Tubular Structures VIII, Choo & Van der Vegte (eds) © 1998 Balkema, Rotterdam, ISBN 90 5809 001 9

Optimum design and comparison of hollow flange beams

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ABSTRACT: A hollow flange beam (HFB) consists of a straight web and two hollow flanges. The shape of flanges can be triangular, circular or square one. To compare these structural versions with welded I-beams an optimization procedure is developed. The optimum cross-sectional dimensions are determined which minimize the cross-sectional area and fulfil the design constraints on stress due to bending and on local buckling of web and compression flange. The comparison shows that a HFB has smaller cross-sectional area (weight), larger moment of inertia (smaller deflection) and larger critical bending moment of lateral-torsional buckling.

1 INTRODUCTION

Hollow flanges can be used instead of simple plate flanges in welded beams (Fig.1). The shape of hollow flanges can be triangular, circular or rectangular (square). A special triangular hollow flange beam (TFB), called also „dogbone", was developed by the Australian firm Palmer Tube Technologies Ltd (Dempsey 1993), but this steel section was subsequently cancelled and is not manufactured.

The section is cold-formed from flat strip. The triangular flanges are closed by two electric resistance welded seam. They have used higher-strength steel of ultimate tensile strength 520 MPa and a yield strength 450 MPa.

They have not used any optimization procedure. We have worked out an optimization method to compare the TFB-s with circular CFB and square SFB and with welded I-beam.

The main advantages of HFB over simple welded I-beams are as follows: (a) the local buckling strength of beam parts is higher, therefore the thicknesses can be smaller; (b) the whole beam is higher, therefore the beam deflection is smaller; (c) the torsional stiffness is much larger, therefore the lateral-torsional buckling strength is larger.

The problem of lateral-torsional buckling of TFB is investigated by Pi & Trahair (1997). They have proposed a reduction of the torsional stiffness due to web distortion. We show that the Eurocode 3 (EC3) formulae give smaller values for lateral-torsional buckling factors, thus, the EC3 method can be used for comparison.

In the optimization the optimum cross-sectional dimensions are sought which minimize the cross-sectional area and fulfil the design constraints on maximum stress due to bending as well as on local buckling of the web and the compression flange.

First the cross-sectional characteristics are derived for an arbitrarily hollow flange shape and the optimization procedure is described, then the optimum cross-sectional areas and moments of inertia are expressed and compared for the above mentioned four beam shapes.

The lateral-torsional buckling strengths are characterized by buckling factors in the function of L/h for simply supported beams of span length L and web height h subject to uniformly distributed normal load.

It should be mentioned that Avery & Mahendron (1997) have investigated the effect of transverse stiffeners on the lateral-torsional buckling of TFB.

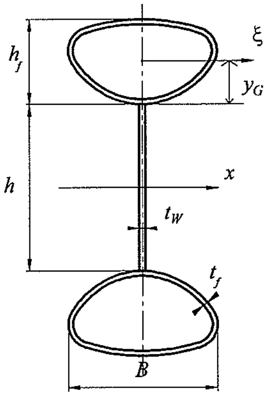

Figure 1. General hollow flange beam

A graphical optimization procedure is worked out for the design of welded I-beams against lateral-torsional buckling by Farkas (1997).

The calculation of the torsional constants of closed thin-walled beams is treated e.g. in the book Farkas & Jármai (1997).

2 SECTIONAL CHARACTERISTICS OF A GENERAL HOLLOW FLANGE BEAM

A general hollow flange beam is shown in Figure 1. Introducing the ratio between the flange depth and web depth as the main variable:

$$\varsigma = \frac{2h_f}{h} \tag{1}$$

and the slenderness:

$$\beta = \frac{t_w}{h} \tag{2}$$

the flange height can be expressed as:

$$h_f = \frac{\varsigma h}{2} \tag{3}$$

The second moment of inertia, the cross-sectional area of the flange and the location of the gravity centre y_G of the flange can be expressed as a function of web depth h or flange depth h_f.

$$I_{f\varsigma} = P_1 h_f^4 = P_1 \varsigma^4 \frac{h^4}{16} \tag{4}$$

$$A_f = P_2 h_f^2 = P_2 \varsigma^2 \frac{h^2}{4} \tag{5}$$

$$y_G = P_3 h_f = P_3 \varsigma \frac{h}{2} \tag{6}$$

where P_1, P_2 and P_3 are constants

The properties of the whole cross-sectional area can be written in the following forms:

$$A = ht_w + 2A_f = ht_w + P_2 \varsigma^2 \frac{h^2}{2} \tag{7}$$

$$I_x = \frac{h^3 t_w}{12} + 2\left[I_{f\varsigma} + A_f \left(\frac{h}{2} + y_G \right)^2 \right] \tag{8}$$

Substituting equations (2), (4), (5) and (6) into equation (8), one obtains the following for the moment of inertia

$$I_x = \frac{\beta h^4}{12} + 2\left[P_1 \varsigma^4 \frac{h^4}{16} + P_2 \varsigma^2 \frac{h^4}{16} (1 + P_3 \varsigma) \right] \tag{9}$$

and the elastic section modulus can be written as:

$$W_x = \frac{I_x}{\frac{h}{2} + h_f} = \frac{I_x}{\frac{h}{2}(1+\varsigma)} \tag{10}$$

From (9) and (10) we get the final form of the elastic section modulus:

$$W_x = \frac{\beta h^3}{6(1+\varsigma)} + \frac{\varsigma^2 h^3}{4(1+\varsigma)} \left[P_1 \varsigma^2 + P_2 (1 + P_3 \varsigma)^2 \right] \tag{11}$$

3 GENERAL OPTIMUM DESIGN

The objective function is the cross-sectional area (7).

The constraint on maximum normal stress due to bending moment M_{max} according to EC3 (1992) is defined by:

$$\sigma_{max} = \frac{M_{max}}{W_{el.x}} \le f_{y.1} = \frac{f_y}{\gamma_{M.1}} \tag{12}$$

$$\gamma_{M.1} = 1.1 \tag{13}$$

where $\gamma_{M.1}$ is a safety factor,
or expressed by the required section modulus W_0:

$$W_{el.x} \ge W_0 = \frac{M_{max}}{f_{y.1}} \tag{14}$$

Local buckling constraints
For the web:

$$t_w \ge \beta h \tag{15}$$

where

$$\frac{1}{\beta} = 124 \sqrt{\frac{235}{\sigma_f}} \tag{16}$$

where β is the ultimate plate slenderness for the web and σ_f is the absolute value of the normal stress at the upper and the lower end of the web. Expressing the ratio of σ_f and σ_{max},

$$\sigma_f = \sigma_{\max} \frac{\frac{h}{2}}{\frac{h}{2} + h_f} = \sigma_{\max} \frac{1}{1+\varsigma} \tag{17}$$

The final form of the plate slenderness for the web provided that the maximum normal stress σ_{max} in the extreme fibre is equal to the yield stress:

$$\frac{1}{\beta} = 124\sqrt{\frac{235}{f_y}(1+\varsigma)} = 124\varepsilon\sqrt{1+\varsigma} \tag{18}$$

$$\varepsilon = \sqrt{\frac{235}{f_y}} \tag{19}$$

For the upper part of the flange:

$$t_f \geq \frac{B}{\delta} \tag{20}$$

where B depends on the shape of the flange and δ is the ultimate plate slenderness for a flange according to EC3.

Converting the objective function (7)

$$A = \beta h^2 + P_2\varsigma^2 \frac{h^2}{2} = h^2\left(\beta + \frac{P_2\varsigma^2}{2}\right) \tag{21}$$

$$h^3 = \frac{A^{\frac{3}{2}}}{\left(\beta + \frac{P_2\varsigma^2}{2}\right)^{\frac{3}{2}}} \tag{22}$$

and substituting (22) into (11) the stress constraint gets the following form:

$$\frac{A}{W_0^{\frac{2}{3}}} = \frac{\beta + \frac{P_2\varsigma^2}{2}}{\left\{\frac{\beta}{6(1+\varsigma)} + \frac{\varsigma^2}{4(1+\varsigma)}\left[P_1\varsigma^2 + P_2(1+P_3\varsigma)^2\right]\right\}^{\frac{2}{3}}} \tag{23}$$

As W_0 is a constant the only variable is ζ, so the task is to find the optimum ζ which gives the minimum cross-sectional area.

4 TRIANGULAR FLANGE BEAM (TFB)

The dimensions of a TFB can be seen in Figure 2. The flange details of TFB are shown in Figure 3. Because of the production technology of TFB its thickness has to be a constant along its circumference. Instead of using both buckling constraints (local buckling for the web and for the flange) we introduce only one buckling constraint:

$$t \geq \beta h \tag{24}$$

For the web:

$$\frac{1}{\beta_w} = 124\varepsilon\sqrt{1+\varsigma} \tag{25}$$

For the upper part of the flange:

$$t \geq \beta_{f0} B \tag{26}$$

Expressing the flange width B by means of the web depth h we get another form of (26):

$$B = \frac{2h_f}{\tan\alpha} = \frac{\varsigma h}{\tan\alpha} \tag{27}$$

$$t \geq \beta_{f0}\frac{\varsigma h}{\tan\alpha} = \beta_f h \tag{28}$$

According to EC3(1992) $\frac{1}{\beta_{f0}} = 42\varepsilon$ (29)

$$\frac{1}{\beta_f} = 42\varepsilon\frac{\tan\alpha}{\varsigma} \tag{30}$$

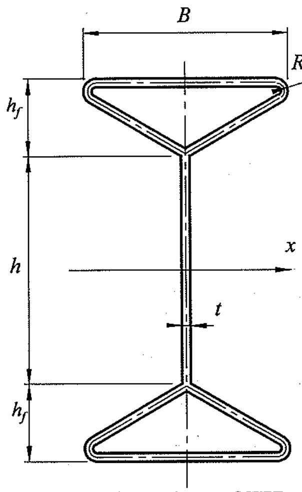

Figure 2. Dimensions of TFB

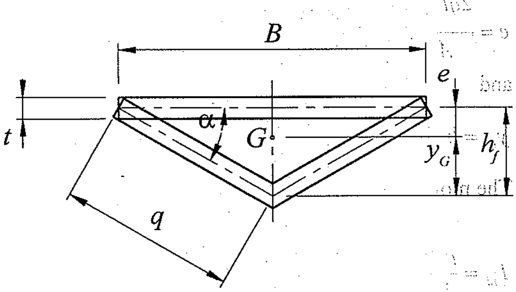

Figure 3. Flange details of TFB

Since the web and flange have the same thickness, the larger value from $\beta_w h$ and $\beta_f h$ is governing. The symbol β is used, but the optimization has to be performed either with $\beta_w h$ and with $\beta_f h$.

The solution of (31) gives the value of ζ at which web and flange buckling constraints are active.

$$\beta_w h = \beta_f h \qquad (31)$$

$$\frac{h}{124\varepsilon\sqrt{1+\varsigma}} = \frac{\varsigma h}{42\varepsilon tan\alpha} \qquad (32)$$

It can be seen from (32) that the solution for ζ does not depend on the yield stress of the material. The only factor which affects the solution of ζ is the angle of the flange α.

Solving (32) for given flange angles for the interval $0.1 \le \zeta \le 1$:

for α=30° $\varsigma_{opt} = 0.180020$ (33)

for α=35° $\varsigma_{opt} = 0.215149$ (34)

for α=40° $\varsigma_{opt} = 0.253818$ (35)

Below the limit of ζ_{opt} the web buckling constraint is active and defines the thickness of TFB. Above ζ_{opt} the flange buckling constraint is active.

The sectional properties can be expressed as follows: the cross-sectional area of a triangular hollow flange:

$$A_f = Bt + 2qt = \frac{2h_f t(1+\cos\alpha)}{\sin\alpha} \qquad (36)$$

considering that the local buckling constraint is active:

$$t = \beta h \qquad (37)$$

$$A_f = \frac{4\beta h_f^2 (1+cos\,\alpha)}{\varsigma\, sin\,\alpha} = P_2 h_f^2 \qquad (38)$$

The location of the gravity centre G of the flange is described by the distance:

$$e = \frac{2qt\dfrac{h_f}{2}}{A_f} = \frac{h_f}{2(1+\cos\alpha)} \qquad (39)$$

and

$$y_G = h_f - e = \frac{h_f(1+2\cos\alpha)}{2(1+\cos\alpha)} = P_3 h_f \qquad (40)$$

The moment of inertia of the flange section is:

$$I_{f\xi} = \frac{t^3 B}{12} + Bte^2 + \frac{2q^3 t \sin^2\alpha}{12} + \left(\frac{h_f}{2} - e\right)^2 2qt \qquad (41)$$

The first member of (41) can be neglected in comparison with the other members.

$$I_{f\xi} = P_1 h_f^4 \qquad (42)$$

The design constants P_1, P_2 and P_3 can be expressed from (41), (38) and (40), respectively:

$$P_1 = \frac{\beta}{\varsigma \sin\alpha(1+\cos\alpha)^2}\left[\cos\alpha + \cos^2\alpha + \frac{(1+\cos\alpha)^2}{3}\right] \qquad (43)$$

$$P_2 = \frac{4\beta(1+\cos\alpha)}{\varsigma \sin\alpha} \qquad (44)$$

$$P_3 = \frac{1+2\,cos\,\alpha}{2(1+cos\,\alpha)} \qquad (45)$$

Substituting the design constants into (23), and iterating this equation we can get the optimum value of ζ.

The cross-sectional area of TFB can be calculated as follows:

For steel with a yield stress of 355 MPa:

α=30° $A_{optTFB} = 0.536347 \cdot W_0^{\frac{2}{3}}$ (46)

α=35° $A_{optTFB} = 0.531289 \cdot W_0^{\frac{2}{3}}$ (47)

α=40° $A_{optTFB} = 0.526035 \cdot W_0^{\frac{2}{3}}$ (48)

The optimum dimensions for TFB are as follows:

$$h_{opt} = \sqrt{\frac{A_{optTFB}}{\beta + P_2\dfrac{\varsigma_{opt}^2}{2}}} \qquad (49)$$

$$h_f = \frac{\varsigma_{opt} h_{opt}}{2} \qquad (50)$$

$$B_{opt} = \frac{2h_f}{\tan\alpha} \qquad (51)$$

$$t_{opt} = \beta h_{opt} \qquad (52)$$

5 CIRCULAR HOLLOW FLANGE BEAM (CFB)

The dimensions of a CFB can be seen in Figure 4. The design constants (P_1, P_2 and P_3) can be expressed by means of the ultimate plate slenderness of the flange.

The ultimate plate slenderness according to EC3 (1992) for the flange, considering that the local buckling constraint for the flange is active:

$$\delta_c = \frac{2r}{t} = \frac{D}{t} = 90\varepsilon^2 = 90\frac{235}{f_y} \quad (53)$$

$$h_f = 2r \quad (54)$$

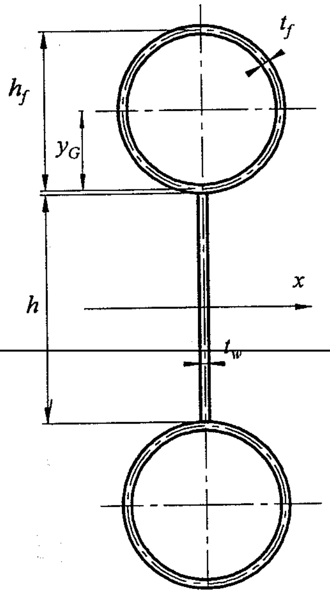

Figure 4. Dimensions of a CFB

The P_1, P_2, P_3 design constants can be expressed from (4):

$$I_{f\xi} = \pi r^3 t = \frac{2\pi r^4}{\delta_c} = P_1 h_f^4 = P_1(2r)^4 = 16P_1 r^4 \quad (55)$$

$$P_1 = \frac{\pi}{8\delta_c} \quad (56)$$

from (5):

$$A_f = P_2 h_f^2 = P_2 4r^2 = 2\pi rt = \frac{4\pi r^2}{\delta_c} \quad (57)$$

$$P_2 = \frac{\pi}{\delta_c} \quad (58)$$

from (6):

$$y_G = P_3 h_f = P_3 2r = r \quad (59)$$

$$P_3 = \frac{1}{2} \quad (60)$$

Substituting P_1, P_2, P_3 into (23) we can get its final form

$$\frac{A}{W_0^{\frac{2}{3}}} = \frac{\beta + \frac{\pi}{2\delta_c}\varsigma^2}{\left\{\frac{\beta}{6\beta(1+\varsigma)} + \frac{\varsigma^2}{4(1+\varsigma)}\left[\frac{\pi}{8\delta_c}\varsigma^2 + \frac{\pi}{\delta_c}\left(1+\frac{\varsigma}{2}\right)^2\right]\right\}^{\frac{2}{3}}} \quad (61)$$

The optimum value of ζ can be calculated by the iteration of ζ. The optimum value of ζ is 0.92, 0.81, 0.75 for steel with a yield stress of 235 MPa, 355 MPa, 450 MPa, respectively.

The minimum cross-sectional area of CFB can be calculated as follows:
For steel with a yield stress of 355 MPa:

$$A_{opt.CFB} = 0.513517 \cdot W_0^{\frac{2}{3}} \quad (62)$$

The optimum dimensions of CFB are as follows:

$$h_{opt} = \sqrt{\frac{A_{opt CFB}}{\beta + P_2 \frac{\varsigma_{opt}^2}{2}}} \quad (63)$$

$$t_{w,opt} = \beta h_{opt} = \frac{h_{opt}}{124\varepsilon\sqrt{1+\varsigma_{opt}}} \quad (64)$$

$$D_{opt} = \frac{1}{2}\varsigma h \quad (65)$$

$$t_{opt} = \frac{D_{opt}}{\delta_c} \quad (66)$$

6 SQUARE HOLLOW FLANGE BEAM (SFB)

The dimensions of a SFB can be seen in Figure 5. The design constants P_1, P_2 and P_3 can be expressed by means of the ultimate plate slenderness of the flange.

The ultimate plate slenderness, according to EC3 (1992) for the flange, considering that the local buckling constraint for the flange is active, is:

$$\delta_L = \frac{b_s}{t} = 42\varepsilon = 42\sqrt{\frac{235}{f_y}} \quad (67)$$

$$h_f = b_s \quad (68)$$

The P_1, P_2, P_3 design constants can be expressed from (4):

$$I_{f\xi} = \frac{2b_s^3 t}{3} = \frac{2b_s^4}{3\delta_L} = P_1 h_f^4 = P_1 b_s^4 \quad (69)$$

$$P_1 = \frac{2}{3\delta_L} \tag{70}$$

from (5):

$$A_f = P_2 h_f^2 = 4b_s t = \frac{4h_f^2}{\delta_L} \tag{71}$$

$$P_2 = \frac{4}{\delta_L} \tag{72}$$

from (6):

$$y_G = P_3 h_f = P_3 b_s = \frac{b_s}{2} \tag{73}$$

$$P_3 = \frac{1}{2} \tag{74}$$

Substituting P_1, P_2, P_3 into (23), we can get its final form

$$A / W_0^{2/3} = \frac{\beta + \frac{2}{\delta_L}\varsigma^2}{\left\{\frac{\beta}{6\beta(1+\varsigma)} + \frac{\varsigma^2}{4(1+\varsigma)}\left[\frac{2}{3\delta_L}\varsigma^2 + \frac{4}{\delta_L}\left(1+\frac{\varsigma}{2}\right)^2\right]\right\}^{\frac{2}{3}}} \tag{75}$$

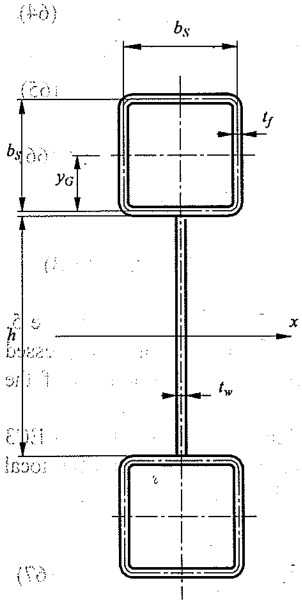

Figure 5. Dimensions of a SFB

The optimum value of ζ can be calculated by the iteration of ζ. The optimum value of ζ for steels of yield stress of 235 MPa, 355 MPa, 450 MPa is 0.5.

The cross-sectional area of SFB can be calculated as follows:

For steel with a yield stress of 355 MPa:

$$A_{opt.SFB} = 0.536256 \cdot W_0^{\frac{2}{3}} \tag{76}$$

The optimum dimensions for HFB with CHS flange:

$$h_{opt} = \sqrt{\frac{A_{optSFB}}{\beta + P_2 \frac{\varsigma_{opt}^2}{2}}} \tag{77}$$

$$t_{w,opt} = \beta h_{opt} = \frac{h_{opt}}{124\varepsilon\sqrt{1+\varsigma_{opt}}} \tag{78}$$

$$b_{sopt} = \frac{1}{2}\varsigma_{opt} h_{opt} \tag{79}$$

$$t_{fopt} = \frac{b_{sopt}}{\delta_L} \tag{80}$$

7 WELDED I-SECTION

The objective function is

$$A = ht_w + 2bt_f \tag{81}$$

the stress constraint is

$$\frac{M}{W_x} \le \frac{f_y}{\gamma_{M1}} \tag{82}$$

the web buckling constraint is

$$\frac{t_w}{h} \ge \beta = \frac{1}{124\varepsilon} \tag{83}$$

the flange buckling constraint is

$$\frac{t_f}{b} \ge \delta = \frac{1}{42\varepsilon} \tag{84}$$

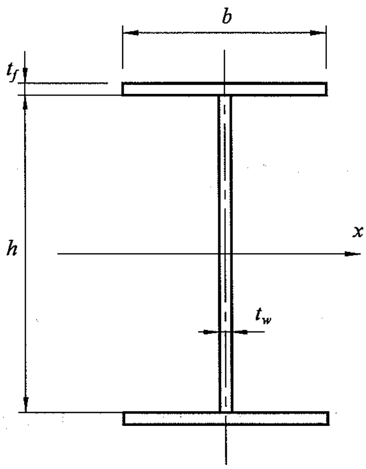

Figure 6. Dimensions of a welded I-section

The optimal solution of the problem is the following (Farkas 1984, Farkas & Jármai 1997):

$$A_{opt} = 2\beta h_{opt}^2 = \sqrt[3]{18\beta W_0^2} \quad (85)$$

$$h_{opt} = \sqrt[3]{\frac{1.5W_0}{\beta}} \quad (86)$$

$$t_{wopt} = \beta h_{opt} \quad (87)$$

$$t_{fopt} = h_{opt}\sqrt{\frac{\beta\delta}{2}} \quad (88)$$

$$b_{opt} = \frac{t_{fopt}}{\delta} \quad (89)$$

The minimum cross-sectional area of welded I-section for steel of 355 MPa yield stress

$$A_{optI} = 0.554165 \cdot W_0^{\frac{2}{3}} \quad (90)$$

The above obtained results are summarized in Table 1.

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi^2 - \bar{\lambda}^2}} \quad \text{but } \chi_{LT} \le 1 \quad (92)$$

in which

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right] \quad (93)$$

The values of the imperfection factor α_{LT} for lateral torsional buckling should be taken as: $\alpha_{LT} = 0.49$ for welded sections.

The formula of $\bar{\lambda}_{LT}$ is as follows:

$$\bar{\lambda}_{LT} = \frac{\lambda_{LT}}{\lambda_1}\beta_w^{\frac{1}{2}} \quad (94)$$

where

$$\lambda_1 = \pi\left(\frac{E}{f_y}\right)^{\frac{1}{2}} = 93.9\varepsilon \quad (95)$$

$$\varepsilon = \sqrt{\frac{235}{f_y}} \quad (96)$$

Table 1. Cross-sectional area and moment of inertia of optimized TFB, CFB, SFB and welded I-beams

beam type	$A_{min} / W_0^{2/3}$	$I_x/W_0^{4/3}$
TFB	0.536347	2.95464
CFB	0.513517	4.12001
SFB	0.536256	3.64304
I-beam	0.554165	2.70677

It can be seen that the HFB-s have smaller cross-sectional area and larger moment of inertia than the welded I-beam. The CFB has the smallest cross-sectional area and the largest moment of inertia.

$$\lambda_{LT} = \frac{L\left(\frac{W_{pl.x}^2}{I_y I_\omega}\right)^{0.25}}{C_1^{0.5}\left(1 + \frac{GL^2 I_t}{\pi^2 EI_\omega}\right)^{0.25}} \quad (97)$$

for beams with pinned ends with uniformly disributed loads $C_1 = 1.132$, I_t is the torsion constant, I_ω is the warping constant, I_y is the moment of inertia about the minor axis, L is the beam length between points which have lateral restraint.

8 LATERAL-TORSIONAL BUCKLING STRENGTH

8.1 *The EC3 method*

The design buckling resistance moment of a laterally unrestrained beam shall be taken as:

$$M_{b,Rd} = \chi_{LT}\beta_w W_{pl,y}\frac{f_y}{\gamma_{M1}} \quad (91)$$

where

$$\beta_w = \frac{W_{el,x}}{W_{pl,x}} \quad \text{for Class 3 cross-sections}$$

and χ_{LT} is the reduction factor for lateral-torsional buckling

8.2 *The method of Pi & Trahair (1997)*

They have proposed a calculation method which considers also the effect of web distortion. Their formula for the design bending moment is

$$\frac{M_{bdx}}{M_{Sx}} = \alpha_{Sd}\alpha_m = 0.6\alpha_m^2\left(\sqrt{\lambda_d^4 + \frac{2.8}{\alpha_m^2}} - \lambda_d^2\right) \quad (98)$$

where α_m is the factor for the effect of loading, for uniformly distributed normal load it is 1.1662. Values of $\alpha_{Sd}\alpha_m$ in the function of λ_d are compared with the lateral-torsional buckling factors χ_{LT} given by EC3 in function of $\bar{\lambda}_{LT}$ in Table 2.

Table 2. Comparison of lateral-torsional buckling factors given by Pi & Trahair as well as by EC3

λ_d or $\overline{\lambda}_{LT}$	$\alpha_{Sd}\alpha_m$	χ_{LT}
0.5	0.9845	0.8430
1.0	0.6111	0.5399
1.5	0.3416	0.3145
2.0	0.2036	0.1962

It can be seen that the buckling factors given by EC3 are smaller than those of Pi & Trahair, thus, the EC3 method is suitable for comparison.

8.3 *Comparison of lateral-torsional buckling factors using the EC3 formulae*

We calculate the buckling factors in the function of $\varphi = L/10h$ in the range of $\varphi = 1-10$.

(a) Using the formulae given in Section 4, the characteristics of a TFB, expressed in terms of the web height are as follows:

with $\varsigma_{opt} = 0.18$, $f_y = 355$ MPa and $\alpha = 30^0$ one obtains

$$W_{pl.x} = \beta h^3\left[0.25 + \frac{\varsigma(1+\varsigma)}{\tan\alpha} + \frac{\varsigma(1+\varsigma/2)}{\sin\alpha}\right] =$$

$$= 0.9218 * 10^{-2} h^3,$$

$$W_{el.x} = 0.79608 * 10^{-2} h^3 \; ; \; \beta_w^{0.5} = 0.9293$$

$$I_y = \frac{\varsigma^3 h^3 \beta}{64\tan^3\alpha}\left(1 + \frac{1}{\cos\alpha}\right) = 9.9279 * 10^{-5} h^4$$

$$I_t = \frac{\varsigma^3 \beta \cos^2\alpha}{2\sin\alpha(\sin\alpha + \cos\alpha)} h^4 = 2.9218 * 10^{-5} h^4$$

$$I_\varpi = \frac{\varsigma^3 \beta}{24\tan^3\alpha}\left(1 + \frac{1}{\cos\alpha}\right)\left(1 - \frac{\varsigma}{1+\cos\alpha}\right)^2 =$$

$$=2.0263 * 10^{-5} h^6$$

It should be mentioned that we use the symbol ϖ instead of ω , since the hollow flanges are closed sections.

According to (94) and (97)

$$\overline{\lambda}_{LT} = \frac{1.63875\varphi}{\left(1 + 5.6193\varphi^2\right)^{025}}$$

(b) Formulae for CFB with $\varsigma_{opt} = 0.81$ and f_y = 355 MPa:

$$W_{pl.x} = \frac{\beta h^4}{4} + \frac{\pi\varsigma^2 h^3(1+\varsigma/2)}{4\delta_C} = 1.3994 * 10^{-2} h^3$$

$$W_{el.x} = \frac{2I_x}{1+\varsigma} = 1.0503 * 10^{-2} h^3$$

$$I_y = \frac{\pi\varsigma^4 h^4}{64\delta_C} = 0.35466 * 10^{-3} h^4$$

$$I_t = \frac{\pi\varsigma^4 h^4}{32\delta_C} = 7.09315 * 10^{-4} h^4$$

$$I_\varpi = \frac{2}{3} h_f t_f\left(3\varpi_{T1}^2 + 2\varpi_{T1}\varpi_{T2} + 3\varpi_{T2}^2\right)$$

where

$$\varpi_{T1} = -\frac{h_f^2}{2}\left(\frac{1}{\varsigma} + \frac{1}{2}\right); \; \varpi_{T2} = -\frac{h_f^2}{2}\left(\frac{1}{\varsigma} - 1\right)$$

Since the exact calculation of the warping constant for the closed sectional parts of CFB leads to very complicated integrals, we use the formula of the warping constant derived for SFB with the approximation that $h_f = D\sqrt{2}/2$, this means that, instead of the circle, a square is used inscribed inside the circle. This approximation gives the following value:

$$I_\varpi = 2.18333 * 10^{-5} h^6.$$

$$\overline{\lambda}_{LT} = \frac{1.34385\varphi}{\left(1 + 126.6051\varphi^2\right)^{0.25}}$$

(c) Formulae for SFB with $\varsigma_{opt} = 0.5$ and f_y = 355 MPa:

$$W_{pl.x} = \frac{\beta h^4}{4} + \varsigma h^2 t_f(2+\varsigma) = 1.11688 * 10^{-2} h^3$$

$$W_{el.x} = 0.872368 * 10^{-2} h^3$$

$I_y = \varsigma^3 h^3 t_f / 6 = 1.52425 * 10^{-4} h^4$

$I_t = \varsigma^3 h^3 t_f / 4 = 2.28637 * 10^{-4} h^4$

The formula for warping constant is the same as for CFB:

$I_\omega = 6.99612 * 10^{-5} h^6$;

$$\bar{\lambda}_{LT} = \frac{2.01048\varphi}{\left(1 + 12.7356\varphi^2\right)^{0.25}}.$$

The results of calculations are summarized in Table 3.

It can be seen that the lateral-torsional buckling strength of HFB-s is larger than that of welded I-beam. The high values for CFB show that the torsional stiffness of circular hollow flanges is very large because of the high value of $\varsigma_{opt} = 0.81$ compared with the lower ς_{opt}-values for TFB and SFB.

9 CONCLUSIONS

The minimum cross-sectional area design of four welded beam types considering the maximum stress due to bending and the limiting local buckling slendernesses gives a basis of comparison relating to the beam weight, deflection and lateral-torsional buckling strength.

The cross-sectional area (weight) of HFB-s is smaller than that of I-beams. The moment of inertia about the major axis of HFB-s is larger, therefore the beam deflection is smaller than that of I-beams (Table 1).

The lateral-torsional buckling factor in function of $\varphi = L/10h$ is larger for HFB-s than that for I-beams (Table 3). These realistic comparisons give designers a basis for selection of suitable structural versions.

Table 3. Comparison of lateral-torsional buckling factors

φ	χ_{LT} TFB	χ_{LT} CFB	χ_{LT} SFB	χ_{LT} I-beam
1	0.5275	0.8974	0.5146	0.3536
2	0.3181	0.8052	0.3153	0.1346
3.33	0.2075	0.7052	0.2068	0.0702
10	0.0769	0.4031	0.0769	0.0216

ACKNOWLEDGEMENTS

This work has been supported by grants OTKA 19003 and 22846 of the Hungarian Fund for Scientific Research.

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