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The effect of instrumental precision on optimisation of displacement monitoring networks

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Abstract In order to detect the geo-hazards, different deformation monitoring networks are usually established. It is of importance to design an optimal monitoring network to fulfil the requested precision and reliability of the network. Generally, the same observation plan is considered during different time intervals (epochs of observation). Here, we investigate the case that instrumental improvements in sense of precision are used in two successive epochs. As a case study, we perform the optimisation procedure on a GPS monitoring network around the Lilla Edet village in the southwest of Sweden. The network was designed for studying possible displacements caused by landslides. The numerical results show that the optimisation procedure yields an observation plan with significantly fewer baselines in the latter epoch, which leads to saving time and cost in the project. The precision improvement in the second epoch is tested in several steps for the Lilla Edet network. For instance, assuming two times better observation precision in the second epoch decreases the number of baselines from 215 in the first epoch to 143 in the second one.

Keywords Optimal design · GPS network · Landslide · Observation plan

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1 Introduction

Since a few decades ago, the Global Positioning System (GPS) measurements are commonly used in geodetic monitoring networks. Such networks are established to investigate and detect possible deformations that can take place on the Earth due to crustal movements, landslides, etc., which in some cases threaten human life. Through many studies that have been carried out in this field, we can mention a work by Sjöberg et al. (2004), where they studied the possibility of using the GPS technology in detecting very small crustal deformations of the Äspö region in the southeast of Sweden. The importance of data evaluation for estimating the precise deformation models was studied by Setan and Singh (2001). They performed trend analysis of the net points in a displacement field to acquire its deformation model.

It is of importance to design and perform an optimal monitoring network in order to achieve high precision and reliability with low cost, as well as high sensitivity in detecting deformations (see e.g. Schmitt 1982; Gerasimenko et al. 2000; Even-Tzur 2002; Shestakov et al. 2005). Amongst several design stages pioneered by Grafarend (1974) that lead to optimal networks, we work with Second Order Design (SOD) in this paper. No zero order design is required to be performed on GPS monitoring networks (Kuang 1996, p. 260). This is because the problem in displacement networks is not to define the optimum datum for the reference network, but stabilising the datum at the reference frame to provide the same position and orientation of the network in subsequent epochs. Moreover, the configuration of the GPS network is defined as the formed geometry of the ground stations and satellites. On the one hand, we cannot change the configuration of the satellite constellation, and on the other hand, planning the optimum location of the ground points has less significant effect on designing the optimum observation plan in the small-scale GPS networks. Therefore, the first order design stage is not performed either. However, in the SOD the weight matrix of the observations is subjected to be optimally determined. Kuang (1992) proposed an approach to solve the SOD problem, where an optimal solution for weights is obtained by the best approximation of a defined criterion matrix. In another work, Even-Tzur and Papo (1996) solved the SOD problem by linear programming method, where a simulated GPS network was optimised to fulfil the requirements of the defined criterion matrix.

A criterion matrix is a representative of an ideal Variance–Covariance (VC) matrix for a geodetic network, and since it represents an ideal situation, there is no need for the network VC matrix to be equal to the criterion (Koch 1985). The criterion matrix is introduced to the optimisation procedure to push the current precision of the network towards the desired or required ones. In studying the deformation of a geodetic network, it is important to distinguish between measurement errors and displacements. Therefore, a reasonable criterion matrix for a deforming network should have the capability of protecting the analysis from measurement failures (Grafarend and Sanso 1985).

In addition to the precision criterion, an optimal network is supposed to be reliable enough in order to detect the gross errors and minimise the effects of undetected ones. Baarda (1968) introduced the concept of reliability to perform a quality control for the geodetic networks. He used statistical hypothesis to test if the outliers are detectable or not. The effect of less reliable observations on a deformation monitoring network of a dam was inspected by Amiri-Simkooei (2001). To confront the probable distortions in the network due to weak observations from a reliability point of view, he came up with a solution to decrease their weights in the SOD stage to reach the reasonable range of reliability.

The cost of a project is another criterion to be considered in optimising a geodetic network. Using the GPS measurements, one can reduce the cost by considering a distance between net points and the length of observation times. Although there are many monitoring projects, carried out using continuous GPS stations (e.g. Naito et al. 1998), it is very common to establish GPS receivers temporarily at pre-set net points to perform such surveys. Dare and Saleh (2000) performed an epoch-wise survey consisting of many observation sessions in which GPS receivers were moved between the net points. In their work, the instrument shifts were addressed as more costly than the observation time of each session.

In optimisation of a geodetic network we try to minimise or maximise any of the network quality criteria as an object function (Kuang 1996, p. 245) by making a Single Objective Optimisation Model (SOOM) subject to a number of constraints. However, considering multiple criteria in one object function is used widely for different purposes. The multi-objective optimisation technique was the tool that Xu and Grafarend (1995) took the benefit from and designed an optimal deforming network, and Bagherbandi et al. (2009) implemented the single- and multi-objective optimisation models in a simulated geodetic network and concluded that the SOOM of reliability is the best model in providing high reliability and precision.

Usually in monitoring networks the assumption is to use the measuring devices with the same precision during the different epochs, where as in this paper, it is assumed that more precise instruments are used in the second observation epoch. A GPS displacement monitoring network will be optimised by the SOOM of precision, constrained to reliability to obtain the optimal observation plans for epoch-wise measurements.

2 Methodology

To express our idea and implement it on a GPS monitoring network, first we need to define some basic equations for observations in a GPS monitoring network. The following subsections include also explanation about the sensitivity criterion in a displacement network and the concept of optimisation procedure.

2.1 Basic equations

In a GPS network, single baselines are acquired from any pairs of stations. The observation equations for such a network, which consists of all possible combinations of independent baselines, can be written as:

$$\boldsymbol{\Delta} - \boldsymbol{\varepsilon} = \mathbf{A}\mathbf{x}, \tag{1}$$

where

$$\boldsymbol{\Delta} = \begin{bmatrix} \boldsymbol{\Delta}_1 \\ \boldsymbol{\Delta}_2 \\ \vdots \\ \boldsymbol{\Delta}_n \end{bmatrix}_{n\times 1}, \quad \boldsymbol{\Delta}_i = \mathbf{x}_j - \mathbf{x}_k \quad i = 1, 2, \ldots, n \tag{2}$$

and $\mathbf{A}$ is the design matrix, containing the zero and plus/minus ones. The vector $\mathbf{x}$ carries the coordinates of all involved station points in forming n observations, and $\boldsymbol{\varepsilon}$ is the vector of residuals. In Eq. (2), $\mathbf{x}_j$ and $\mathbf{x}_k$ are the three-dimensional coordinate vectors of the observation stations j, k.

Although, from a theoretical point of view, we have correlations between the observed baselines, it has occasionally been neglected to take account of this issue. Therefore, in introducing a weight matrix, the off-diagonal elements of the matrix are considered as zero. To start the optimisation procedure, we need to introduce initial weight values for the observations. The weight matrix is being updated during the process according to the desired and defined criteria, and hopefully, turns into an optimum weight matrix, and provides us with decisively needed baselines. In GPS measured baselines, we have three components for each baseline, thus, three weights should be defined for each baseline. Mathematically, we form the weight matrix as:

$$\mathbf{P} = \mathrm{diag}(\mathbf{p}_i), \quad i = 1, 2, \ldots, n \tag{3}$$

where

$$\mathbf{p}_i = \sigma_0^2 \begin{bmatrix} \sigma_{\Delta x_i}^2 & 0 & 0 \\ 0 & \sigma_{\Delta y_i}^2 & 0 \\ 0 & 0 & \sigma_{\Delta z_i}^2 \end{bmatrix}^{-1} \tag{4}$$

and the precisions of baseline components are shown by $\sigma_{\Delta x_i}$, $\sigma_{\Delta y_i}$ and $\sigma_{\Delta z_i}$, and σ_0^2 is the a priori variance of unit weight, which is often assumed as 1. Less accuracy in the up-direction of the GPS measurements in each baseline i can be tuned up in Eq. (4) by considering

$$\sigma_{\Delta x_i}^2 = \sigma_{\Delta y_i}^2 < \sigma_{\Delta z_i}^2. \tag{5}$$

In order to prevent the matrix product $\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}$, i.e. the normal equation matrix, to become singular, the datum defect should be resolved by either minimum or inner constraints.

Here, the inner constraints are used to fix the coordinates of the network centroid by introducing the matrix $\mathbf{H}$ as:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 1 \end{bmatrix}_{3\times 3m}^{\mathrm{T}} \tag{6}$$

with m being the number of observation stations. Now, the VC matrix of the network can be written as (Kuang 1996, p. 221):

$$\mathbf{C}_{\mathbf{x}} = \sigma_0^2\left[\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A} + \mathbf{H}\mathbf{H}^{\mathrm{T}}\right)^{-1} - \mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\right], \tag{7}$$

where its components are already defined in the previous equations.

2.2 Instrumental precision improvement

Monitoring a geodetic network is performed within different time intervals, i.e. observation epochs, to assess the possible deformation of that network. It has been usually assumed that the same types of instruments are used in this procedure. Here, we are about to investigate the case that more precise measurements can be performed in the next epoch of observations. It is highly expected to measure with more accurate devices such as total stations during the time intervals, but using the GPS instruments, we can also consider the longer observation time to obtain a better accuracy. Suppose that the observations in the first epoch have the weight matrix $\mathbf{P}_1$ and therefore the VC matrix $\mathbf{Cx}_1$. If we increase the weight of observations in the second epoch $\mathbf{P}_2$, we will acquire a more precise network by the second observation plan, so we can write:

$$\mathbf{P}_2 = \frac{1}{k}\mathbf{P}_1 \rightarrow \ \mathbf{C}_{\mathbf{x}_2} = k\,\mathbf{C}_{\mathbf{x}_1} \quad \text{with } k < 1 \tag{8}$$

where $\mathbf{C}_{\mathbf{x}_2}$ represents the VC matrix of the network in the second epoch. In Eq. (8), k is a coefficient that scales down the variances of the net points in the first epoch and makes a rigorous observation plan for the second epoch. In other words, the VC matrix of the coordinates in the latter epoch becomes k times smaller than the former one that leads to a more precise plan.

2.3 A monitoring network sensitive to displacements

We commence with defining the displacement of a net point in a geodetic network, which is caused by changes in the observed coordinate value of the point. By denoting the coordinate vector of the point j in the first and second epochs by $\mathbf{x}_j^1$ and $\mathbf{x}_j^2$, respectively, the displacement $\mathbf{d}_j$ of that point can be written as:

$$\mathbf{d}_j = \mathbf{x}_j^2 - \mathbf{x}_j^1. \tag{9}$$

It is of importance to statistically test the obtained displacement in order to figure out whether it is significant enough to be detected in the network or it is placed amongst the random errors. Kuang (1996, p. 302) performed a Chi square (χ^2) distribution to test the hypotheses with the significance level of $1 - \alpha$, where typically $\alpha = 0.05$, and the degrees of freedom, df, as below:

$$\mathbf{d}_j^{\mathrm{T}}\mathbf{C}_{\mathrm{d}_j}^{-1}\mathbf{d}_j \sim \chi^2_{1-\alpha}(df). \tag{10}$$

To facilitate the computation of the displacements of all the net points, we rewrite the Eq. (9) to gather all the displacements in a $3m \times 1$ vector $\mathbf{d}$ as:

$$\mathbf{d} = \mathbf{x}_2 - \mathbf{x}_1, \tag{11}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the coordinate vectors of all net points in the network in epochs 1 and 2, respectively. Now, considering independent observations and applying the error propagation law to Eq. (11), the VC matrix of the displacements $\mathbf{C}_\mathbf{d}$ is defined as:

$$\mathbf{C}_\mathbf{d} = \mathbf{C}_{\mathbf{x}_1} + \mathbf{C}_{\mathbf{x}_2} \tag{12}$$

where $\mathbf{C}_{\mathbf{x}_1}$ and $\mathbf{C}_{\mathbf{x}_2}$ are the VC matrix of net points in the first and second epochs, respectively.

2.4 Single-objective optimisation model with reliability constraint

The observation plan in the second epoch of a two-epoch measuring plan needs to be optimised to fulfil the desired precision criterion and provide a network to detect the possible displacements. Substituting Eq. (8) into Eq. (12), we can write:

$$\mathbf{C_d} = \mathbf{C}_{\mathbf{x}_1} + k\,\mathbf{C}_{\mathbf{x}_1}. \tag{13}$$

Considering Eq. (7), the above equation can be linearised by expanding to a Taylor series as:

$$\mathbf{C_d} - k\,\mathbf{C}_{\mathbf{x}_1} = \mathbf{C}_{\mathbf{x}_1} = \mathbf{C}^0_{\mathbf{x}_1} + \sum_{i=1}^{n} \frac{\partial \mathbf{C}_{\mathbf{x}_1}}{\partial \mathbf{P}_i} \Delta \mathbf{P}_i, \tag{14}$$

where $\mathbf{C}^0_{\mathbf{x}_1}$ can be computed as:

- $\left(\mathbf{A}_1^{\mathrm{T}}\mathbf{P}\mathbf{A}_1 + \mathbf{H}\mathbf{H}^{\mathrm{T}}\right)^{-1} - \mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}$ with $\mathbf{A}_1$ defined as the coefficient matrix of the coordinates for existing baselines in the first epoch, or
- $\left(\mathbf{A}_0^{\mathrm{T}}\mathbf{P}\mathbf{A}_0 + \mathbf{H}\mathbf{H}^{\mathrm{T}}\right)^{-1} - \mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}$ with $\mathbf{A}_0$ defined as the coefficient matrix of all possible baselines that can be observed in the network,

and $\Delta\mathbf{P}_i$ is the improvement of the weight values.

In order to design an optimal observation plan for the next epoch, the optimal weight values for the observations should be estimated in a way that the difference of the VC matrix and the criterion becomes a minimum. Based on Eq. (14) the criterion matrix ($\mathbf{C}_{\mathrm{s}}$) for optimisation procedure can be defined as:

$$\mathbf{C}_{\mathrm{s}} = \mathbf{C_d} - k\,\mathbf{C}_{\mathbf{x}_1}, \tag{15}$$

where $\mathbf{C_d}$ can be numerically obtained from Eq. (10) by considering the desired and definitely statistically possible displacement value. In addition to the capability of the above criterion matrix in dealing with displacement sensitivity, it can be controlled for the precision improvement in the next epoch by k. Moreover, it is required to transform the criterion matrix with the same datum parameters as the VC matrix of the network. It should be noted that considering the left hand side of Eq. (14) and Eq. (15) leads to $\mathbf{C}_{\mathrm{s}} = \mathbf{C}_{\mathbf{x}_1}$, which is meaningless for an optimisation purpose. Therefore, $\mathbf{C}_{\mathbf{x}_1}$ in the middle part of Eq. (14) is linearised to be somehow related to the observation weights. Through the optimisation process, some improvements are estimated to the observation weights in order to fit $\mathbf{C}^0_{\mathbf{x}_1}$ to $\mathbf{C}_{\mathrm{s}}$.

As already mentioned, the goal in Eq. (14) is to fit the VC matrix of the first epoch to a defined ideal criterion in Eq. (15). Therefore, we try to minimise this difference, namely

$$\left\|\mathbf{C}_{\mathrm{s}} - \mathbf{C}^0_{\mathbf{x}_1}\right\|_2 = \min \tag{16}$$

subjected to precision, reliability and physical constraints. Equation (16) can be reformulated in a vectorised matrix model by considering its linearised form presented in Eq. (14) as (Kuang 1996, p. 227):

$$\|\mathbf{T}\mathbf{w} - \mathbf{u}\|_2 = \min \tag{17}$$

subject to constraints

$$\mathbf{S}_1\mathbf{w} \le \mathbf{S}_2, \tag{18}$$

where

$$\mathbf{T} = \left[\operatorname{vec}\left(\frac{\partial \mathbf{C_x}}{\partial \mathbf{P}_1}\right) \quad \operatorname{vec}\left(\frac{\partial \mathbf{C_x}}{\partial \mathbf{P}_2}\right) \quad \cdots \quad \operatorname{vec}\left(\frac{\partial \mathbf{C_x}}{\partial \mathbf{P}_n}\right)\right] \tag{19}$$

$$\mathbf{u} = \mathrm{vec}(\mathbf{C}_{\mathrm{s}}) - \mathrm{vec}\left(\mathbf{C}_{\mathbf{x}}^{0}\right) \tag{20}$$

$$\mathbf{w} = \left[\, \Delta\mathbf{P}_1 \quad \Delta\mathbf{P}_2 \quad \cdots \quad \Delta\mathbf{P}_n \,\right]^{\mathrm{T}} \tag{21}$$

and

$$\mathbf{S}_1 = \begin{bmatrix} \mathbf{T}_1 \\ -\mathbf{R}_2 \\ \mathbf{A}_{00} \end{bmatrix} \quad \text{and} \quad \mathbf{S}_2 = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{R}_1 - \mathbf{r}_{\mathrm{o}} \\ \mathbf{b}_{00} \end{bmatrix}. \tag{22}$$

Using the vec$(\cdot)$ operator in the above equations stacks the columns of a matrix one below each other to build a vector and $\|\cdot\|_2$ stands for the L_2-norm. The submatrices of $\mathbf{S}_1$ and $\mathbf{S}_2$ are the representatives of the precision, reliability and physical constraints, where the precision control constraints are:

$$\mathbf{T}_1 = (\mathbf{I}_{3m} \Theta\, \mathbf{I}_{3m})^{\mathrm{T}} \mathbf{T} \quad \text{and} \quad \mathbf{u}_1 = (\mathbf{I}_{3m} \Theta\, \mathbf{I}_{3m})^{\mathrm{T}} \mathbf{u}, \tag{23}$$

where $\mathbf{I}_{3m}$ represents a $3m \times 3m$ unit matrix and Θ is the Khatri-Rao product (Khatri and Rao 1968). In Eq. (22), $\mathbf{R}_2$ is a $3n \times n$ matrix consisting of the derivatives of the reliability matrix with respect to the weight matrix of each baseline; $\mathbf{R}_1$ is a $3n \times 1$ vector including the diagonal elements of the reliability matrix of the observations in the network (cf. Alizadeh-Khameneh et al. 2015), and finally, $\mathbf{r}_{\mathrm{o}}$ is introduced to consider the desired value of the reliability in the optimisation process, which receives a value between 0 to 1, representing low to high reliability of a network in detecting the blunders.

The submatrix $\mathbf{A}_{00}$ in Eq. (22) is the coefficient matrix of the inequality constraints and $\mathbf{b}_{00}$ is a matrix including the weights and their improvements. Generally, the physical constraint tries to bind the weights in order to avoid achieving negative weights.

Theoretically, Eqs. (17) and (18) define a SOOM with precision and reliability constraints. It has been previously studied in Eshagh and Kiamehr (2007) and Alizadeh-Khameneh et al. (2015) that a reliability constraint plays a significant role in designing a robust network against outliers, while a single precision model cannot provide such a reliable network. Consequently, a SOOM of precision that is constrained to the precision and reliability yields an optimised network fulfilling these major criteria.

3 Numerical studies

In this section, we implemented the explained methodology in a real application. First the specifications of the study area will be defined, and then the experimentation results will be described in the next subsection.

3.1 Study area

It is of great interest to experiment the introduced methodology on a real case study. Despite applying the explained method to a simulated network and obtaining an acceptable result, we prefer to test the idea in reality. For this purpose, the GPS monitoring network of Lilla Edet region in the southwest of Sweden is chosen. This area is well-known for its landslides, which have been happening during the years, and since the year 2000, the risky area has been settled under monitoring controls in different time intervals (epochs).

The existing GPS monitoring network of the area consists of 35 observation stations, where 6 of them are fixed points. The coordinate system of the network is SWEREF 99, which is the Swedish realisation of the European Terrestrial Reference System (ETRS 89) at epoch 1999.5. Totally, 245 independent baselines are observed in this network by neglecting the correlation influence amongst the GPS receivers. It has been assumed that each observation session includes two GPS receivers and the instruments are moved to perform the next session. The current observation plan provides us with initial data to accomplish our aim in designing two sequential epochs of measurements, where it is assumed that in the second epoch we can observe the baseline components with a higher precision by increasing the observation duration.

3.2 Results

Before starting the optimisation procedure, we need to fill in the basic matrices introduced in Sect. 2.1 with their corresponding values from the existing information of the network. The initial weight matrix **P**, the VC matrix of the displacements $\mathbf{C_d}$ and the precision criterion matrix $\mathbf{C}_s$ are required to be defined first. As it has been explained before, the weight matrix is formed by precision of the baseline components for each observation. In this network that initially consists of 245 baselines, the weight matrix will be a (245 × 3) by (245 × 3) square matrix. We use the report of the consultant (Nordqvist 2012), who performed the monitoring measurements for defining the baseline variances as:

$$\begin{cases} \sigma^2_{\Delta x_i} = \sigma^2_{\Delta y_i} = 5^2 + [1\,\text{ppm} \times S_i(\text{km})]^2 & [\text{mm}^2] \\ \sigma^2_{\Delta z_i} = 7^2 + [1\,\text{ppm} \times S_i(\text{km})]^2 & [\text{mm}^2] \end{cases} \tag{24}$$

where S_i is the distance between the net points. It can be seen in Eq. (24) that the z-components of the baselines have larger variances due to less accurate result of GPS in up-direction. Moreover, inserting the numerical values from Eq. (24) into Eqs. (3) and (4) yields the initial weight matrix of the observations.

The displacement VC matrix $\mathbf{C_d}$ is defined by using the statistical method explained in Eq. (10), where the minimum displacement of 5 millimetres in all directions is assumed to be detected for each point j in this network i.e. $\mathbf{d}_j = [5 \quad 5 \quad 5]^T$.

Defining the criterion matrix as mentioned in Eq. (15) is the practical and productive step in all and specifically in this optimisation procedure. In this work, two criteria are involved in defining the precision criterion matrix. First, we aim to consider the sensitivity of the network in detecting the displacement of the points by $\mathbf{C_d}$, and second we deal with the effect of better instrumental precision by multiplying k to the VC matrix of the existing network. The applied coefficient k in this paper ranges from 0.5 to 1 with the increment of 0.1.

The SOOM of precision and reliability is used to optimise a monitoring network in order to redesign observation plans for two subsequent epochs considering better precision for instruments in the second one. We proceed the optimisation procedure with solving Eq. (17), which is subjected to the constraints in Eq. (18). It has been explained in the previous section that the necessity of having reliable observations in the designed plan is the reason of constraining the SOOM to the reliability criterion in addition to precision criterion. In the used optimisation model, the procedure tries to optimally change the weight values to minimise the difference between the matrices of criterion and VC of the network, namely it tries to fulfil the criterion.

Here, we assume different values for k to investigate its effect on the optimised observation plan. The number of remaining baselines after all proposed values for k in the optimisation procedure is shown in Table 1.

As can be seen in Table 1, in the first step, we assume the equal weight matrix for the both epochs i.e. no instrumental precision improvement is considered. In order to have a network to be able to detect 5 millimetre displacements, it is enough to observe 215 baselines in each epoch of measurements. In the next steps, we try to indicate the effect of precision improvements on the second epoch of a monitoring observation plan. The larger we make the weight matrix in second epoch, the less number of baselines we require for measuring. It should be clarified here that it is improbable these days to double the weight matrix for the next epoch. In other words, with available precise measuring devices in the market, it is very rare to be able to increase the precision of the instruments very much within a time interval of some months. However, it has been investigated theoretically in this paper to express the idea and bring up the thoughts around this issue. Moreover, the effect of even small improvements on the number of baselines is recognisable enough to consider this method as an efficient one.

Table 1 is illustrated in Figs. 1 and 2, where the observation plans of the network before and after optimisation are depicted. The figures are generated for two cases in which k is 0.5 (Fig. 1) and 0.7 (Fig. 2). In both figures, the left panel shows the unoptimised network with all possible 245 baselines, and the outcome of the optimisation process for $k = 1$ is presented in the middle as the observation plan for the first epoch. The right panel in the figures shows the optimised network for the second epoch. This panel is different in two figures due to different coefficients that are used for improving the precision in the second epoch.

The precision of the net points after the optimisation procedure are plotted as graphs in Fig. 3 for one attempt, where $k = 0.7$. It is clearly evident in the figure that the network achieves better accuracy after optimising procedure. The lower graph shows the precision of the net points by considering k equals to 1. By this assumption, we obtained the first observation plan, which provides the higher precision for the net points. Despite our expectation to achieve even higher precision in the second epoch compared with the first one, we obtained lower precision. The reason can be found in the definition of the criterion matrix. The first term in the criterion matrix has fixed values ($\mathbf{C_d}$), implying that its difference with $k\mathbf{C}_x$ becomes larger for smaller k. In other words, the higher instrumental precision that we demand, the larger criterion we receive. When we introduce a smaller

Table 1 Number of required observations in the first and second epochs after optimisation procedure according to precision improvements. The number of baselines before optimisation is 245

$k < 1$	$P_2 = \frac{1}{k} P_1$	No. of observations in	
		First epoch	Second epoch
1	$P_2 = P_1$	215	215
0.9	$P_2 = 1.1P_1$	215	204
0.8	$P_2 = 1.2P_1$	215	193
0.7	$P_2 = 1.4P_1$	215	175
0.6	$P_2 = 1.7P_1$	215	154
0.5	$P_2 = 2P_1$	215	143

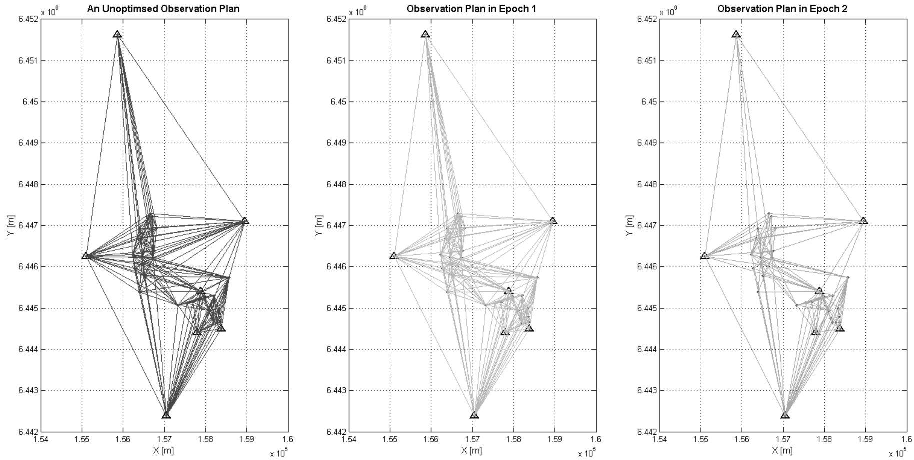

Fig. 1 The panels in the figure display the unoptimised and optimised networks in first epoch, and observation plan for second epoch considering $k = 0.5$, respectively, from *left* to *right*. The coordinates on the X and Y axes are, respectively, distances from the central meridian of zone 12° and the equator in SWEREF 99 12 00

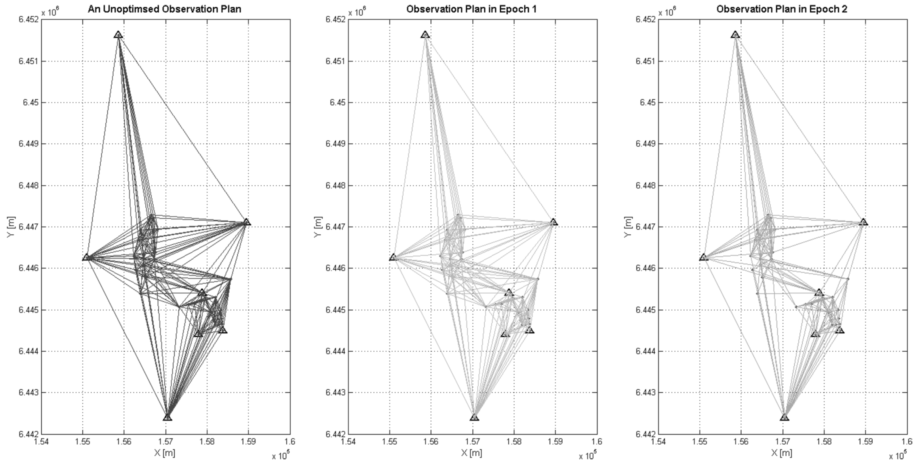

Fig. 2 The panels in the figure display the unoptimised and optimised networks in first epoch and observation plan for second epoch considering $k = 0.7$, respectively, from *left* to *right*. The coordinates on the X and Y axes are, respectively, distances from the central meridian of zone 12° and the equator in SWEREF 99 12 00

value for k, the criterion matrix gives more flexibility to the network to ignore the baselines that cannot significantly diminish the precision of the network, and consequently we get less number of baselines to observe in the second epoch. Increasing the instrumental precision by a specific scale does not necessarily lead to a network with higher precision of the same scale. However, observing the baselines with more precise instruments provides the required precision of the network with less number of baseline measurements.

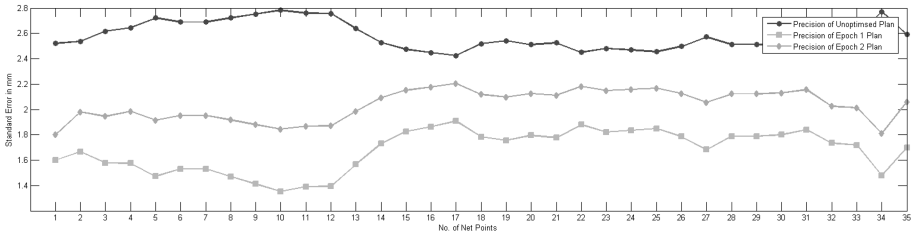

Fig. 3 Precision of the net points before and after optimisation. k is assumed to be 0.7 in the second epoch. The *upper graph* shows the precision of the points before optimisation

4 Conclusion

In this paper, the effect of observation precision is investigated in the optimisation of the Lilla Edet GPS displacement monitoring network. It has been assumed that the precision of GPS observations can be increased in the subsequent epochs. The enhancement of GPS observation precisions is achievable by increasing the observation time, using forced centring pillars and the combined use of GPS and other satellite systems. However, this study is conducted to numerically present the results of such an assumption. The existing monitoring network of Lilla Edet comprises 245 single baselines. We start the optimisation procedure by defining a criterion matrix to fulfil both sensitivity and precision of the network. The sensitivity is introduced as capability of the network in detecting 5 mm displacement at each net point. Considering the similar precisions for both epochs has yielded two optimum observation plans with 215 baselines in each. Increasing the precision for the second epoch is performed within several increments. In the extreme case, we assume two times larger observation weights in the second epoch in the optimisation procedure. An observation plan with 143 baselines in the second epoch versus 215 in the first one is designed as the process result. Regardless of the fact that the aforementioned assumption is unrealistic, the other slight improvements in the observation weights can efficiently and practically decrease the number of observation demands. In the medium scale networks such as Lilla Edet, removing unnecessary baselines from the observation plan, whilst the quality requirements are preserved, can save a considerable amount of time and cost in the project.

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