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ON FUZZY α-CONTINUOUS MULTIFUNCTIONS

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Abstract. In this paper we use fuzzy α-sets in order to obtain certain characterizations and properties of upper (or lower) fuzzy α-continuous multifunctions.

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1. INTRODUCTION

In 1968 Chang [3] introduced fuzzy topological spaces by using fuzzy sets [12]. Since then several workers have contributed to this area: various types of functions play a significant role in the theory of classical point set topology. A great number of papers dealing with such functions have appeared, and a good many of them have been extended to the setting of multifunctions.

In 1988 Neubrunn [6] and others [9]] introduced the concept of α-continuous multifunctions. Njasted [7] and Mashhour [4] introduced α-open (α-closed) sets, respectively. Bin Shahna in [2] defined these concepts in the fuzzy setting. In this paper our purpose is to define upper (lower) fuzzy α-continuous multifunctions and to obtain several characterizations of upper (lower) fuzzy α-continuous multifunctions.

Fuzzy sets on a universe X will be denoted by μ, ρ, η, etc. Fuzzy points will be denoted by x_ϵ, y_ν, etc. For any fuzzy points x_ϵ and any fuzzy set μ, we write $x_\epsilon \in \mu$ iff $\epsilon \leq \mu(x)$. A fuzzy point x_ϵ is called quasi-coincident with a fuzzy set ρ, denoted by $x_\epsilon \text{ q } \rho$, iff $\epsilon + \rho(x) > 1$.

A fuzzy set μ is called quasi-coincident with a fuzzy set ρ, denoted by $\mu \text{ q } \rho$, iff there exists a $x \in X$ such that $\mu(x) + \rho(x) > 1$. [10, 11]

In this paper we use the concept of fuzzy topological space as introduced in [3]. By int (μ) and cl (μ), we mean the interior of μ and the closure of μ, respectively.

Let (X, τ) be a topological space in the classical sense and (Y, ν) be a fuzzy topological space. $F : X \to Y$ is called a fuzzy multifunction iff for each $x \in X$, $F(x)$ is a fuzzy set in Y. [8]

Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space X to a fuzzy topological space Y. For any fuzzy set $\mu \leq X$, $F^+(\mu)$ and $F^-(\mu)$ are defined by $F^+(\mu) = \{x \in X : F(x) \leq \mu\}$, $F^-(\mu) = \{x \in X : F(x) q \mu\}$. [5]

2. FUZZY α-CONTINUOUS MULTIFUNCTION

Definition 1. Let (X, τ) be a fuzzy topological space and let $\mu \leq X$ be a fuzzy set. Then it is said that:

(i) μ is fuzzy α-open set [2] if $\mu \leq \text{int cl int } \mu$.
(ii) μ is fuzzy α-closed set [2] if $\mu \geq \text{cl int cl } \mu$.
(iii) μ is fuzzy semiopen set [1] if $\mu \leq \text{cl int } \mu$.
(iv) μ is fuzzy preopen set [2] if $\mu \leq \text{int cl } \mu$.

Definition 2. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, ν). Then it is said that F is :

(1) Upper fuzzy α-continuous at $x_\epsilon \in X$ iff for each fuzzy open set μ of Y containing $F(x_\epsilon)$, there exists a fuzzy α-open set ρ containing x_ϵ such that $\rho \leq F^+(\mu)$.
(2) Lower fuzzy α-continuous at $x_\epsilon \in X$ iff for each fuzzy open set μ of Y such that $x_\epsilon \in F^-(\mu)$ there exists a fuzzy α-open set ρ containing x_ϵ such that $\rho \leq F^-(\mu)$.
(3) Upper (lower) fuzzy α-continuous iff it has this property at each point of X.

We know that a net $(x^\alpha_{\epsilon_\alpha})$ in a fuzzy topological space (X, τ) is said to be eventually in the fuzzy set $\rho \leq X$ if there exists an index $\alpha_0 \in J$ such that $x^\alpha_{\epsilon_\alpha} \in \rho$ for all $\alpha \geq \alpha_0$.

The following theorem states some characterizations of upper fuzzy α-continuous multifunction.

Definition 3. A sequence (x_{ϵ_n}) is said to α-converge to a point X if for every fuzzy α-open set μ containing x_ϵ there exists an index n_0 such that for $n \geq n_0$, $x_{\epsilon_n} \in \mu$. This is denoted by $x_{\epsilon_n} \to_\alpha x_\epsilon$.

Theorem 1. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological (X, τ) to a fuzzy topological space (Y, ν). Then the following statements are equivalent:*

(i) *F is upper fuzzy α-continuous.*
(ii) *For each $x_\epsilon \in X$ and for each fuzzy open set μ such that $x_\epsilon \in F^+(\mu)$ there exists a fuzzy α -open set ρ containing x_ϵ such that $\rho \leq F^+(\mu)$.*
(iii) *$F^+(\mu)$ is a fuzzy α-open set for any fuzzy open set $\mu \leq Y$.*
(iv) *$F^-(\mu)$ is a fuzzy α-closed set for any fuzzy open set $\mu \leq Y$.*
(v) *For each $x_\epsilon \in X$ and for each net $(x^\alpha_{\epsilon_\alpha})$ which α-converges to x_ϵ in X and for each fuzzy open set $\mu \leq Y$ such that $x_\epsilon \in F^+(\mu)$, the net $(x^\alpha_{\epsilon_\alpha})$ is eventually in $F^+(\mu)$.*

Proof. (i)$\Leftrightarrow$(ii) this statement is obvious.

(i)$\Leftrightarrow$(iii). Let $x_\epsilon \in F^+(\mu)$ and let μ be a fuzzy open set. It follows from (i) that there exists a fuzzy α-open set ρ_{x_ϵ} containing x_ϵ such that $\rho_{x_\epsilon} \leq F^+(\mu)$. It follows that $F^+(\mu) = \bigvee_{x_\epsilon \in F^+(\mu)} \rho_{x_\epsilon}$ and hence $F^+(\mu)$ is fuzzy α-open.

The converse can be shown easily.

(iii)$\Rightarrow$(iv) Let $\mu \leq Y$ be a fuzzy open set. We have that $Y \setminus \mu$ is a fuzzy open set. From (iii), $F^+(Y \setminus \mu) = X \setminus F^-(\mu)$ is a fuzzy α-open set. Then it is obtained that $F^-(\mu)$ is a fuzzy α-closed set.

(i)$\Rightarrow$(v). Let $(x^\alpha_{\epsilon_\alpha})$ be a net which α -converges to x_ϵ in X and let $\mu \leq Y$ be any fuzzy open set such that $x_\epsilon \in F^+(\mu)$. Since F is an upper fuzzy α-continuous multifunction, it follows that there exists a fuzzy α-open set $\rho \leq X$ containing x_ϵ such that $\rho \leq F^+(\mu)$. Since $(x^\alpha_{\epsilon_\alpha})$ α-converges to x_ϵ, it follows that there exists an index $\alpha_o \in J$ such that $(x^\alpha_{\epsilon_\alpha}) \in \rho$ for all $\alpha \geq \alpha_o$ from here, we obtain that $x^\alpha_{\epsilon_\alpha} \in \rho \leq F^+(\mu)$ for all $\alpha \geq \alpha_o$. Thus the net $(x^\alpha_{\epsilon_\alpha})$ is eventually in $F^+(\mu)$.

(v)$\Rightarrow$(i). Suppose that is not true. There exists a point x_ϵ and a fuzzy open set μ with $x_\epsilon \in F^+(\mu)$ such that $\rho \not\leq F^+(\mu)$ for each fuzzy α-open set $\rho \leq X$ containing x_ϵ. Let $x_{\epsilon_\rho} \in \rho$ and $x_\epsilon \notin F^+(\mu)$ for each fuzzy α-open set $\rho \leq X$ containing x_ϵ. Then for the α-neighborhood net (x_{ϵ_ρ}), $x_{\epsilon_\rho} \rightarrow_\alpha x_\epsilon$, but (x_{ϵ_ρ}) is not eventually in $F^+(\mu)$. This is a contradiction. Thus, F is an upper fuzzy α-continuous multifunction. □

Remark 1. For a fuzzy multifunction $F : X \to Y$ from a fuzzy topological (X, τ) to a fuzzy topological space (Y, ν), the following implication holds:
Upper fuzzy continuous $\Longrightarrow$ Upper fuzzy α-continuous.

The following example show that the reverse need not be true.

Example 1. Let $X = \{x, y\}$ with topologies $\tau = \{X, \phi, \mu\}$ and $\nu = \{X, \phi, \rho\}$, where the fuzzy sets μ, ρ are defined as:

$$\mu(x) = 0.3, \quad \mu(y) = 0.6$$
$$\rho(x) = 0.7, \quad \rho(y) = 0.4$$

A fuzzy multifunction $F : (X, \tau) \to (Y, \nu)$ given by $x_\epsilon \to F(x_\epsilon) = \{x_\epsilon\}$ is upper α-continuous, but it is not upper continuous.

The following theorem states some characterizations of a lower fuzzy α-continuous multifunction.

Theorem 2. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological (X, τ) to a fuzzy topological space (Y, ν). Then the following statements are equivalent.*

(i) *F is lower fuzzy α-continuous.*

(ii) *For each $x_\epsilon \in X$ and for each fuzzy open set μ such that $x_\epsilon \in F^-(\mu)$ there exists a fuzzy α-open set ρ containing x_ϵ such that $\rho \leq F^-(\mu)$.*

(iii) *$F^-(\mu)$ is a fuzzy α-open set for any fuzzy open set $\mu \leq Y$,*

(iv) *$F^+(\mu)$ is a fuzzy α-closed set for any fuzzy open set $\mu \leq Y$,*
(v) *For each $x_\epsilon \in X$ and for each net $(x^{\alpha}_{\epsilon_\alpha})$ which α-converges to x_ϵ in X and for each fuzzy open set $\mu \leq Y$ such that $x_\epsilon \in F^-(\mu)$, the net $(x^{\alpha}_{\epsilon_\alpha})$ is eventually in $F^-(\mu)$.*

Proof. It can be obtained similarly as Theorem 1. □

Theorem 3. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological (X,τ) to a fuzzy topological space (Y,ν) and let $F(X)$ be endowed with subspace fuzzy topology. If F is an upper fuzzy α-continuous multifunction, then $F : X \to F(X)$ is an upper fuzzy α-continuous multifunction.*

Proof. Since F is an upper fuzzy α-continuous, $F(X \wedge F(X)) = F^+(\mu)\wedge$ $\wedge F^+(F(X)) = F^+(\mu)$ is fuzzy α-open for each fuzzy open subset μ of Y. Hence $F : X \to F(X)$ is an upper fuzzy α-continuous multifunction. □

Definition 4. Suppose that $(X,\tau),(Y,\nu)$ and (Z,ω) are fuzzy topological spaces. It is known that if $F_1 : X \to Y$ and $F_2 : Y \to Z$ are fuzzy multifunctions, then the fuzzy multifunction F_1o $F_2 : X \to Z$ is defined by $(F_1 \text{ o } F_2)(x_\epsilon) = F_2(F_1(x_\epsilon))$ for each $x_\epsilon \in X$.

Theorem 4. *Let $(X,\tau),(Y,\nu)$ and (Z,ω) be fuzzy topological space and let $F : X \to Y$ and $G : Y \to Z$ be fuzzy multifunction. If $F : X \to Y$ is an upper (lower) fuzzy continuous multifunction and $G : Y \to Z$ is an upper (lower) fuzzy α-continuous multifunction. Then G o $F : X \to Z$ is an upper (lower) fuzzy α-continuous multifunction.*

Proof. Let $\lambda \leq Z$ be any fuzzy open set. From the definition of G o F, we have $(G \text{ o } F)^+(\lambda) = F^+(G^+(\lambda))((G \text{ o } F)^-(\lambda) = F^-(G^-(\lambda)))$, since G is an upper (lower) fuzzy α-continuous, it follows that $G^+(\lambda)(G^-(\lambda))$ is a fuzzy open set. Since F is an upper (lower) fuzzy continuous, it follows that $F^+(G^+(\lambda))(F^-(G^-(\lambda)))$ is a fuzzy α-open set, this shows that G o F is an upper (lower) fuzzy α-continuous. □

Theorem 5. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological (X,τ) to a fuzzy topological space (Y,ν). If F is a lower(upper) fuzzy α-continuous multifunction and $\mu \leq X$ is a fuzzy set, then the restriction multifunction $F|_\mu : \mu \to Y$ is an lower (upper) fuzzy α-continuous multifunction.*

Proof. Suppose that $\beta \leq Y$ is a fuzzy open set. Let $x_\epsilon \in \mu$ and let $x_\epsilon \in F^-|_\mu(\beta)$. Since F is a lower fuzzy α-continuous multifunction, if follows that there exists a fuzzy open set $x_\epsilon \in \rho$ such that $\rho \leq F^-(\beta)$. From here we obtain that $x_\epsilon \in \rho \wedge \mu$ and $\rho \wedge \mu \leq F|_\mu(\beta)$. Thus, we show that the restriction multifunction $F|_\mu$ is lower fuzzy α-continuous multifunction. □

The proof for the case of the upper fuzzy α-continuity of the multifunction $F|_\mu$ is similar to the above.

Theorem 6. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological (X,τ) to a fuzzy topological space (Y,ν), let $\{\lambda_\gamma : \gamma \in \Phi\}$ be a fuzzy open cover of X. If the restriction multifunction $F_\gamma = F_{\lambda_\gamma}$ is lower (upper) fuzzy α-continuous multifunction for each $\gamma \in \Phi$, then F is lower (upper) fuzzy α-continuous multifunction.*

Proof. Let $\mu \leq Y$ be any fuzzy open set. Since F_γ is lower fuzzy α-continuous for each γ, we know that $F_\gamma^-(\mu) \leq int_{\lambda_\gamma}(F_\gamma^-(\mu))$ and from here $F^-(\mu) \wedge \lambda_\gamma \leq int_{\lambda_\gamma}(F^-(\mu) \wedge \lambda_\gamma)$ and $F^-(\mu) \wedge \lambda_\gamma \leq int(F^-(\mu)) \wedge \lambda_\gamma$. Since $\{\lambda_\gamma : \gamma \in \Phi\}$ is a fuzzy open cover of X. It follows that $F^-(\mu) \leq int(F^-(\mu))$. Thus, we obtain that F is lower(upper) fuzzy α-continuous multifunction. □

The proof of the upper fuzzy α-continuity of F is similar to the above.

Definition 5. Suppose that $F : X \to Y$ is a fuzzy multifunction from a fuzzy topological space X to a fuzzy topological space Y. The fuzzy graph multifunction $G_F : X \to X \times Y$ of F is defined as $G_F(x_\epsilon) = \{x_\epsilon\} \times F(x_\epsilon)$.

Theorem 7. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological (X,τ) to a fuzzy topological space (Y,ν). If the graph function of F is lower(upper) fuzzy α-continuous multifunction, then F is lower(upper) fuzzy α-continuous multifunction.*

Proof. For the fuzzy sets $\beta \leq X, \eta \leq Y$, we take

$$(\beta \times \eta)(z,y) = \begin{cases} 0 & \text{if } z \notin \beta \\ \eta(y) & \text{if } z \in \beta \end{cases}$$

Let $x_\epsilon \in X$ and let $\mu \in Y$ be a fuzzy open set such that $x_\epsilon \in F^-(\mu)$. We obtain that $x_\epsilon \in G_F^-(X \times \mu)$ and $X \times \mu$ is a fuzzy open set. Since fuzzy graph multifunction G_F is lower fuzzy α-continuous, it follows that there exists a fuzzy α-open set $\rho \leq X$ containing x_ϵ such that $\rho \leq G_F^-(X \times \mu)$. From here, we obtain that $\rho \leq F^-(\mu)$. Thus, F is lower fuzzy α-continuous multifunction. □

The proof of the upper fuzzy α-continuity of F is similar to the above.

Theorem 8. *Suppose that (X,τ) and (X_α,τ_α) are fuzzy topological space where $\alpha \in J$. Let $F : X \to \prod_{\alpha \in J} X_\alpha$ be a fuzzy multifunction from X to the product space $\prod_{\alpha \in J} X_\alpha$ and let $P_\alpha : \prod_{\alpha \in J} X_\alpha \to X_\alpha$ be the projection multifunction for each $\alpha \in J$ which is defined by $P_\alpha((x_\alpha)) = \{x_\alpha\}$. If F is an upper (lower) fuzzy α-continuous multifunction, then P_α o F is an upper (lower) fuzzy α-continuous multifunction for each $\alpha \in J$.*

Proof. Take any $\alpha_o \in J$. Let μ_{α_o} be a fuzzy open set in (X_α, τ_α). Then $(P_{\alpha_o}$ o $F)^+(\mu_{\alpha_o}) = F^+(P_{\alpha_o}^+(\mu_{\alpha_o})) = F^+(\mu_{\alpha_o} \times \prod_{\alpha \neq \alpha_o} X_\alpha)$ (resp.,$(P_{\alpha_o}$ o $F)^-(\mu_{\alpha_o}) = F^-(P_{\alpha_o}^-(\mu_{\alpha_o})) = F^-(\mu_{\alpha_o} \times \prod_{\alpha \neq \alpha_o} X_\alpha)$).

Since F is upper (lower) fuzzy α-continuous multifunction and since $\mu_{\alpha_o} \times \prod_{\alpha \neq \alpha_o} X_\alpha$ is a fuzzy open set, it follows that $F^+(\mu_{\alpha_o} \times \prod_{\alpha \neq \alpha_o} X_\alpha)$ (resp., $F^-(\mu_{\alpha_o} \times \prod_{\alpha \neq \alpha_o} X_\alpha)$) is fuzzy α- open in (X, τ). It shows that P_{α_o} o F is upper (lower) fuzzy α-continuous multifunction.

Hence, we obtain that P_α o F is an upper (lower) fuzzy α-continuous multi function for each $\alpha \in J$. □

Theorem 9. *Suppose that for each $\alpha \in J$, (X_α, τ_α) and (Y_α, ν_α) are fuzzy topological spaces. Let $F_\alpha : X_\alpha \to Y_\alpha$ be a fuzzy multifunction for each $\alpha \in J$ and let $F : \prod_{\alpha \in J} X_\alpha \to \prod_{\alpha \in J} Y_\alpha$ be defined by $F((x_\alpha)) = \prod_{\alpha \in J} F_\alpha(x_\alpha)$ from the product space $\prod_{\alpha \in J} X_\alpha$ to product space $\prod_{\alpha \in J} Y_\alpha$. If F is an upper (lower) fuzzy α-continuous multifunction, then each F_α is an upper (lower) fuzzy α-continuous multifunction for each $\alpha \in J$.*

Proof. Let $\mu_\alpha \leq Y_\alpha$ be a fuzzy open set. Then $\mu_\alpha \times \prod_{\alpha \neq \beta} Y_\beta$ is a fuzzy open set. Since F is an upper (lower) fuzzy α-continuous multifunction, it follows that $F^+(\mu_\alpha \times \prod_{\alpha \neq \beta} Y_\beta) = F^+(\mu_\alpha) \times \prod_{\alpha \neq \beta} X_\beta$, $(F^-(\mu_\alpha \times \prod_{\alpha \neq \beta} Y_\beta) = F^-(\mu_\alpha) \times \prod_{\alpha \neq \beta} X_\beta)$ is a fuzzy α-open set. Consequently, we obtain that $F^+(\mu_\alpha)$ $(F^-(\mu_\alpha))$ is a fuzzy α-open set. Thus, we show that F_α is an upper (lower) fuzzy α-continuous multifunction. □

Theorem 10. *Suppose that (X_1, τ_1), (X_2, τ_2), (Y_1, ν_1) and (Y_2, ν_2) are fuzzy topological spaces and $F_1 : X_1 \to Y_1$, $F_2 : X_2 \to Y_2$ are fuzzy multifunctions and suppose that if $\eta \times \beta$ is fuzzy α-open set then η and β are fuzzy α-open sets for any fuzzy sets $\eta \leq Y_1$, $\beta \leq Y_2$. Let $F_1 \times F_2 : X_1 \times X_2 \to Y_1 \times Y_2$ be a fuzzy multifunction which is defined by $(F_1 \times F_2)(x_\epsilon, y_\nu) = F_1(x_\epsilon) \times F_2(y_\nu)$. If $F_1 \times F_2$ is an upper (lower) fuzzy α-continuous multifunction, then F_1 and F_2 are upper (lower) fuzzy α-continuous multifunctions.*

Proof. We know that $(\mu^* \times \beta^*)(x_\epsilon, y_\nu) = min\{\mu^*(x), \beta^*(y)\}$ for any fuzzy sets μ^*, β^* and for any fuzzy point x_ϵ, y_ν.

Let $\mu \times \beta \leq Y_1 \times Y_2$ be a fuzzy open set. It known that $(F_1 \times F_2)^+(\mu \times \beta) = F_1^+(\mu) \times F_2^+(\beta)$. Since $F_1 \times F_2$ is an upper fuzzy α-continuous multifunction, it follows that $F_1^+(\mu) \times F_2^+(\beta)$ is a fuzzy α-open set. From here, $F_1^+(\mu)$ and $F_2^+(\beta)$ are fuzzy α-open sets. Hence, it is obtain that F_1 and F_2 are upper fuzzy α-continuous multifunctions. □

The proof of the lower fuzzy α-continuity of the multifunctions F_1 and F_2 is similar to the above.

Theorem 11. *Suppose that (X, τ), (Y, ν) and (Z, ω) are fuzzy topological spaces and $F_1 : X \to Y$, $F_2 : X \to Z$ are fuzzy multifunction and suppose that if $\eta \times \beta$ is a fuzzy α-open set, then η and β are fuzzy α-open sets for any fuzzy sets $\eta \leq Y$, $\beta \leq Z$. Let $F_1 \times F_2 : X \to Y \times Z$ be a fuzzy multifunction which is defined by*

$(F_1 \times F_2)(x_\epsilon) = F_1(x_\epsilon) \times F_2(x_\epsilon)$. If $F_1 \times F_2$ is an upper (lower) fuzzy α-continuous multifunction, then F_1 and F_2 are upper (lower) fuzzy α-continuous multifunctions.

Proof. Let $x_\epsilon \in X$ and let $\mu \leq Y$, $\beta \leq Z$ be fuzzy α-open sets such that $x_\epsilon \in F_1^+(\mu)$ and $x_\epsilon \in F_2^+(\beta)$. Then we obtain that $F_1(x_\epsilon) \leq \mu$ and $F_2(x_\epsilon) \leq \beta$ and from here, $F_1(x_\epsilon) \times F_2(x_\epsilon) = (F_1 \times F_2)(x_\epsilon) \leq \mu \times \beta$. We have $x_\epsilon \in (F_1 \times F_2)^+(\mu \times \beta)$. Since $F_1 \times F_2$ is an upper fuzzy α-continuous multifunction, it follows that there exist a fuzzy α-open set ρ containing x_ϵ such that $\rho \leq (F_1 \times F_2)^+(\mu \times \beta)$. We obtain that $\rho \leq F_1^+(\mu)$ and $\rho \leq F_2^+(\beta)$. Thus we obtain that F_1 and F_2 are fuzzy α-continuous multifunctions. □

The proof of the lower fuzzy α-continuity of the multifunctions F_1 and F_2 is similar to the above.

Lemma 1 ([2]). *A fuzzy set in fuzzy topological space X is a fuzzy α-open set if and only if it is fuzzy semiopen and fuzzy preopen.*

Theorem 12. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological (X, τ) to a fuzzy topological space (Y, ν). Then F is an upper fuzzy α-continuous if and only if it is an upper fuzzy semicontinuous and upper fuzzy precontinuous.*

Proof. Let F be upper fuzzy semicontinuous and upper fuzzy precontinuous, and let μ be a fuzzy open set in Y. Then $F^+(\mu)$ is fuzzy semiopen and fuzzy preopen, it follows from lemma 1 that $F^+(\mu)$is a fuzzy α-open set, and hence F is an upper fuzzy α-continuous multifunction. The converse is immediate. □

Theorem 13. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, ν). Then F is a lower fuzzy α-continuous if and only if it is lower fuzzy semicontinuous and lower fuzzy precontinuous.*

Proof. Similar to that of Theorem 12 and is omitted. □

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