On the Persistence of the Electromagnetic Field

Márton Gömöri* and László E. Szabó**

*Institute of Philosophy, Research Center for the Humanities, Hungarian Academy of Sciences, Budapest

**Department of Logic, Institute of Philosophy, Eötvös University, Budapest

Abstract

According to the standard realistic interpretation of classical electrodynamics, the electromagnetic field is conceived as a real physical entity existing in space and time. The problem we address in this paper is how to understand this spatiotemporal existence, that is, how to describe the persistence of a field-like physical entity like electromagnetic field.

First, we provide a formal description of the notion of persistence: we derive an "equation of persistence" constituting a necessary condition that the spatiotemporal distributions of the fundamental attributes of a persisting physical entity must satisfy. We then prove a theorem according to which the vast majority of the solutions of Maxwell's equations, describing possible spatiotemporal distributions of the fundamental attributes of the electromagnetic field, violate the equation of persistence. Finally, we discuss the consequences of this result for the ontology of the electromagnetic field.

1 Introduction

There is a long debate in contemporary metaphysics whether and in what sense instantaneous velocity can be regarded as an intrinsic property of an object at a given moment of time (Butterfield 2006; Hawley 2001, pp. 76–80; Sider 2001, pp. 34–35; Arntzenius 2000; Tooley 1988). What is important from this debate to our present concern is—in which there seems to be a consensus—that

> [T]he notion of velocity presupposes the persistence of the object concerned. For average velocity is a quotient, whose numerator must be the distance traversed by the given persisting object [...] So presumably, average velocity's limit, instantaneous velocity, also presupposes persistence. (Butterfield 2005, p. 257).

We will argue in this paper that the opposite is also true: *persistence presupposes velocity*. More precisely, as we will see, in case of a spatially extended physical object, persistence presupposes, at least, the existence of a field of local and instantaneous velocity; regardless if this local instantaneous velocity is considered as an intrinsic property of the object concerned, or not. Velocity occurs as a "kinematic" feature of the way in which the object persists. This is in accordance with our natural intuition: if a material object like a small particle exists, it exists in space and time; if it persists, it must occur somewhere in space at different moments of time. So it must have some (not necessarily non-zero) average velocity characterizing its spatiotemporal existence.

In section 2 we give a formal description of persistence in terms of the spatiotemporal distributions of quantities that track the identity of the persisting object through time. We derive an "equation of persistence" constituting a necessary condition the distributions of tracking quantities must satisfy in every space-time point where the extended object persists. It turns out however that this condition is not necessarily satisfied by some field-like physical entities. In section 3 we discuss the case of electromagnetic field. We show that the equation of persistence is satisfied in some particular states of the electromagnetic field. This is however the exception: we will prove a theorem according to which the vast majority of the solutions of Maxwell's equations violate the equation of persistence. In section 4 we discuss the possible consequences concerning the ontology of electromagnetic field.

2 Formal description of persistence

Physics literature has practically no explicit discussion of the problem of persistence. There must be however an implicit intuition of persistence behind such everyday notions of physics as the velocity of a moving object, the world line or world tube of a material body, the lifetime of a particle, the track of a particle in a cloud chamber, the speed of a neutrino, the deformation of a rod, the cooling of a hot iron ball, etc. If these notions make sense at all – without which there would be no meaningful physical theory – there must be a sense of persistence; some criteria, some facts of the physical world, that entitle the physicist to say that the substance occupying region A at moment t and the one occupying region A^* at t^* constitute the same physical object. Claims of this sort, in which we assert that an object existing at one time is the same object as the one existing at some other time, are called claims of diachronic sameness.

Metaphysicians offer two major interpretations of persistence: endurantism and perdurantism. According to endurantism a concrete particular persists through time by existing wholly and completely at each moment of time of its existence. In other words, persistence through time is construed as the *numerical identity* of a thing existing at one time with a thing existing at another time. As opposed to endurantism, according to perdurantism assertions of diachronic sameness are not assertions of literal identity at all; a concrete particular is constituted by a sequence of *numerically different temporal parts* (phases, stages, or temporal slices). Endurantism is usually combined with presentism or three dimensionalism, while perdurantism with eternalism or four dimensionalism.

Although it is not at all obvious what the final ontology of the world is according to our best physical theories (e.g. Kuhlmann 2015), it is not far from the truth to say that the world view of contemporary physics is closer to perdurantism + four dimensionalism.

Perdurantism + four dimensionalism (also called "worm-view") has to explain how temporal "parts", as independent entities, constitute a four-

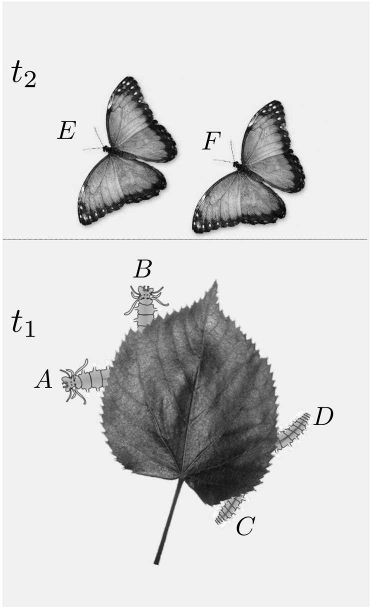

Figure 1: *Which head belongs to which tail? Entity A is obviously not similar to entity D in all respects; as A is a head and D is a tail, for example. But the fact that A belongs to the same caterpillar as D entails that some intrinsic properties of A and D are common; for example, the cells in A and D have the same genome. What is true about the spatial parts is true about the temporal parts. The fact that caterpillar AD is the same being as butterfly F entails that they have the same genome.*

dimensional "whole". In other words: what makes the "temporal parts" parts of one and the same four-dimensional object? The dominant views are based on three components: spatiotemporal continuity, qualitative similarity, and causal relations. The crucial question is how qualitative similarity should be understood. Qualitative identity, the maximal level of qualitative similarity, can be too strong a requirement for diachronic sameness; arbitrary kind and level of qualitative similarity can be too weak. Further restrictions to qualitative similarity might be needed (e.g. Swartz 1991, pp. 337–357).

It is not our intention, however, to give a general criterion *sufficient* for diachronic sameness. For our present purposes we only need to assume that, in every particular case, there is *some* level of qualitative similarity that is *necessary* for diachronic sameness. To make explicit what we mean by the "necessary level of qualitative similarity", we stipulate the following.

Tracking principle *If two things in region A at moment t and in region A^* at t^* are the same physical object, then there is a certain package of tracking properties intrinsic to both things, in region A at time t and in region A^* at time t^*.*

That is to say, diachronic sameness is tracked by sameness in some key properties. This claim quadrates with the wider metaphysical view that qualitative similarity should be understood as "partial identity" (e.g. Heil 2003, p. 156).

Literally speaking, the tracking principle, as a necessary condition for diachronic sameness, is also compatible with endurantism. It simply follows from the Leibniz principle: if two things in two different spatiotemporal regions are numerically identical, then, by the indiscernibility of identicals, they must have *all* properties in common; *a fortiori,* they must share *some* of their properties. (It is not our concern here to discuss the tension between diachronic identity, the Leibniz principle, and change in time.)

First of all, however, the tracking principle can be learned from the practice of physics. For when the physicist asserts that the particle emitted here is the same particle as the one detected over there, what she asserts is the existence of some key properties—the particle's mass, charge, spin, lepton number, etc.—instantiated both at the events of emission and detection. (Consider the analogy depicted in Fig. 1.) When the physicist talks about velocity, world line, lifetime, deformation, etc.—notions that presuppose the persistence of a physical object—, there always exists a package of key intrinsic properties characterizing the object in question in terms of which the physicist can express its diachronic sameness. In fact, from a physics point of view, each particular application of the tracking principle can be conceived as a part of the constitutive *a priori*[1] that provides the *very meaning* of a physical term like velocity, world line, lifetime, deformation, etc. Again, it must be emphasized that the tracking principle, even when conceived as a constitutive principle for physics, is only supposed to specify a *necessary* condition for identity through time, but *not* a sufficient one. Two electrons have the same intrinsic properties, but in many cases the physicist is able to discern them based on other considerations like spatiotemporal continuity.

From metaphysical point of view, the difference between necessity and sufficiency is even more fundamental. Imagine a particle moving along a path. Assume that the particle is annihilated at a moment of time; and, immediately, *another* particle is created with exactly the same intrinsic properties, continuing along the same path. ('Another' means 'numerically distinct' for the endurantist, and 'belonging to a different worm' for the perdurantist.) As this example shows, even if spatiotemporal continuity is provided, and even if the package of tracking properties covers all intrinsic properties (qualitative identity), the tracking condition does not constitute a sufficient metaphysical criterion of diachronic sameness.

Note also that the principle says nothing about the content of the package of tracking properties in question. In particular, it is not necessarily identical with the complete package of properties determining the synchronic identity of the object; neither in the sense of a four-dimensional worm, nor in the sense of its temporal parts in region A at time t or in region A^* at time t^*. Further, the existence of such a package does not imply that the object cannot change in time: it can change in all the various properties not contained in the tracking package. (Consider again the analogy depicted in Fig. 1.)

In what follows we shall translate the condition provided by the tracking principle to mathematical terms. Without loss of generality we may assume that each of the tracking properties in question can be characterized as such that a certain (real valued) quantity f_i takes a certain value; more precisely, the spatiotemporal distribution of this quantity, $f_i(\mathbf{r}, t)$, takes a certain local value at a spatiotemporal locus. Accordingly, we are going to express a necessary

[1] In the sense of Reichenbach 1965.

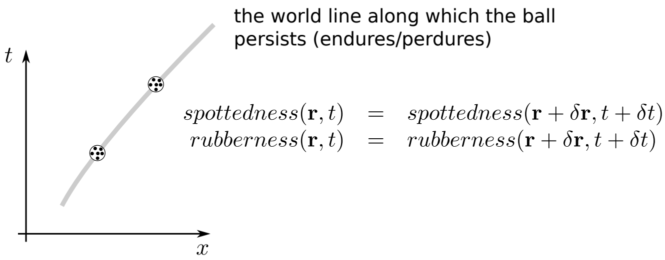

Figure 2: *A small, "point-like" ball can be tracked by its spottednes, rubberness, etc.*

condition for diachronic sameness in terms of these distributions of tracking quantities. We proceed in three heuristic steps.

I.

First we consider the persistence of a point-like physical object.

Let $f_1, f_2, ..., f_n$ be the package of tracking quantities. In line with the tracking principle we assume that

$$f_i(\mathbf{r}, t) = f_i(\mathbf{r}^*, t^*) \qquad (i = 1, 2, \ldots, n) \tag{1}$$

for any two points $(\mathbf{r}, t)$ and $(\mathbf{r}^*, t^*)$ along the world-line of the persisting object (Fig. 2). Introducing the *average velocity* as $\mathbf{v} = \frac{\mathbf{r}^* - \mathbf{r}}{t^* - t}$, we can write:

$$f_i(\mathbf{r}, t) = f_i(\mathbf{r} + \mathbf{v}\delta t, t + \delta t) \qquad (i = 1, 2, \ldots, n) \tag{2}$$

with $\delta t = t^* - t$.

Assume that all functions $f_1, f_2, ..., f_n$ are smooth (if not, they can be approximated as closely as required for physics by smooth functions). Taking (2) for a small, infinitesimal interval of time, and expressing it in a differential form—by which, moreover, we comply with the required spatiotemporal continuity—we have

$$-\partial_t f_i(\mathbf{r}, t) = \nabla f_i(\mathbf{r}, t) \cdot \mathbf{v}(t) \qquad (i = 1, 2, \ldots, n) \tag{3}$$

where $\mathbf{v}(t)$ is the instantaneous velocity. In components:

$$-\partial_t f_i(\mathbf{r}, t) = V_x \partial_x f_i(\mathbf{r}, t) + V_y \partial_y f_i(\mathbf{r}, t) + V_z \partial_z f_i(\mathbf{r}, t) \qquad (i = 1, 2, \ldots, n) \tag{4}$$

Of course, the concrete world-line along which the object persists may be varied. Thus, equations (3) with some instantaneous velocity constitute a *necessary* condition the tracking quantities must satisfy in every space-time point where the object persists. Let us call them the *equations of point-like persistence*.

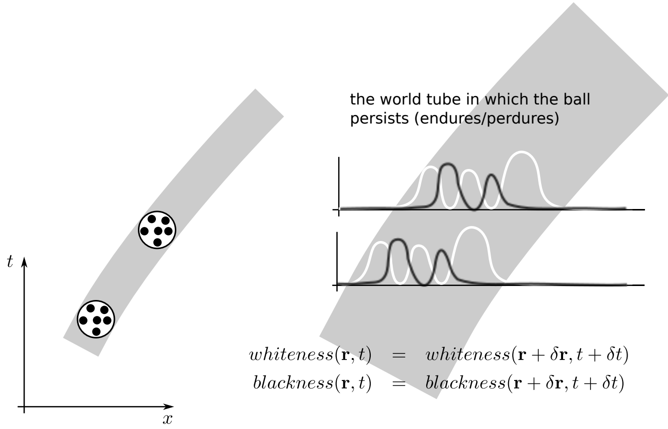

Figure 3: *A spotted ball, as an extended object, can be characterized by the distributions of whiteness and blackness*

II.

Now we make a straightforward extension of the above results to the case of a spatially extended object. Assume that the fine-grained structure of an extended object also can be described in terms of the distributions of some, probably more fundamental, quantities (Fig. 3). And, therefore, the diachronic sameness of the persisting object can be tracked in terms of a suitable package of these distributions, $f_1, f_2, ..., f_n$. It is a straightforward generalization of the idea expressed in equation (3) to say that if an extended object persists then there must exist a velocity vector $\mathbf{v}(t)$ for every moment of time, such that equation (3) is satisfied in all space-time points $(\mathbf{r}, t)$ belonging to the space-time tube swept by the extended object.

However, this describes only a particular situation when the extended object persists like a rigid body in translational motion. The instantaneous velocity $\mathbf{v}(t)$ is the same everywhere in the spatial region occupied by the object. Consequently, the spatial distributions $f_i(\mathbf{r}, t = const)$ are simply translating with a universal velocity, without deformation. Of course, generally this is not necessarily the case. For example, the ball in Fig. 4 preserves its identity even though it rotates and inflates.

III.

Concerning the general case, imagine an extended object with a more complex behavior. Let Σ_t and $\Sigma_{t+\delta t}$ denote the spatial regions occupied by the object at time t and $t + \delta t$. The object can change in various senses. Even if $\Sigma_t = \Sigma_{t+\delta t}$, the spatial distributions of its local properties may change, in the sense that for several distributions $f_i(\mathbf{r}, t) \not\equiv f_i(\mathbf{r}, t + \delta t)$. Moreover, Σ_t and $\Sigma_{t+\delta t}$ may differ not only in their location but also in size and shape. All these changes manifest themselves in the spatiotemporal distributions of local properties, that is, in the distributions $f_i(\mathbf{r}, t)$. For example, all changes, the translation, the rotation, and the inflation of the ball in Fig. 4 are expressible in terms of the distributions

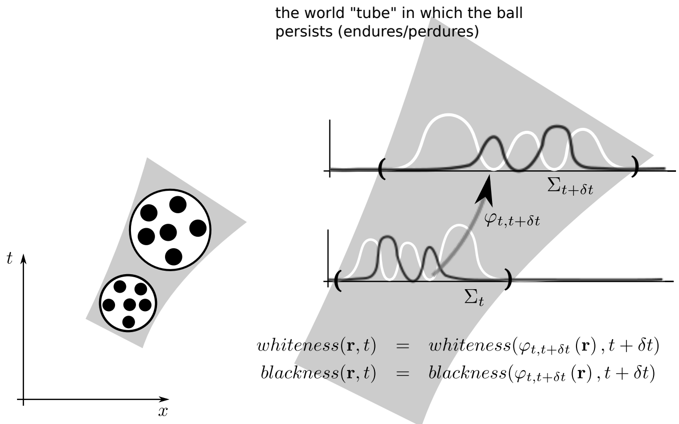

Figure 4: *The ball preserves its identity even though it may rotate or inflate*

like $whiteness(\mathbf{r}, t)$ and $blackness(\mathbf{r}, t)$.

Now, how can we describe the persistence of such an object? What conditions the distributions $f_i(\mathbf{r}, t)$ must satisfy in order to count the two things in Σ_t and $\Sigma_{t+\delta t}$ as identical, or as two temporal parts of the same object? In line with the tracking principle, the conditions we are looking for have to express, in terms of a relation between the values of $f_i(\mathbf{r}, t)$ in Σ_t and the values of $f_i(\mathbf{r}, t+\delta t)$ in $\Sigma_{t+\delta t}$, that Σ_t and $\Sigma_{t+\delta t}$ share some key properties. On the basis of our previous considerations in points **I** or **II** and the examples like the inflating-rotating ball, we claim that the general form of such conditions is the following. There must exist a package of relevant, tracking distributions $f_1, f_2, ..., f_n$ and a mapping $\varphi_{t,t+\delta t} : \Sigma_t \to \Sigma_{t+\delta t}$, such that

$$f_i(\mathbf{r}, t) = f_i(\varphi_{t,t+\delta t}(\mathbf{r}), t+\delta t) \qquad (i = 1, 2, \ldots, n) \tag{5}$$

(5) expresses the metaphysical intuition that the extended object can be conceived as the mereological sum of its "local parts", each of which itself being a persisting entity, whose identity over time is described by function $\varphi_{t,t+\delta t}$. Notice that the only non-trivial requirement imposed on $\varphi_{t,t+\delta t}$ by (5) is that it must be common for all tracking distributions $f_i(\mathbf{r}, t)$. Intuitively this means that if a local part of the object at $\mathbf{r}$ instantiates some local tracking properties then its counterpart at point $\varphi_{t,t+\delta t}(\mathbf{r})$ instantiates the same local tracking properties—in harmony with the tracking principle's requirement. Note however that this fact by no means implies that the extended object necessarily consists of atomic entities—pointlike or non-pointlike—persisting in the sense of points **I** or **II**. Just the contrary, the general notion of persistence defined by (5) satisfies a kind of downward mereological principle: if the whole extended object persists in the sense of (5) then all (arbitrarily small) local parts of the object persist in the same sense.

Assuming that $\varphi_{t,t+\delta t}(\mathbf{r})$ is smooth and $\varphi_{t,t} = id_{\Sigma_t}$—by which we, again, comply with spatiotemporal continuity—, one can express (5) in the following differential form:

$$-\partial_t f_i(\mathbf{r},t) = \nabla f_i(\mathbf{r},t)\cdot \mathbf{v}(\mathbf{r},t) \qquad (i=1,2,\ldots,n) \tag{6}$$

where $\mathbf{v}(\mathbf{r},t) = \partial_{t^*}\varphi_{t,t^*}\Big|_{t^*=t}(\mathbf{r})$. $\mathbf{v}(\mathbf{r},t)$ can be interpreted as the instantaneous velocity field characterizing the motion of the local part of the extended entity at the spatiotemporal locus $(\mathbf{r},t)$.

Taking into account that the concrete mapping $\varphi_{t,t+\delta t} : \Sigma_t \to \Sigma_{t+\delta t}$ may be varied, equations (6) with some suitable instantaneous velocity field $\mathbf{v}(\mathbf{r},t)$ constitute a *necessary* condition the distributions of tracking quantities must satisfy in every space-time point where the extended object persists. Let us call them the *equations of persistence*.[2]

3 Covariance

To get closer to real physical examples, we bring a new perspective. Without serious loss of generality we assume that not only the spatiotemporal quantities, $\mathbf{r}, t$, but all other physical quantities, including the tracking quantities $f_1, f_2, ..., f_n$ are defined relative to a given inertial frame of reference K.[3] Let K' be another inertial frame of reference. All physical quantities $\xi_1, \xi_2, \ldots \xi_m$ in K have a counterpart in K', denoted by $\xi'_1, \xi'_2, \ldots \xi'_m$ respectively. Let Λ denote the transformation law, that is a one-to-one map from the space of quantities $(\xi_1, \xi_2, \ldots \xi_m)$ to the space of quantities $\left(\xi'_1, \xi'_2, \ldots \xi'_m\right)$, interconnecting the physically equivalent points in the two spaces. (Lorentz transformation is a typical example.) That is to say, the values of $\xi_1, \xi_2, \ldots \xi_m$—all together—uniquely determine the values of $\xi'_1, \xi'_2, \ldots \xi'_m$, and vice versa.

Consider a subset of physical quantities $\left\{\xi_{s_1}, \xi_{s_2}, \ldots \xi_{s_k}\right\} \subset \{\xi_1, \xi_2, \ldots \xi_m\}$. We will say that $\left\{\xi_{s_1}, \xi_{s_2}, \ldots \xi_{s_k}\right\}$ is closed against the transformation law, if Λ generates a one-to-one map between the space of $\left(\xi_{s_1}, \xi_{s_2}, \ldots \xi_{s_k}\right)$ and the space of $\left(\xi'_{s_1}, \xi'_{s_2}, \ldots \xi'_{s_k}\right)$; that is, the values of $\xi_{s_1}, \xi_{s_2}, \ldots \xi_{s_k}$ uniquely determine the values of $\xi'_{s_1}, \xi'_{s_2}, \ldots \xi'_{s_k}$, and vice versa.[4]

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[2]In a mathematical sense the equation of persistence (6) is of the same form as a continuity equation without source and conductive current densities,

$$\partial_t f_i(\mathbf{r},t) + \nabla\left(f_i(\mathbf{r},t)\mathbf{v}(\mathbf{r},t)\right) = 0$$

in the particular case when $\nabla \mathbf{v}(\mathbf{r},t) = 0$, that is the velocity field describes an "incompressible" flow. It must be emphasized however that the two equations have different contents. Conceptually, the equation of persistence is about the quantities tracking the object in question. Such a quantity is not necessarily a density-like quantity, in the sense that its volume integral is not necessarily a meaningful physical quantity, especially not a conserved one. Moreover, the equation of persistence and the continuity equation are independent: for a given set of quantities, one equation may hold without the other. The coarse-grained density of the spreading gas we will discuss in section 5 is an example where the continuity equation holds but not the equation of persistence. In contrast, the whiteness and blackness of an inflating spotted ball (Fig. 4) are quantities that satisfy the equation of persistence but not the continuity equation (the velocity field describing an inflating object is not divergence free).

[3]In the present analysis we restrict ourselves to classical (Galileo covariant) and special relativistic physics, and set aside the possible generalization for general relativity.

[4]For example, consider $\left\{t, x, y, z, E_x, E_y, E_z, B_x, B_y, B_z\right\}$ where E_x, E_y, E_z and B_x, B_y, B_z are the

It is a universal principle of contemporary physics that all natural laws are covariant; which means that the physical equations preserve their forms against the transformation Λ.[5] Accordingly, the equation of persistence must be covariant. As the following considerations show, this requirement is automatically satisfied, given that the set of tracking quantities $f_1, f_2, ..., f_n$ is closed against the transformation law.

Indeed, the notion of persistence we arrived at is independent of the choice of the reference frame, and in fact it is fully compatible with the principle of covariance. One way to see this is the following. Let A and B denote those points of spacetime whose coordinates in frame K are $(\mathbf{r}, t)$ and $(\varphi_{t,t+\delta t}(\mathbf{r}), t+\delta t)$, respectively. Equation (5) says that at points A and B each quantity f_i takes the same value. Assume that the transformation of quantities in package $f_1, f_2, ..., f_n$ is closed against the transformation law, that is the values of $f_1, f_2, ..., f_n$ together (taken at a given spacetime point) uniquely determine the values of the corresponding quantities $f_1', f_2', ..., f_n'$ in another inertial frame K' (at the same spacetime point). Due to the fact that the transformation law is a one-to-one map, the f_i'-s will also take the same values in spacetime points A and B, given that the f_i -s do so. This in turn is nothing but saying that equation (5) also holds in inertial frame K'—with a suitable map $\varphi'_{t',t'+\delta t'} : \Sigma'_{t'} \to \Sigma'_{t'+\delta t'}$, where $\Sigma'_{t'}$ and $\Sigma'_{t'+\delta t'}$ are the spatial regions occupied by the object in question at time t' and $t'+\delta t'$ in frame K'.

In order to show another way of demonstrating that the equations of persistence (6) are covariant we focus on the most important particular case: special relativistic physics. Applying the basic notions of Minkowski spacetime, we can rewrite (6) in terms of four-velocity and four-gradient:

$$V_\alpha \partial^\alpha f_i = 0 \qquad (i = 1, 2, \ldots, n) \tag{7}$$

Assume again that the transformation of quantities $f_1, f_2, ..., f_n$ is closed against Lorentz transformation, that is

$$f_i' = \sum_{j=1}^{n} L_{ij} f_j \tag{8}$$

with suitable coefficients L_{ij}. (Here we assume, as it is usually the case, that the Lorentz transformation is linear.) Then, for the primed expression $V'_\alpha \partial'^\alpha f_i'$ one receives

$$V'_\alpha \partial'^\alpha f_i' = V'_\alpha \partial'^\alpha \left(\sum_{j=1}^{n} L_{ij} f_j \right) = \sum_{j=1}^{n} L_{ij} \left(V_\alpha \partial^\alpha f_j \right) = 0 \qquad (i = 1, 2, \ldots, n) \tag{9}$$

where we used the invariance of the Minkowksi scalar product and (7). This means that the equations of persistence also hold in frame K', given that they

electric and magnetic field strengths in K. Now, for example, $\{E_x, E_y, E_z\}$ is not closed against the Lorentz transformation, while subset $\{E_x, E_y, E_z, B_x, B_y, B_z\}$ is closed; the values of $E_x, E_y, E_z, B_x, B_y, B_z$ in K uniquely determine the values of $E_x', E_y', E_z', B_x', B_y', B_z'$. Similarly, $\{t, x, y, z\}$ is a closed subset, while $\{x, y, z\}$ is not.

[5] For a more precise formulation see Gömöri–Szabó 2015.

do so in K; that is, they are covariant. Further, assuming that the transformations of the f_i-s are such that they can be identified with components of suitable four-tensors, one can cast the equations of persistence in a manifestly covariant form. For example, the equations of persistence (24)–(25) written down for the electromagnetic field in the next section have the covariant form

$$V_\alpha \partial^\alpha F^{\beta\gamma} = 0 \qquad (\beta, \gamma = 0,1,2,3) \tag{10}$$

where $F^{\beta\gamma}$ is the Faraday tensor.

Thus, the upshot of the above analysis is that persistence admits a covariant formulation iff *the tracking quantities $f_1, f_2, ..., f_n$ constitute a closed set against the transformation laws.* With this result in mind in what follows we shall continue to use the 3+1 notation.

4 The Case of a General Electrodynamic System

As a concrete physical example, we will deal with electromagnetic field. According to the standard realistic interpretation of classical electrodynamics, "the interaction between charged particles are mediated by the electromagnetic field, which is ontologically on a par with charged particles and the state of which is given by the values of the field strengths" (Frisch 2005, p. 28). Thus, electromagnetic field is conceived as a real extended physical substance possessing two intrinsic physical properties, the field strengths **E** and **B**, in every space point at every moment of time.

We will consider the electromagnetic field as a part of a coupled system of charged particles and the field; described by the Maxwell–Lorentz equations (for this form of the equations, see for example Gömöri and Szabó 2013):

$$\nabla \cdot \mathbf{E}(\mathbf{r},t) = \sum_{i=1}^{n} q^i \delta\left(\mathbf{r} - \mathbf{r}^i(t)\right) \tag{11}$$

$$c^2 \nabla \times \mathbf{B}(\mathbf{r},t) - \partial_t \mathbf{E}(\mathbf{r},t) = \sum_{i=1}^{n} q^i \delta\left(\mathbf{r} - \mathbf{r}^i(t)\right) \mathbf{v}^i(t) \tag{12}$$

$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0 \tag{13}$$

$$\nabla \times \mathbf{E}(\mathbf{r},t) + \partial_t \mathbf{B}(\mathbf{r},t) = 0 \tag{14}$$

$$m^i \gamma\left(\mathbf{v}^i(t)\right) \mathbf{a}^i(t) = q^i \Big\{ \mathbf{E}\left(\mathbf{r}^i(t), t\right) + \mathbf{v}^i(t) \times \mathbf{B}\left(\mathbf{r}^i(t), t\right) - c^{-2}\mathbf{v}^i(t)\left(\mathbf{v}^i(t) \cdot \mathbf{E}\left(\mathbf{r}^i(t), t\right)\right)\Big\} \qquad (i = 1, 2, \ldots, n) \tag{15}$$

where, $\gamma(\ldots) = \left(1 - \frac{(\ldots)^2}{c^2}\right)^{-\frac{1}{2}}$, q^i is the electric charge and m^i is the rest mass of the i-th particle.

If electromagnetic field is conceived as a persisting physical entity, some of its intrinsic properties, as tracking properties, have to satisfy the equation of persistence. Now, **E** and **B** are fundamental, in the sense that all other known physical properties of the field (energy, momentum, etc.) supervene on **E** and

B. Moreover, **E** and **B** together constitute a closed set of quantities against the Lorentz transformation. So, the natural expectation is that the six field strength components $E_x, E_y, E_z, B_x, B_y, B_z$ constitute a package of tracking quantities; and, therefore, they satisfy equation (6), in addition to the Maxwell–Lorentz equations (11)–(15).

Let us first investigate an example where this works well: the static and uniformly moving 'charged particle + the coupled electromagnetic field' system. Consider the static solution when the charge q is at *rest* at point (x_0, y_0, z_0) in a given inertial frame of reference K:

$$\begin{aligned} E_x(t,x,y,z) &= \frac{q\,(x-x_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_y(t,x,y,z) &= \frac{q\,(y-y_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_z(t,x,y,z) &= \frac{q\,(z-z_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ B_x(t,x,y,z) &= 0 \\ B_y(t,x,y,z) &= 0 \\ B_z(t,x,y,z) &= 0 \end{aligned} \tag{16}$$

The stationary field of a charge q moving at constant velocity $\mathbf{V} = (V, 0, 0)$ relative to K can be obtained (Jackson 1999, pp. 661–665) by solving the equations of electrodynamics with the time-depending source. The solution is the following:

$$\begin{aligned} E_x(t,x,y,z) &= \frac{qX_0}{\left(X_0^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_y(t,x,y,z) &= \frac{\gamma q\,(y-y_0)}{\left(X_0^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_z(t,x,y,z) &= \frac{\gamma q\,(z-z_0)}{\left(X_0^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ B_x(t,x,y,z) &= 0 \\ B_y(t,x,y,z) &= -c^{-2}VE_z(t,x,y,z) \\ B_z(t,x,y,z) &= c^{-2}VE_y(t,x,y,z) \end{aligned} \tag{17}$$

where (x_0, y_0, z_0) is the initial position of the particle at $t = 0$, $X_0 = \gamma\,(x-(x_0+Vt))$ and $\gamma = \left(1-\frac{V^2}{c^2}\right)^{-\frac{1}{2}}$.

Now, it is easy to verify that both the static solution (16) and the stationary solution (17) satisfy the equations of persistence (6) with constant and homogeneous velocity field $\mathbf{V} = (0,0,0)$ and $\mathbf{V} = (V,0,0)$,[6] respectively, in the

[6]It must be pointed out that velocity **V** conceptually differs from the speed of light c. Basically, c

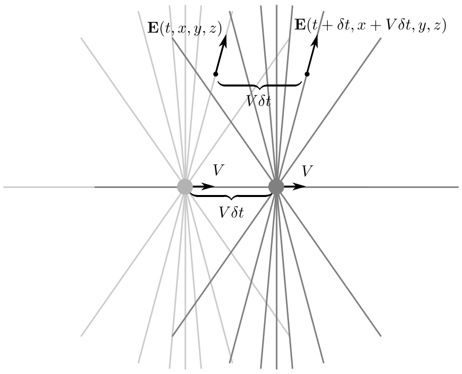

Figure 5: *The stationary field of a uniformly moving point charge is in collective motion together with the point charge*

following sense:[7]

$$-\partial_t \mathbf{E}(\mathbf{r}, t) = \mathsf{D}\mathbf{E}(\mathbf{r}, t)\mathbf{V} \tag{18}$$

$$-\partial_t \mathbf{B}(\mathbf{r}, t) = \mathsf{D}\mathbf{B}(\mathbf{r}, t)\mathbf{V} \tag{19}$$

Or, in the form of (5),

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r} - \mathbf{V}\delta t, t - \delta t) \tag{20}$$

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}(\mathbf{r} - \mathbf{V}\delta t, t - \delta t) \tag{21}$$

Thus, in this particular example the necessary conditions of the persistence of the electromagnetic field are clearly satisfied.

But, this example obviously represents a special electrodynamic configura-

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is a constant of nature in the Maxwell–Lorentz equations, which can emerge in the solutions of the equations; and, in some cases, it can be interpreted as the velocity of propagation of changes in the electromagnetic field. For example, in our case, the stationary field of a uniformly moving point charge, in collective motion with velocity $\mathbf{V}$, can be constructed from the superposition of retarded potentials, in which the retardation is calculated with velocity c. Nevertheless, the two velocities are different concepts. To illustrate the difference, consider the fields of a charge at rest (16), and in motion (17). The speed of light c plays the same role in both cases. Both fields can be constructed from the superposition of retarded potentials in which the retardation is calculated with velocity c. Also, in both cases, a small local perturbation in the field configuration would propagate with velocity c. But still there is a consensus to say that the system described by (16) is at rest while the one described by (17) is moving with velocity $\mathbf{V}$ (relative to K.) A good analogy would be a Lorentz contracted moving rod: $\mathbf{V}$ is the velocity of the rod, which differs from the speed of sound in the rod.

[7] In $\mathsf{D}\mathbf{E}(\mathbf{r}, t)$ and $\mathsf{D}\mathbf{B}(\mathbf{r}, t)$, D denotes the spatial derivative operator (Jacobian for variables x, y and z). That is, in components we have:

$$\begin{aligned}
-\partial_t E_x(\mathbf{r}, t) &= V_x \partial_x E_x(\mathbf{r}, t) + V_y \partial_y E_x(\mathbf{r}, t) + V_z \partial_z E_x(\mathbf{r}, t) \\
-\partial_t E_y(\mathbf{r}, t) &= V_x \partial_x E_y(\mathbf{r}, t) + V_y \partial_y E_y(\mathbf{r}, t) + V_z \partial_z E_y(\mathbf{r}, t) \\
&\vdots \\
-\partial_t B_z(\mathbf{r}, t) &= V_x \partial_x B_z(\mathbf{r}, t) + V_y \partial_y B_z(\mathbf{r}, t) + V_z \partial_z B_z(\mathbf{r}, t) \\
-\partial_t \varrho(\mathbf{r}, t) &= V_x \partial_x \varrho(\mathbf{r}, t) + V_y \partial_y \varrho(\mathbf{r}, t) + V_z \partial_z \varrho(\mathbf{r}, t)
\end{aligned}$$

tion. Indeed, equations (18)–(19) imply that

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r}-\mathbf{V}t) \quad (22)$$
$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0(\mathbf{r}-\mathbf{V}t) \quad (23)$$

with some time-independent $\mathbf{E}_0(\mathbf{r})$ and $\mathbf{B}_0(\mathbf{r})$. In other words, the field must be a stationary one, that is, a translation of a static field with velocity $\mathbf{V}$. In fact, this corresponds to the very special "rigid" way of persistence we described in point **II** of the previous section. But, (22)–(23) is certainly not the case for a general solution of the equations of classical electrodynamics. The behavior of the field can be much more complex. Whatever this complex behavior is, one might hope that it satisfies the general form of persistence described in point **III**; that is, the equations of persistence are satisfied with a more general local and instantaneous velocity field $\mathbf{v}(\mathbf{r},t)$:

$$-\partial_t\mathbf{E}(\mathbf{r},t) = \mathsf{D}\mathbf{E}(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) \quad (24)$$
$$-\partial_t\mathbf{B}(\mathbf{r},t) = \mathsf{D}\mathbf{B}(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) \quad (25)$$

In other words, if electromagnetic field is a real persisting physical entity, existing in space and time, then for all possible solutions of the Maxwell–Lorentz equations (11)–(15) there must exist, at least, a local instantaneous velocity field $\mathbf{v}(\mathbf{r},t)$ satisfying (24)–(25). That is, substituting an arbitrary solution[8] of (11)–(15) into (24)–(25), the overdetermined system of equations must have a solution for $\mathbf{v}(\mathbf{r},t)$.

One encounters however the following difficulty:

Theorem 1. *There exists a solution of the coupled Maxwell–Lorentz equations (11)–(15) for which there cannot exist a local instantaneous velocity field* $\mathbf{v}(\mathbf{r},t)$ *satisfying the persistence equations (24)–(25).*

Proof. As a proof, we give a surprisingly simple example. Consider the electric field in a parallel-plate capacitor being charged up by a constant current. The electric field strength is:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 t \quad (26)$$

where $\mathbf{E}_0$ is a constant vector determined by the current and the properties of the capacitor (Fig. 6). It is easy to check that there is no space-time point $(\mathbf{r},t)$ where $\mathbf{E}(\mathbf{r},t)$ would satisfy the equation of persistence (24) with some velocity $\mathbf{v}(\mathbf{r},t)$. □

One might think that this is an exceptional case, due to the idealization of the real physical situation. But, as the next theorem shows, this is not so exceptional.

[8]Without entering into the details, it must be noted that the Maxwell–Lorentz equations (11)–(15), exactly in this form, have *no* solution. The reason is that the field is singular at precisely the points where the coupling happens: on the trajectories of the particles. The generally accepted answer to this problem is that the real source densities are some "smoothed out" Dirac deltas, determined by the physical laws of the internal worlds of the particles—which are, supposedly, outside of the scope of classical electrodynamics. With this explanation, for the sake of simplicity we leave the Dirac deltas in the equations. Since our considerations here focus on the electromagnetic field, satisfying the four Maxwell equations, we only have to assume that there is a coupled dynamics—approximately described by equations (11)–(15)—and that it constitutes an initial value problem. In fact, Theorem 2 could be stated in a weaker form, by leaving the concrete form and dynamics of the source densities unspecified.

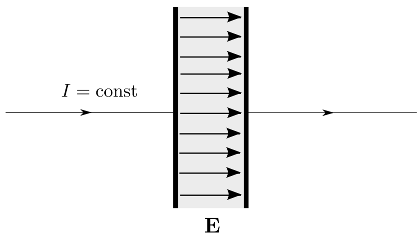

Figure 6: *Linearly increasing electric field strengths in a parallel-plate capacitor charged up by a constant current*

Theorem 2. *There is a dense subset of solutions of the coupled Maxwell–Lorentz equations (11)–(15) for which there cannot exist a local instantaneous velocity field* $\mathbf{v}(\mathbf{r},t)$ *satisfying the persistence equations (24)–(25).*

Proof. The proof is almost trivial for a locus $(\mathbf{r},t)$ where there is a charged point particle. However, in order to avoid the eventual difficulties concerning the physical interpretation, we are providing a proof for a point $(\mathbf{r}_*,t_*)$ where there is assumed no source at all.

Consider a solution $\left(\mathbf{r}^1(t),\mathbf{r}^2(t),\ldots,\mathbf{r}^n(t),\mathbf{E}(\mathbf{r},t),\mathbf{B}(\mathbf{r},t)\right)$ of the coupled Maxwell–Lorentz equations (11)–(15), which satisfies (24)–(25). At point $(\mathbf{r}_*,t_*)$, the following equations hold:

$$\begin{aligned}
-\partial_t\mathbf{E}(\mathbf{r}_*,t_*) &= \mathrm{D}\mathbf{E}(\mathbf{r}_*,t_*)\mathbf{v}(\mathbf{r}_*,t_*) && (27)\\
-\partial_t\mathbf{B}(\mathbf{r}_*,t_*) &= \mathrm{D}\mathbf{B}(\mathbf{r}_*,t_*)\mathbf{v}(\mathbf{r}_*,t_*) && (28)\\
\partial_t\mathbf{E}(\mathbf{r}_*,t_*) &= c^2\nabla\times\mathbf{B}(\mathbf{r}_*,t_*) && (29)\\
-\partial_t\mathbf{B}(\mathbf{r}_*,t_*) &= \nabla\times\mathbf{E}(\mathbf{r}_*,t_*) && (30)\\
\nabla\cdot\mathbf{E}(\mathbf{r}_*,t_*) &= 0 && (31)\\
\nabla\cdot\mathbf{B}(\mathbf{r}_*,t_*) &= 0 && (32)
\end{aligned}$$

Without loss of generality we can assume—at point $\mathbf{r}_*$ and time t_*—that operators $\mathrm{D}\mathbf{E}(\mathbf{r}_*,t_*)$ and $\mathrm{D}\mathbf{B}(\mathbf{r}_*,t_*)$ are invertible and $v_z(\mathbf{r}_*,t_*)\neq 0$.

Now, consider a 3×3 matrix J such that

$$J=\begin{pmatrix}\partial_xE_x(\mathbf{r}_*,t_*) & J_{xy} & J_{xz}\\ \partial_xE_y(\mathbf{r}_*,t_*) & \partial_yE_y(\mathbf{r}_*,t_*) & \partial_zE_y(\mathbf{r}_*,t_*)\\ \partial_xE_z(\mathbf{r}_*,t_*) & \partial_yE_z(\mathbf{r}_*,t_*) & \partial_zE_z(\mathbf{r}_*,t_*)\end{pmatrix} \tag{33}$$

with

$$\begin{aligned}
J_{xy} &= \partial_yE_x(\mathbf{r}_*,t_*)+\lambda && (34)\\
J_{xz} &= \partial_zE_x(\mathbf{r}_*,t_*)-\lambda\frac{v_y(\mathbf{r}_*,t_*)}{v_z(\mathbf{r}_*,t_*)} && (35)
\end{aligned}$$

by virtue of which

$$\begin{aligned}
J_{xy}v_y(\mathbf{r}_*,t_*)+J_{xz}v_z(\mathbf{r}_*,t_*) &= v_y(\mathbf{r}_*,t_*)\partial_yE_x(\mathbf{r}_*,t_*)\\
&\quad +v_z(\mathbf{r}_*,t_*)\partial_zE_x(\mathbf{r}_*,t_*) && (36)
\end{aligned}$$

Therefore, $J\mathbf{v}(\mathbf{r}_*, t_*) = \mathrm{D}\mathbf{E}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*)$. There always exists a vector field $\mathbf{E}^{\#}_{\lambda}(\mathbf{r})$ such that its Jacobian matrix at point $\mathbf{r}_*$ is equal to J. Obviously, from (31) and (33), $\nabla \cdot \mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*) = 0$. Therefore, there exists a solution of the Maxwell–Lorentz equations, such that the electric and magnetic fields $\mathbf{E}_{\lambda}(\mathbf{r}, t)$ and $\mathbf{B}_{\lambda}(\mathbf{r}, t)$ satisfy the following conditions:[9]

$$\mathbf{E}_{\lambda}(\mathbf{r}, t_*) = \mathbf{E}^{\#}_{\lambda}(\mathbf{r}) \tag{37}$$

$$\mathbf{B}_{\lambda}(\mathbf{r}, t_*) = \mathbf{B}(\mathbf{r}, t_*) \tag{38}$$

At $(\mathbf{r}_*, t_*)$, such a solution obviously satisfies the following equations:

$$\partial_t \mathbf{E}_{\lambda}(\mathbf{r}_*, t_*) = c^2 \nabla \times \mathbf{B}(\mathbf{r}_*, t_*) \tag{39}$$

$$-\partial_t \mathbf{B}_{\lambda}(\mathbf{r}_*, t_*) = \nabla \times \mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*) \tag{40}$$

therefore

$$\partial_t \mathbf{E}_{\lambda}(\mathbf{r}_*, t_*) = \partial_t \mathbf{E}(\mathbf{r}_*, t_*) \tag{41}$$

As a little reflection shows, if $\mathrm{D}\mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*)$, that is J, happened to be not invertible, then one can choose a *smaller* λ such that $\mathrm{D}\mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*)$ becomes invertible (due to the fact that $\mathrm{D}\mathbf{E}(\mathbf{r}_*, t_*)$ is invertible), and, at the same time,

$$\nabla \times \mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*) \neq \nabla \times \mathbf{E}(\mathbf{r}_*, t_*) \tag{42}$$

Consequently, from (41) , (35) and (27) we have

$$-\partial_t \mathbf{E}_{\lambda}(\mathbf{r}_*, t_*) = \mathrm{D}\mathbf{E}_{\lambda}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) = \mathrm{D}\mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*)\mathbf{v}(\mathbf{r}_*, t_*) \tag{43}$$

and $\mathbf{v}(\mathbf{r}_*, t_*)$ is uniquely determined by this equation. On the other hand, from (40) and (42) we have

$$-\partial_t \mathbf{B}_{\lambda}(\mathbf{r}_*, t_*) \neq \mathrm{D}\mathbf{B}_{\lambda}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) = \mathrm{D}\mathbf{B}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) \tag{44}$$

because $\mathrm{D}\mathbf{B}(\mathbf{r}_*, t_*)$ is invertible, too. That is, for $\mathbf{E}_{\lambda}(\mathbf{r}, t)$ and $\mathbf{B}_{\lambda}(\mathbf{r}, t)$ there is no local and instantaneous velocity at point $\mathbf{r}_*$ and time t_*.

At the same time, λ can be arbitrary small, and

$$\lim_{\lambda \to 0} \mathbf{E}_{\lambda}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \tag{45}$$

$$\lim_{\lambda \to 0} \mathbf{B}_{\lambda}(\mathbf{r}, t) = \mathbf{B}(\mathbf{r}, t) \tag{46}$$

Therefore solution $\left(\mathbf{r}^1_{\lambda}(t), \mathbf{r}^2_{\lambda}(t), \ldots, \mathbf{r}^n_{\lambda}(t), \mathbf{E}_{\lambda}(\mathbf{r}, t), \mathbf{B}_{\lambda}(\mathbf{r}, t)\right)$ can fall into an arbitrary small neighborhood of $\left(\mathbf{r}^1(t), \mathbf{r}^2(t), \ldots, \mathbf{r}^n(t), \mathbf{E}(\mathbf{r}, t), \mathbf{B}(\mathbf{r}, t)\right)$.[10] □

[9] $\mathbf{E}^{\#}_{\lambda}(\mathbf{r})$ and $\mathbf{B}_{\lambda}(\mathbf{r}, t_*)$ can be regarded as the initial configurations at time t_*; we do not need to specify a particular choice of initial values for the sources.

[10] Notice that our investigation has been concerned with the general laws of Maxwell–Lorentz electrodynamics of a coupled particles + electromagnetic field system. The proof was essentially based on the presumption that all solutions of the Maxwell–Lorentz equations, determined by *any* initial state of the particles + electromagnetic field system, corresponded to physically possible configurations of the electromagnetic field. It is sometimes claimed, however, that the solutions must be restricted by the so called retardation condition, according to which all physically admissible field configurations must be generated from the retarded potentials belonging to some pre-histories of the charged particles (Jánossy 1971, p. 171; Frisch 2005, p. 145). There is no obvious answer to the question of how Theorem 2 is altered under such additional condition.

5 Ontology of Classical Electrodynamics

The consequence of this result is embarrassing: the two *fundamental* electrodynamic quantities, the field strengths $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$, do not satisfy the equations of persistence (6). Therefore, the electromagnetic field tracked by the field strengths cannot be regarded as a persisting physical object; in other words, electromagnetic field—for example, the field within the capacitor in Fig. 6—cannot be regarded as being a real physical entity existing in space and time. This seems to contradict the usual realistic interpretation of classical electrodynamics.

So, there are three options.

(i) One can abandon the realist understanding of electrodynamics: There is no such a persisting physical entity as "electromagnetic field".

(ii) Although, we think, in point **III** we formulated the most general form of how an extended physical object can persist, one may try to imagine a more sophisticated way of persistence.[11]

(iii) Electromagnetic field is a real physical entity, persisting in the sense we formulated persistence in point **III**, but it cannot be tracked by the field strengths $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$. That is, there must exist some quantities other than the field strengths, perhaps outside of the scope of classical electrodynamics, tracking the electromagnetic field. This suggests that classical electrodynamics is an ontologically incomplete theory.

How to conceive properties, different from the field strengths, which are capable of tracking the electromagnetic field? One might think of them as some "finer", more fundamental, properties of the field, not only tracking it as a persisting extended object, but also determining the values of the field strengths. However, the following easily verifiable theorem shows that this determination cannot be so simple:

[11] In point **III** the equations of persistence were based on the metaphysical intuition that an extended object can be conceived as the mereological sum of its local parts, each of which itself being a persisting entity. One might object that in case of the electromagnetic field this intuition is not justified: the electromagnetic field should rather be seen as one single indivisible entity spreading over the whole of space, whose persistence simply means that the field, as a whole, is present at all instants of time. This fact then might be translated as the condition that the field strengths take *some* values in all spatiotemporal regions, which is clearly respected by all solutions of the Maxwell–Lorentz equations.

We believe nonetheless that this is not the way we usually think about the electromagnetic field, and in fact one has good physical grounds to talk about the local parts of the field as entities themselves. We make three observations: 1) In electrodynamics we attribute properties to the local parts of the electromagnetic field—the parts of the field occupying certain spatial regions—that we attribute to *entities* in other cases. Such properties, for example, are energy and momentum. 2) Part of the reason why one believes that the electromagnetic field is a real physical entity is that it makes manifest the idea of *local* action—that of the continuous propagation of physical actions in space and time. The idea of local action makes no sense unless there exists a local entity, the local part of the field, that mediates the physical action. 3) Another aspect of locality in electrodynamics is that the state of the electromagnetic field given on a segment of Cauchy surface determines the state of the field in the future dependence domain of the surface in question. Clearly, this idea requires that we must be able to assign local states of the electromagnetic field to spatiotemporal regions (to the surface and domain of dependence in question). Such an assignment only makes sense if there exists something, a local entity, that is capable of being in those local states.

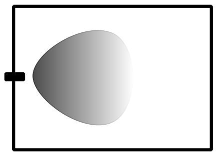

Figure 7: *A puff of gas is sprayed into an empty room through a little pipe*

Theorem 3. *Let $f_1, f_2, ..., f_n$ be a package of quantities for which there exists a local instantaneous velocity field $\mathbf{v}(\mathbf{r}, t)$ satisfying the equations of persistence* (6) *in a given space-time region. If a quantity Φ is a functional of the quantities $f_1, f_2, \ldots, f_n$ in the following form:*

$$\Phi(\mathbf{r}, t) = \Phi\left(f_1(\mathbf{r}, t), f_2(\mathbf{r}, t), ..., f_n(\mathbf{r}, t)\right)$$

then Φ also obeys the equation of persistence

$$-\partial_t \Phi(\mathbf{r}, t) \quad = \quad \nabla \Phi(\mathbf{r}, t) \mathbf{v}(\mathbf{r}, t)$$

with the same local instantaneous velocity field $\mathbf{v}(\mathbf{r}, t)$, within the same space-time region.

Therefore, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ cannot supervene pointwise upon some more fundamental tracking quantities satisfying the persistence equations. However, they might supervene in some non-local sense. For example, imagine that $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ provide only a course-grained characterization of the field, but there exist some more fundamental fields $\tilde{\mathbf{E}}(\mathbf{r}, t)$ and $\tilde{\mathbf{B}}(\mathbf{r}, t)$, such that

$$\mathbf{E}(\mathbf{r}, t) = \int\limits_{\Omega_{\mathbf{r}}} \tilde{\mathbf{E}}\left(\hat{\mathbf{r}}, t\right) d^3(\hat{\mathbf{r}}) \tag{47}$$

$$\mathbf{B}(\mathbf{r}, t) = \int\limits_{\Omega_{\mathbf{r}}} \tilde{\mathbf{B}}\left(\hat{\mathbf{r}}, t\right) d^3(\hat{\mathbf{r}}) \tag{48}$$

where $\Omega_{\mathbf{r}}$ is a neighborhood of $\mathbf{r}$. In this case, the more fundamental quantities $\tilde{\mathbf{E}}(\mathbf{r}, t)$ and $\tilde{\mathbf{B}}(\mathbf{r}, t)$ may satisfy the equations of persistence, while $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$, supervening on $\tilde{\mathbf{E}}(\mathbf{r}, t)$ and $\tilde{\mathbf{B}}(\mathbf{r}, t)$, may not.

It is worthwhile to mention that one has very similar situation in the case of continuum mechanics. Consider the following simple example. A puff of gas is sprayed into an empty room through a little pipe (Fig. 7). As the gas is spreading, the density of the gas $\varrho(x, t)$ is continuously decreasing in every point of the instantaneous region occupied by the gas (Fig. 8). Consequently, $\varrho(x, t)$ does not satisfy the equation of persistence. This means that the density distribution, which is one of the basic quantities of the continuum mechanical description of the gas, cannot be in the package of intrinsic properties tracking the gas.

In contrast, assuming that the gas consists of a huge number of small rigid particles, the *fine-grained* density distribution $\tilde{\varrho}(x, t)$ looks like as depicted in Fig. 8 and satisfies the equation of persistence (6) with a suitable local and instantaneous velocity field, the value of which at every point in a region occupied by a particle is equal to the instantaneous velocity of the particle concerned. The course-grained density supervenes on the fine-grained density;

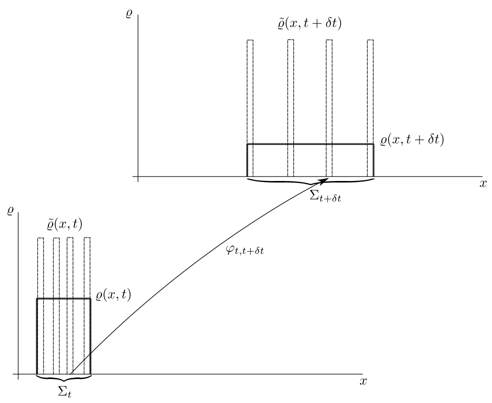

Figure 8: *The density of the gas $\varrho(x, t)$ is continuously decreasing in every point of the region Σ_t occupied by the gas. Consequently, $\varrho(x, t)$ does not satisfy the equation of persistence. In contrast, the fine-grained density distribution $\tilde{\varrho}(x, t)$, reflecting the molecular structure of the gas, does satisfy the equation of persistence*

not pointwise, but in the style of (47)–(48):

$$\varrho(\mathbf{r}, t) = \frac{1}{\Omega} \int\limits_{\Omega_{\mathbf{r}}} \tilde{\varrho}(\hat{\mathbf{r}}, t) d^3(\hat{\mathbf{r}})$$

where $\Omega_{\mathbf{r}}$ denotes a sphere of volume Ω with center $\mathbf{r}$, large enough relative to the fine-grained structure, but small enough to have a meaningful smooth approximation.

It is worth noting that while continuum mechanics alone is thus incapable of accounting for the persistence of the gas, the continuum mechanical description *itself* also tacitly assumes that the gas constitutes a persisting entity. The reason is that the continuum mechanical description refers to a velocity field, as one of its fundamental quantities. For example, the coarse-grained density $\varrho(x, t)$ of the spreading gas obeys a continuity equation

$$\partial_t \varrho(\mathbf{r}, t) + \nabla \left(\varrho(\mathbf{r}, t) \mathbf{v}(\mathbf{r}, t) \right) = 0$$

expressing that the quantity of the gas remains constant upon spreading out. Here $\varrho(\mathbf{r}, t)\mathbf{v}(\mathbf{r}, t)$ is the convection current density attached to the local motion of the gas at space-time point $(\mathbf{r}, t)$. Now *whose* velocity is $\mathbf{v}(\mathbf{r}, t)$? One might think that within continuum mechanics $\mathbf{v}(\mathbf{r}, t)$ can be interpreted as the velocity of the local part of a persisting continuum located at $\mathbf{r}$ at time t. However, continuum mechanics itself fails to support such an interpretation: the coarse-grained quantities featuring the continuum mechanical description, among them $\varrho(x, t)$, fail to satisfy the equations of persistence, as the example of the spreading gas demonstrates, and hence $\mathbf{v}(\mathbf{r}, t)$ is not definable in terms of the properties of the continuum described by continuum mechanics. It is thus no surprise that this is not the way the fundamental field $\mathbf{v}(\mathbf{r}, t)$ is finally explicated (cf. Truesdell and Toupin p. 227 vs. p. 327). Instead, $\mathbf{v}(\mathbf{r}, t)$ is explicated by going beyond the domain of the continuum mechanical description, in terms of the molecular structure of the gas: $\mathbf{v}(\mathbf{r}, t)$ is defined on the basis of the velocities of the particles of the gas, located around $\mathbf{r}$ at time t, by means of some averaging procedures (see Murdoch 2012, Section 3.6).

The upshot of all this is that the continuum mechanical description of the gas in terms of the course-grained quantities is ontologically incomplete. This incomplete description can be completed by appealing to the fine-grained structure of the gas (cf. Murdoch 2012, Chapter 3; Batterman 2006). The perplexing question is: what could be a similar fine-grained structure of a classical electromagnetic field?

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