes a strict limitation of the horizontal sway at the middle of the beams. This sway has four components as follows: the sway of the vertical frames, deformation of the beam due to bending in horizontal plane, displacement of the beam due to angular deformation of the frame corner and another displacement caused by torsion.

Optimization means a search for better solutions, which better fulfil the requirements. Requirements for a modern load-carrying structure are the safety, fitness for production and economy. In an optimum design procedure the safety and fitness for production are guaranteed by design and fabrication constraints, the economy is achieved by minimization of a cost function (Farkas & Jármai 1997, Farkas & Jármai 2003).

The fabrication (assembly and welding) cost of frame corner joints is proportional to the size of columns and beams, thus the minimum cost design is identical to the minimum mass design. Since the investigated frame is symmetric, the unknowns to be optimized are the thicknesses $t_1$ and $t_2$ for columns and beams, respectively as well as a common width $h_1 = h_2$ of the square hollow section (SHS) of the columns and beams.

The constraints relate to the sway limitation and to the stability of frame members against compression and bending according to Eurocode 3 (2002).

# 2 CALCULATION OF THE SEISMIC FORCE

According to Eurocode 8 (2003) (EC8)

$$F_b = S_d(T_1)m\lambda \quad (1)$$

where $S_d(T_1)$ = the ordinate of the design spectrum at period $T_1$, $m$ = the pressure vessel mass, $\lambda$ = correction factor. Values of the parameters describing the recommended Type 1 elastic response spectra (EC8 Table 3.2) are as follows: ground type C is selected, $S$ = 1.15, $T_B$ = 0.20, $T_C$ = 0.60, $T_D$ = 2.0.

$T_1$ (s) is approximated by the expression:

$$T_1 = C_1 H^{0.75}, C_1 = 0.085, H = 4, T_1 = 0.24 \text{ s} \quad (2)$$

For $T_B<T_1<T_C$ $\quad S_d = \alpha S \frac{2.5}{q} \quad (3)$

We use the highest value applied for Japan $\alpha = 0.40$, the behaviour factor according to EC8 Table 6.2, Fig.6.1 $q$ = 1.1x5 = 5.5. Thus $S_d$ = 0.4x1.15x2.5/5.5 = 0.2091, required cross-section class 1 (plastic).

For $T_1 < 2T_C$ $\quad \lambda = 0.85$.

Thus, the pressure vessel mass $m$ should be multiplied by 0.85x0.2091 = 0.1777. The pressure vessel mass is 300 kN, the seismic horizontal force acting on a beam is

$F_b$ = 0.1777x75 = 13.3 kN.

Load combination: $\sum G_k + \psi_E Q_k ; \psi_E = \varphi \psi_{21} = 1$, since, for storage structures, $\varphi = \psi_{21} = 1$.

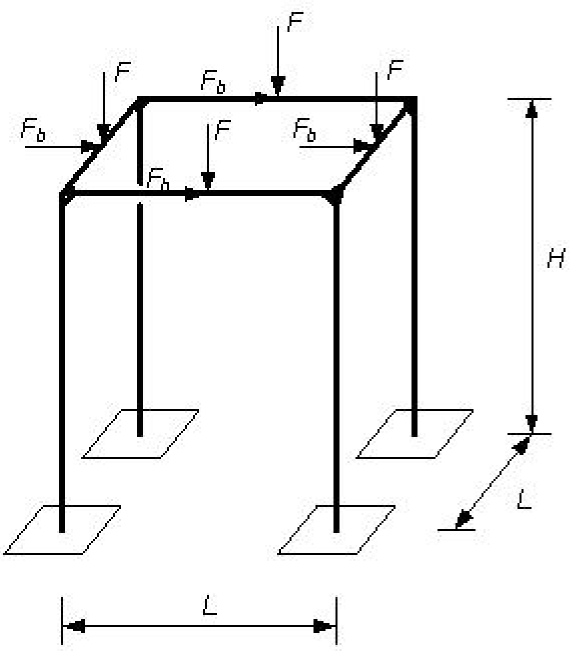


Figure 1. Supporting frame structure with vertical and horizontal forces

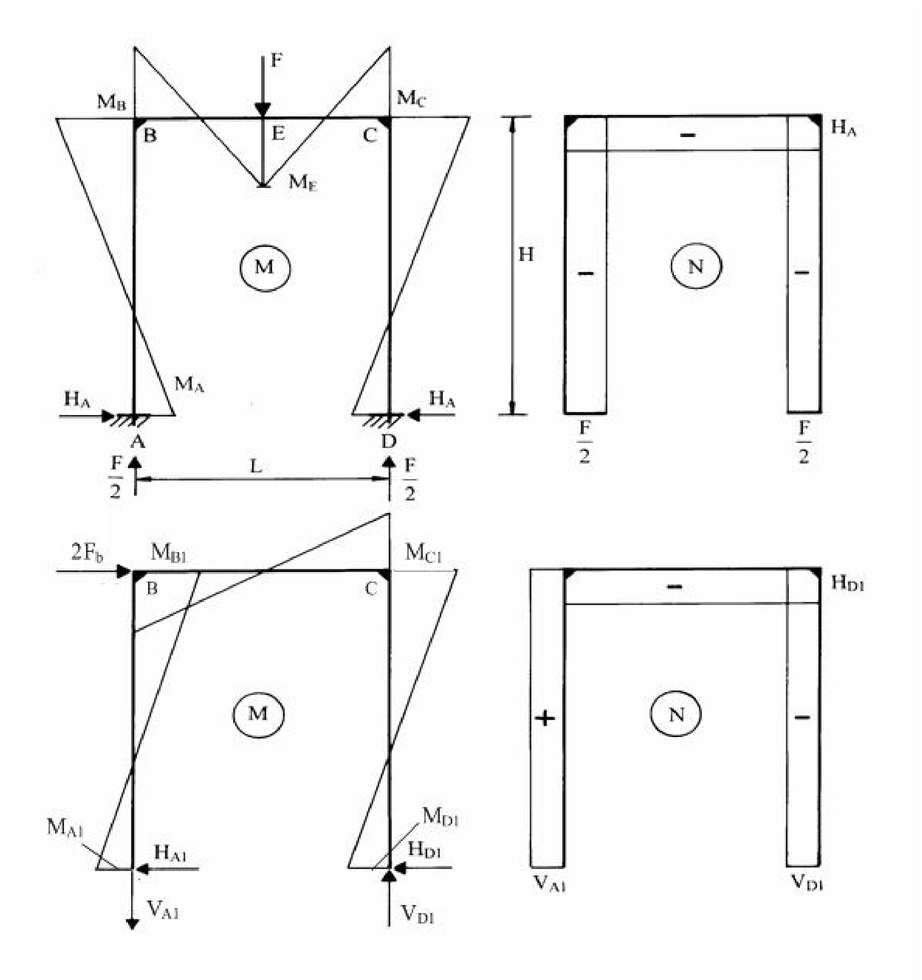


Figure 2. Diagrams for the bending moments and normal forces of a frame

# 3 NORMAL FORCES AND BENDING MOMENTS IN VERTICAL FRAMES (Fig.2)

**According to Glushkov et al. (1975)**

$$H_A = \frac{3M_A}{H}; M_A = \frac{M_B}{2}; M_B = \frac{FL}{4(k+2)}; k = \frac{I_{x2}H}{I_{x1}L} \quad (4)$$

$$M_E = \frac{FL}{4} - M_B\,,\ M_1 = \frac{3F_b Hk}{2(6k+1)},\ V_{D1} = \frac{2M_{B1}}{L} \quad (5)$$

$$N_1 = F + V_{D1}\,,\quad H_{D1} = \frac{k+1}{k+2}F_b \quad (6)$$

$$M_{A1} = \frac{3k+1}{6k+1}F_b H\,,\ M_{B1} = \frac{3k}{6k+1}F_b H\,,$$

$$H_2 = \frac{3k}{6k+1} H \tag{7}$$

$$M_{Bt} = M_B + M_{B1}, \qquad M_{At} = M_A + M_{A1} \tag{8}$$

# 4 GEOMETRIC CHARACTERISTICS OF THE SQUARE HOLLOW SECTION (Fig.3)

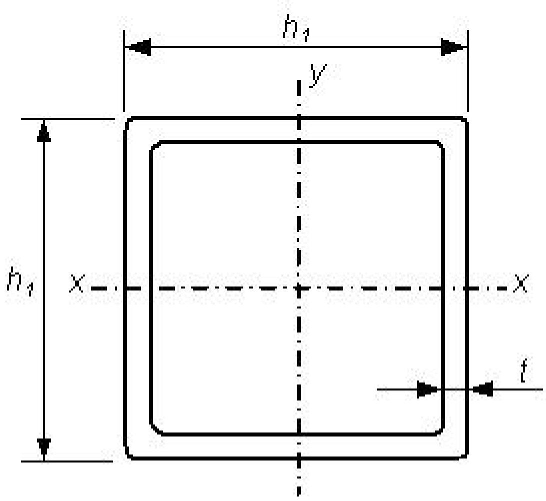


Figure 3. Dimensions of a square hollow section (SHS)

Areas and moments of inertia are calculated according to DASt Richtlinie 016 (1986).

Area of the cross-section for columns

$$A_1 = 4t_1(h_1 - t_1)\left(1 - 0.43\frac{2t_1}{h_1 - t_1}\right), \tag{10}$$

**and for beams**

$$A_2 = 4t_2(h_1 - t_2)\left(1 - 0.43\frac{2t_2}{h_1 - t_2}\right), \tag{11}$$

**moment of inertia for columns**

$$I_{x1} = I_{y1} = \left[\frac{2(h_1 - t_1)^3 t_1}{3}\right]\left(1 - 0.86\frac{2t_1}{h_1 - t_1}\right). \tag{12}$$

**and for beams**

$$I_{x2} = I_{y2} = \left[\frac{2(h_1 - t_2)^3 t_2}{3}\right]\left(1 - 0.86\frac{2t_2}{h_1 - t_2}\right). \tag{13}$$

**section modulus for columns**

$$W_{x1} = W_{y1} = \frac{2I_{x1}}{h_1} \tag{14}$$

**and for beams**

$$W_{x2} = W_{y2} = \frac{2I_{x2}}{h_1} \tag{15}$$

# 5 CALCULATION OF THE ELASTIC SWAY

$$u_e = u_f + u_b + u_t + u_{t1} \tag{16}$$

where $u_f$ = the sway of the frame, $u_b$ = displacement due to bending of a beam in horizontal plane, $u_t$ = beam displacement due to frame corner angle deformation, $u_{t1}$ = beam displacement due to torsion.

$$u_f = \frac{2M_{A1}m_{A1}H_1}{3EI_{x1}} + \frac{2M_{B1}m_{B1}H_2}{3EI_{x1}} + \frac{M_{B1}m_{B1}L}{3EI_{x2}} \tag{17}$$

where $M_{A1} = \frac{3k+1}{6k+1}F_b H; m_{A1} = \frac{3k+1}{6k+1}\frac{H}{2}; H_1 = \frac{3k+1}{6k+1}H$ (18)

$$M_{B1} = \frac{3k}{6k+1}F_b H; m_{B1} = \frac{3k}{6k+1}\frac{H}{2}; H_2 = \frac{3k}{6k+1}H; k = \frac{I_{x2}H}{I_{x1}L} \tag{19}$$

The displacement $u_b$ due to two horizontal forces $F_b$ in the horizontal plane of the frame with rigid corners is calculated as follows. The corner bending moment $M$ can be obtained from the equation of angular deformations (Fig. 4)

$$\varphi_1 = \varphi_2 \tag{20}$$

where

$$EI_{y2}\varphi_1 = \frac{F_b L^2}{16} - \frac{ML}{2} \tag{21}$$

and

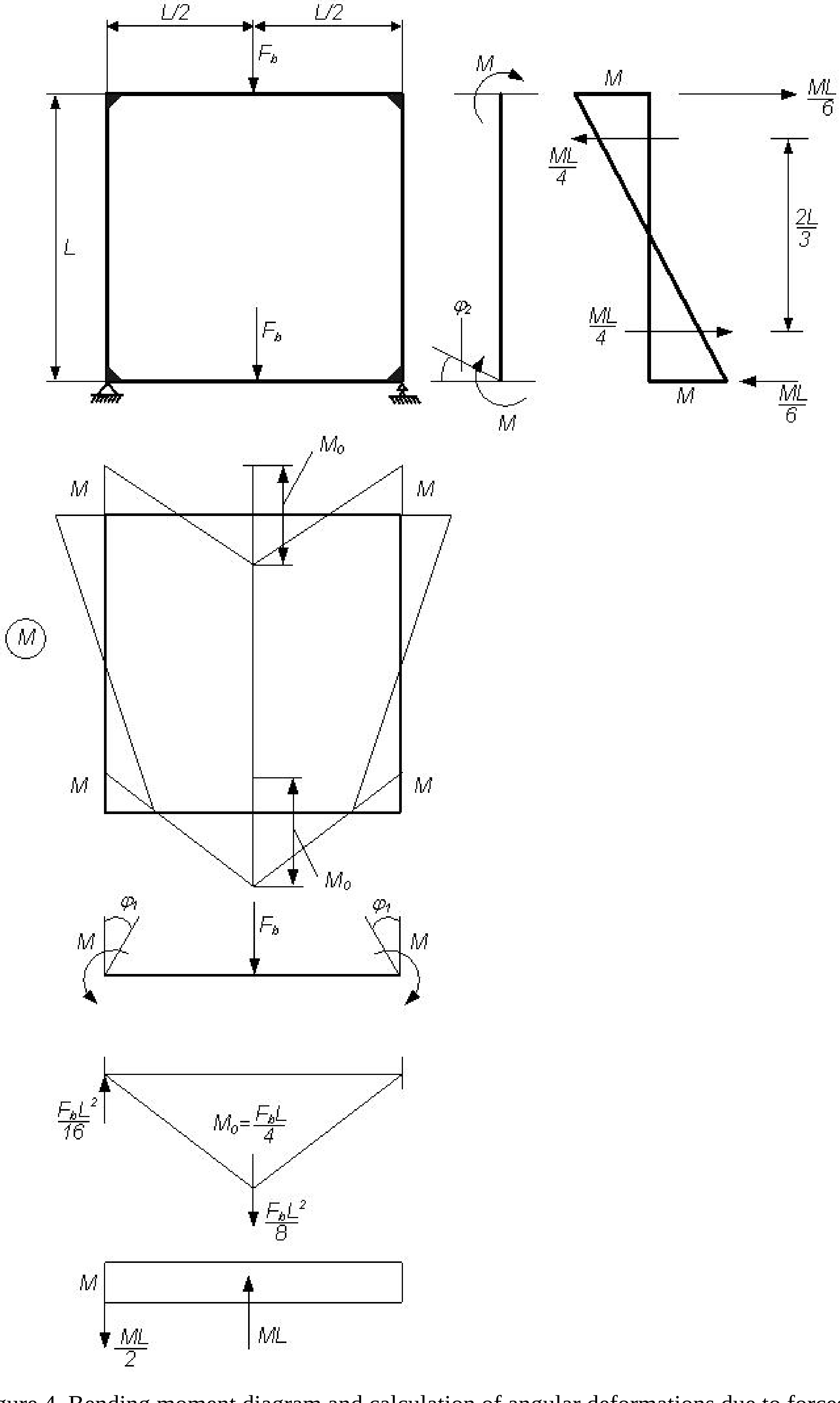


Figure 4. Bending moment diagram and calculation of angular deformations due to forces $F_b$ in the horizontal plane

$$EI_{y2}\varphi_2 = \frac{ML}{6} \tag{22}$$

Considering Eqs (20,21 and 22) one obtains

$$M = \frac{3F_b L}{32} \tag{23}$$

and the displacement from $F_b$ and $M$

$$u_b = \frac{F_b L^3}{48EI_{y2}} - \frac{ML^2}{8EI_{y2}} = \frac{7F_b L^3}{768EI_{y2}} \tag{24}$$

The displacement due to angle deformation of the beam caused by the frame corner angle deformation can be obtained from

$$u_t = \frac{1}{2EI_{x1}}\left(M_{A1}H_1 - M_{B1}H_2\right)\frac{h_1}{2} \tag{25}$$

Finally, the beam deformation due to torsion is

$$u_{t1} = \varphi_t \frac{h_1}{2}; \varphi_t = \frac{F_b h_1 L}{8GI_{t2}}; I_{t2} = h_1^3 t_2; u_{t1} = \frac{F_b L}{16Gh_1 t_2} \tag{26}$$

# 6 CONSTRAINT ON SWAY LIMITATION

The allowable sway according to EC8 (2003) is calculated as follows. The elastic displacement for ductile non-structural elements should fulfil the following limitation

$$u_e \le \frac{0.0075H}{\gamma_1 q \nu} = \frac{0.0075x4000}{1.4x5.5x0.4} = 9.74\,\text{mm.} \tag{27}$$

Importance class for power plants is IV (EC Table 4.3). Structural height $H$ = 4000 mm. The recommended safety factor for importance class IV is $\gamma_1 = 1.4$. The reduction factor $\nu = 0.4$. Behaviour factor $q$ = 1.1x5 = 5.5 according to EC Table 6.2 and Figure 6.1.

# 7 LOCAL BUCKLING CONSTRAINTS

For SHS columns and beams of section class 1 (plastic) the constraint is given by:

$$\frac{h_1 - 3t_i}{t_i} = \frac{h_1}{t_i} - 3 \leq 33\varepsilon, \varepsilon = \sqrt{\frac{235}{f_y}}, i = 1,2 \tag{28}$$

# 8 STRESS CONSTRAINT FOR THE COLUMNS

According to Eurocode 3 (2002) the SHS section is not susceptible to torsional deformations, thus $\chi_{LT} = 1$, $k_{yx} = 0$ and the second constraint in EC3 should not be considered.

$$\frac{N_1}{\chi_{1.min} A_1 f_{y1}} + \frac{k_{xx1}(M_C + M_{B1})}{W_{x1} f_{y1}} + \frac{k_{xy1}(M_C)}{W_{y1} f_{y1}} \leq 1 \tag{29}$$

$$k_{xx1} = \min\left( C_{mx1}\left(1 + \frac{0.6\lambda_{x1}(H_A + H_{D1})}{\chi_{x1} A_1 f_{y1}}\right), C_{mx1}\left(1 + \frac{0.6(H_A + H_{D1})}{\chi_{x1} A_1 f_{y1}}\right)\right) \tag{30}$$

$$C_{mx1} = 0.4$$

$$k_{xy1} = \min\left( C_{my1}\left(1 + \frac{0.6\lambda_{y1}(H_A + H_{D1})}{\chi_{y1} A_1 f_{y1}}\right), C_{my1}\left(1 + \frac{0.6(H_A + H_{D1})}{\chi_{y1} A_1 f_{y1}}\right)\right) \tag{31}$$

$$C_{my1} = 0.4$$

$$r_{x1} = \left(\frac{I_{x1}}{A_1}\right)^{0.5}; r_{y1} = \left(\frac{I_{y1}}{A_1}\right)^{0.5}; \bar{\lambda}_{x1} = \frac{K_{x1} H}{r_{x1}\lambda_E};$$

The value of $K_{x1}$ and $K_{y1}$ are taken according to EC3 (1992)

$$K_{x1} = 2.19; \bar{\lambda}_{y1} = \frac{K_{y1} H}{r_{y1}\lambda_E}; K_{y1} = 0.5$$

$$\bar{\lambda}_{1.\max} = \max\left(\bar{\lambda}_{x1}, \bar{\lambda}_{y1}\right) \tag{32}$$

$$\chi_{i.min} = \frac{1}{\phi_i + \left(\phi_i^2 - \bar{\lambda}_{i.max}^2\right)^{0.5}}; \; \phi_i = 0.5\left[1 + 0.34\left(\bar{\lambda}_{i.max} - 0.2\right) + \bar{\lambda}_{i.max}^2\right] \tag{33}$$

# 9 STRESS CONSTRAINT FOR THE BEAMS

$$\frac{H_A + H_{D1}}{\chi_{2.\min} A_2 f_{y1}} + \frac{k_{xx2} M_E}{W_{x2} f_{y1}} + \frac{k_{xy2} M}{W_{y2} f_{y1}} \leq 1, \; f_{y1} = \frac{f_y}{\gamma_{M1}} \tag{34}$$

The flexural buckling factor is

$$\chi_i = \frac{1}{\phi_i + \left(\phi_i^2 - \bar{\lambda}_i^2\right)^{0.5}};$$

$$\phi_i = 0.5\left[1 + 0.34\left(\bar{\lambda}_i - 0.2\right) + \bar{\lambda}_i^2\right] \tag{35}$$

$\bar{\lambda}_{x2} = \dfrac{K_{x2}L}{r_{x2}\lambda_E}$; the effective length factor is

$$K_{x2} = 0.5 \tag{36}$$

$$r_{x2} = \left(\frac{I_{x2}}{A_2}\right)^{0.5}; \lambda_E = \pi\left(\frac{E}{f_y}\right)^{0.5}; E \text{ is the elastic modulus} \tag{37}$$

$\bar{\lambda}_{y2} = \dfrac{K_{y2}L}{r_{y2}\lambda_E}$; the effective length factor is

$$K_{y2} = 0.5 \tag{38}$$

$$r_{y2} = \left(\frac{I_{y2}}{A_2}\right)^{0.5}$$

$\chi_{2.\min}$ is calculated from $\bar{\lambda}_{2.\max} = \max\left(\bar{\lambda}_{x2}, \bar{\lambda}_{y2}\right)$.

$$k_{xx2} = \min\left(C_{mx2}\left(1 + \frac{0.6\lambda_{x2}\left(H_A + H_{D1}\right)}{\chi_{x2}A_2 f_{y1}}\right), C_{mx2}\left(1 + \frac{0.6\left(H_A + H_{D1}\right)}{\chi_{x2}A_2 f_{y1}}\right)\right) \tag{39}$$

$$C_{my2} = 0.9$$

$$k_{yy2} = \min\left(C_{my2}\left(1 + \frac{0.6\lambda_{y2}\left(H_A + H_{D1}\right)}{\chi_{y2}A_2 f_{y1}}\right), C_{my2}\left(1 + \frac{0.6\left(H_A + H_{D1}\right)}{\chi_{y2}A_2 f_{y1}}\right)\right) \tag{40}$$

$$C_{my2} = 0.9$$

$$k_{xy2} = 0.8k_{xx2}$$

# 10 OPTIMIZATION AND RESULTS

*Numerical data*

$E = 2.1\text{x}10^5$ MPa, $G = 0.8\text{x}10^5$ MPa, $H = 4000$, $L = 4000$ mm, $F = 75$ kN, $F_b = 13.3$ kN.

*The objective function* is the structural volume

$$V = 4A_1H + 4A_2L \tag{41}$$

**or the structural mass**

$m = \rho V, \rho = 7.85x10^{-6}$ **kg/mm$^3$.**

The suitable SHS for columns and beams are selected using a cold-formed SHS catalogue BS EN 10219 (1997). Since the minimum thickness is limited by the local buckling constraint (Eq.28), only that thicknesses can be used, which are larger than this limit, e.g. for $h_1$ = 220 $t$ = 6.3, for $h_1$ = 250 $t$ = 8, for $h_1$ = 260 $t$ = 8 and for $h_1$ = 300 $t$ = 10 mm. Therefore the number of SHS to be investigated is limited.

Table 1 shows the results of the calculations to find the optimum SHS sizes. The governing constraint is that on sway limitation (Eq.27), the stress constraints are always fulfilled. The common width is $h_1$ and the thicknesses are $t_1$ for columns and $t_2$ for beams.

Table 1. Results of the systematic search to find the optimum SHS sizes (in mm)

| $h_1$ | $t_1$ | $t_2$ | **sway constraint** | ***m* (kg)** |
|---|---|---|---|---|
| 220 | 6.3 | 6.3 | 13.6>9.74 | |
| 220 | 8 | 8 | 11.1>9.74 | |
| 220 | 10 | 8 | 9.9>9.74 | |
| 220 | 8 | 10 | 10.6>9.74 | |
| 220 | 10 | 10 | 9.3<9.74 | 2024 |
| 250 | 8 | 8 | 7.434<9.74 | 1890 |
| 260 | 8 | 8 | 6.6<9.74 | 1970 |
| 300 | 10 | 10 | 3.5<9.74 | 2828 |

It can be seen that the optimum sizes are as follows: $h_1$ = 250, $t = t_1 = t_2$ = 8 mm, the minimum mass is $m$ = 1890 kg.

In the case of the optimum solution, the stress constraints are fulfilled as follows:

Eq.(29): 0353<1 and Eq.(34): 0.601<1.

The components of the sway are the following: Eq.(17) $u_f$ = 6.769, Eq.(24) $u_b$ = 0.518, Eq.(25): $u_t$ = 0.127 and Eq.(26): $u_{t1}$ = 0.021 mm, thus, $u_t$ and $u_{t1}$ can be neglected.

Figure 5 shows the welded frame corner.

According to Eurocode 8 (2003) second order (P-$\Delta$) effects need not to be taken account, if the following condition is fulfilled

$$\theta = \frac{P_{tot} d_r}{V_{tot} H} \leq 0.10$$

$P_{tot}$ is the total gravity load,

$V_{tot}$ is the total seismic storey shear,

$d_r$ is the design interstorey drift,

$H$ is the interstorey height.

In our case for the optimum solution $V_{tot}/ P_{tot}$ = 0.1777, $d_r$ = 7.4 mm, $H$ = 4000 mm, thus 0.01 < 0.1, the condition is fulfilled, so the second order effect can be neglected.

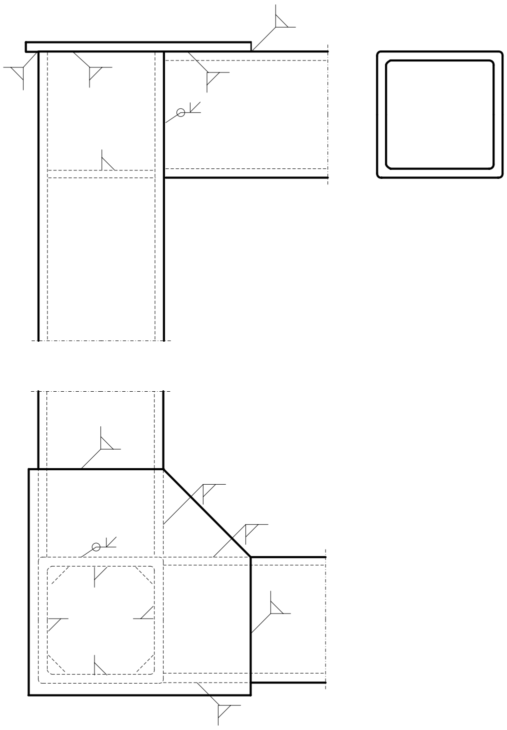

Figure 5. Welded frame corner

# 11 COST CALCULATION

The cost function includes the material and fabrication costs as follows:

$K = K_M + K_F$, $K_M = k_M \rho V_1, \rho = 7.85x10^{-6}$ kg/mm$^3$

$V_1 = V + V_h$, volume of head plates $V_h = 4x3.5h_1^2 t_h, t_h = 8$ mm

$$K_F = k_F \left( \Theta \sqrt{\kappa \rho V_1} + 1.3 \sum_i C_i a_{wi}^n L_{wi} \right) \quad (42)$$

Number of assembled elements $\kappa = 12$,

difficulty factor for a spatial structure $\Theta = 3$, welding time for the connection of a SHS beam to a SHS column with 2 vertical, one overhead and one downhand single bevel (1/2V) butt weld of size $t_2$ and length $h_1$

$$0.9518x10^{-3}x3t_2^2h_1 + 0.5214x10^{-3}t_2^2h_1 \quad (43)$$

welding time for the connection of a head plate to the frame corner with overhead fillet welds of size 5 mm and length $6h_1$ and with downhand fillet welds of length $2h_1$

$$1.667x10^{-3}x5^2x6h_1 + 0.7889x10^{-3}x5^2x2h_1. \quad (44)$$

For the optimum values of $h_1$ = 250, $t_2$ = 8 mm and for cost factors of $k_M$ = 1 \$/kg and $k_F$ = 1 \$/min

$K_M$ = 1944 \$ and $K_F$ = 1395 \$.

It can be seen that the fabrication cost gives a significant part of the total cost.

## 12 CONCLUSIONS

The horizontal seismic forces and the allowable horizontal sway of a simple frame is calculated according to the Eurocode 8 (2003). The frame with rigid joints supports a pressure vessel, the failure of which caused by earthquake can be dangerous. The stress constraints for columns and beams are formulated according to Eurocode 3 (2002). The frame is welded from SHS profiles.

For the fabrication reasons the section width of columns and beams should be equal. Thus, the unknowns are the common width and the different two thicknesses. The minimum thicknesses are limited by the local buckling constraint for section of class 1 (plastic).

The detailed calculation of sway due to bending deformations of the frame in vertical and horizontal plane and due to the torsion of the beams is presented. The objective function is the structural volume or mass, since the minimum cost design coincides with minimum mass design.

The optimum cross-sections are selected from a discrete series for SHS using a systematic search. The sway limitation is the governing constraint. Calculating the sway components it is found that the deformation due to torsion of beams and the sway from the angular deformation of frame corners can be neglected.

## ACKNOWLEDGEMENTS

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